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Micro-resonator loss computation using conformal transformation and active-lasing FDTD approach and applications to tangential/radial output waveguide optimization I: Analytical approach

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ABSTRACT

Understanding the physics of the loss mechanism of optical microresonators is important for one to know how to use them optimally for various applications. In these three-paper series, we utilized both analytical method (Conformal Transformation Approach) and numerical method (Active-Lasing Finite-Difference Time-Domain method) to study the resonator loss and cavity quality "Q" factor and apply them to optimize the radial/tangential waveguide coupling design. Both approaches demonstrate good agreement in their common region of applicability. In Part I, we review and expand on the conformal transformation method to show how exact solution of radiation loss for the case of cylindrical micro-resonator under both TE and TM polarizations can be obtained. We show how the method can be extended to apply to microdisk case.

1. Motivation of paper series

Microdisk [1–24], microring [25–28], and microcylinder [29–60] optical resonators have become a widely used technology to achieve highly compact monolithically integratable high-Q optical cavities on chip. We refer to them collectively as optical microresonators, which can include circular shapes as well as elliptical, race-track, microgear, or irregular curvilinear shapes [61–75].

The propagation of the optical energy in these resonators may be based on whispering gallery [1–24,52–60], channel waveguiding [25–51], or chaotic modes [34,76–81]. These optical micro-resonators can be conveniently integrated on planar photonic integrated circuits and are capable of achieving very narrow resonance spectral widths. As a result, they can be used as highly compact tunable optical filters or optical modulators integrated on chip. They can also be used to form microscopic wavelength-scale optical cavities for realizing microcavity lasers capable of single-frequency lasing with high spectral purity, low lasing thresholds, and ultrafast modulation.

Microcylinder based pulsed lasers and polymer based micro-cavity lasers have also been realized. Beside lasers and filters, there are other applications, including modulators, sensors and all-optical switch, wavelength conversion, logic operation, all-optical memory, and optical delay line. A systematic study of the limitations of these microresonators is important for one to know how to use these optical microresonators optimally for various practical applications.

In these three-paper series, we will systematically study the physics of the loss mechanism of these microresonators via either analytical method based on conformal transformation or numerical method based on numerical solution to Maxwell equations via the Finite Difference Time Domain (FDTD) approach. The different approaches will enable different physical understandings of the loss mechanisms for various applications.

The conformal transformation based approach is useful for understanding the radiation loss but is restricted only to circular geometry and cannot take care of scattering loss. However, it is an exact solution and can be computed very fast. The FDTD based approach is more widely applicable to different geometries that can be non-circular and can also take care of scattering loss. We show that these two approaches agree with each other in the regimes where they are both applicable.

In Paper I we will focus on the conformal transformation approach. While the conformal transformation approach has been introduced before [82,83], it has not been fully formulated or utilized to show its capability to compute the exact cavity Q factors of TE and TM modes in microcylinder resonators, which it is capable of. Hence, it has not been fully validated and compared with the numerical approach. Here, we point out that the conformal transformation approach is exact for the case of cylindrical geometry and show that it can take care of both the

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TE and TM polarizations. We show that the conformal transformation is also capable of computing the radiation loss for microdisk geometry with some minor modifications to take care of wave diffraction outside the disk.

In Paper II we will focus on an "Active-Lasing" FDTD numerical approach. While there were previous computations of radiation losses using FDTD method, the method introduced in Paper II is different from the previous approaches as it directly detects the energy radiated out and the energy stored. As a result, it is capable of obtaining the radiation loss under active-lasing condition. This is useful for computing the radiation loss when the lasing mode is irregular or unpredictable such as in the case of chaotic mode, mode in race-trace waveguide ring resonator or mode in non-circular cavity. The FDTD approach is also capable of computing the scattering and absorption losses. Additionally, we show that the Active-Lasing FDTD approach gives the same radiation loss results as that from the Conformal Transformation Approach for circular geometry, thus validating the two approaches with each other.

In Paper II, we will also discuss the interplay among the main factors affecting the cavity Q factor of the microresonators, namely the radiation loss, scattering loss, and absorption loss. We will show how the radiation loss varies as a function of the resonator diameter, which basically dictates the maximum cavity Q it can achieve when the resonator is small. We will show that larger resonator diameter does not necessarily give higher cavity Q as the etched surface roughness will impose a scattering-loss limitation.

