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Discussion

Inhomogeneous two-dimensional photonic media: A statistical study

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ABSTRACT

Photonic media, in which disorder is introduced, are interesting materials for light management. In this paper, we have performed a statistical study of the average light transmission, over the range of wavelengths 450–1400 nm, for two-dimensional photonic structures with different homogeneity (quantified by the Shannon index). The photonic structure is a square lattice of circular pillars and the homogeneity is varied by clustering pillars in the crystal unit cells. We have calculated the light transmission for 50 different crystal realizations (permutating cluster position in the crystal) for each Shannon index value. Such Monte Carlo Markov Chain method produced the "*a posteriori*" distribution of the light transmission. We have observed a linear trend of the average transmission as a function of the crystal homogeneity. Furthermore, we have found a linear dependence of the average light transmission on the mean distance between pillars in the photonic structures.

1. Introduction

The Shannon index, also known as Shannon entropy, quantifies the homogeneity grade of a system. In 1948, Claude Shannon developed such formalism to measure the entropy in strings of text [1]. Since then the Shannon entropy variation has been also used to assess the stability of a protocol for quantum cryptography based on single photons [2], to measure the electron localization in a molecular system [3], as well as the probability of electronic charge distribution between atoms in a benzene ring (aromaticity measure) [4]. As regards condensed matter, a parallelism between information lattice and subgroup lattices has been exposed [5].

Crystallographic topology is one of the most fascinating areas in the study of light transport control in photonic media. In photonic structures, two different materials give rise to refractive index variations in the order of light wavelength. When different materials form periodic structures, these are called photonic crystals [6–9]. This is a relevant topic within physics and materials science, e.g. for the study of their optical properties and for the fabrication of optical filters, lasers and other devices [10–12]. When two domains have a large-order rotational yet not translational symmetry, they form a quasi-crystal [13] with unusual optical properties [14]. Disordered photonic structures consist of a random mixing of refractive index domains, widely used for the fabrication of random lasers [15–16]. Recent observations, as for instance Levy flights of photons [17] and disordered-enhanced light transport [18], witness that much effort has been devoted to better understand the features of these photonic materials.

In our study, we assumed that there might be a correlation between the grade of disorder of a photonic structure and its light transmission when the Shannon index is used to quantify the homogeneity in a crystal. In this case, the Shannon index can be related to its grade of disorder. It has been recently observed that the average light transmission decreases linearly when the homogeneity (i.e. the Shannon index) of a two-dimensional photonic crystal decreases [19]. Moreover, it has been demonstrated that the average light reflection as a function of the sample length increases sub-linearly in an ideal two-dimensional photonic crystal, while it increases linearly in a non-homogeneous photonic media [20].

In this paper, we give an account of a statistical study of the average light transmission over the range of wavelengths 450–1400 nm for photonic structures with different homogeneity (quantified by the Shannon index). For each crystal corresponding to a Shannon index value, we designed 50 different realizations stochastically, permuting the cluster positions in the crystal unit cells. From such set of crystals, we performed the distribution of the average transmission. We observed a peculiar linear trend of the average transmission as a function of the Shannon index. We have also found that the average transmission linearly increases by increasing the mean distance between pillars in the photonic structures.

2. Methods

Herein, we have considered a two-dimensional photonic crystal composed by a square lattice of dielectric circular pillars [9]. Column

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diameter d is set to 75 nm and the lattice constant a is 300 nm (Fig. 1), the pillars are made of Titanium dioxide ($n_T = 2.45$) and the matrix where the pillars are embedded is Silicon dioxide ($n_S = 1.46$). In this geometrical setting, the condition $n_T d – n_S(a–d)$ is satisfied [9].