In Paper III, we consider the coupling to waveguides and compare the losses and power output efficiency of microresonators coupled to tangential waveguides. We also simulated the resonators coupling to radial waveguide and for the first time examine the physics of coupling to radial waveguide in detail. We show that the radial waveguide coupling method could be advantages over the tangential waveguide case in some situations such as producing unidirectional output. We show how to optimize the tangential/radial output coupling waveguide design when all factors: radiation loss, scattering loss, absorption loss, and waveguide coupling efficiency are considered.

2. Introduction to Paper I

Microdisk, microring and microcylinder resonators are circular shape resonators. Comparing to the conventional straight waveguide, the guided modes in circular shape resonators in which the waveguiding path is curved have different properties. Consider the conventional straight planar waveguide with a high refractive index core and low refractive index claddings, the two low-refractive-index claddings enable the guided mode to propagate along the waveguide without energy loss. However, in microresonators, the waveguiding path is bent with a certain radius of curvature. This bend causes the energy in the waveguide to leak out to infinity at the outer edge of the bend, resulting in what is commonly known as radiation loss due to curved waveguiding. Radiation loss is inherent in microdisk, microring, microcylinder and all dielectric based micro-resonator structures in which the waveguiding path is curved. The analysis and physical understanding of the energy loss for these microresonators is important because it has direct impact on the cavity Q factor as well as the resonance wavelength, which is important for the designs and applications of these microresonators [83–105].

An analysis that could give us a good physical picture and accurate numerical results is based on a "conformal transformation" approach. In this section, we will review and discuss this approach. Conformal transformation converts the circular shape waveguiding structure to a linear shape waveguiding structure in the transformed domain. In the transformed domain, the effective refractive index of the transformed linear waveguide structure has a local maximum at the circumference of the disk. Outside the disk, the effective refractive index increases again and at some distance away from the disk, becomes even higher than the local maximum. Thus the field of the waveguide mode confined by the local refractive index maximum first undergoes exponential decay outside the disk edge. But when the decaying field hits the high effective refractive index region outside the disk, it becomes propagating field leaking energy to infinity, which is the bending radiation loss.

3. Modeling of microcylinder resonators based on conformal transformation: formulation

We consider the microcylinder geometry shown in Fig. 1. The microcylinder has a diameter given by D_{disk} and material refractive index given by n_{disk}. The physical refractive index outside and external to the disk is given by n_{ext}. The Maxwell equations governing the propagation of electromagnetic fields are given by

$$\nabla\times\vec{e}(\vec{r},t)=-\mu\frac{\partial\vec{h}(\vec{r},t)}{\partial t} \tag{1}$$

$$\nabla\times\vec{h}(\vec{r},t)=\varepsilon(\vec{r})\frac{\partial\vec{e}(\vec{r},t)}{\partial t} \tag{2}$$

$$\nabla\cdot\varepsilon(\vec{r})\vec{e}(\vec{r},t)=0 \tag{3}$$

and

$$\nabla\cdot\mu\vec{h}(\vec{r},t)=0 \tag{4}$$

where $\vec{e}(\vec{r},t)$ and $\vec{h}(\vec{r},t)$ denote the electric field and magnetic field, respectively. By applying curl on both sides of Eqs. (1) and (2), we obtain the vectorial wave equations:

$$\nabla^2\vec{e}(\vec{r},t)-\varepsilon(\vec{r})\mu\frac{\partial^2\vec{e}(\vec{r},t)}{\partial t^2}=0 \tag{5}$$

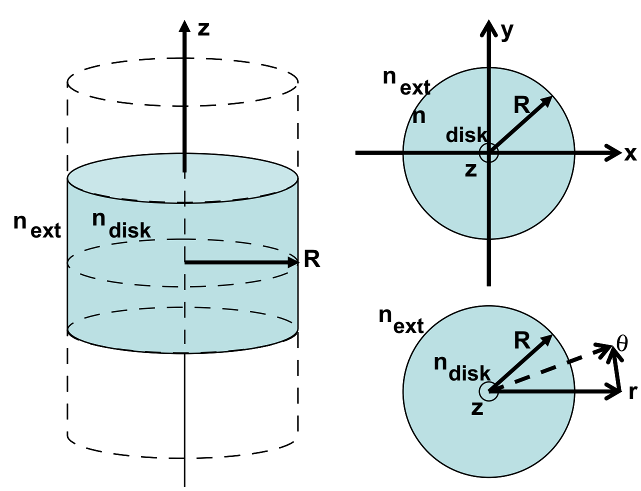

Fig. 1. Microcylinder resonator with radius R and its refractive index distribution in the transverse 2-D plane under rectangular and cylindrical coordinate systems.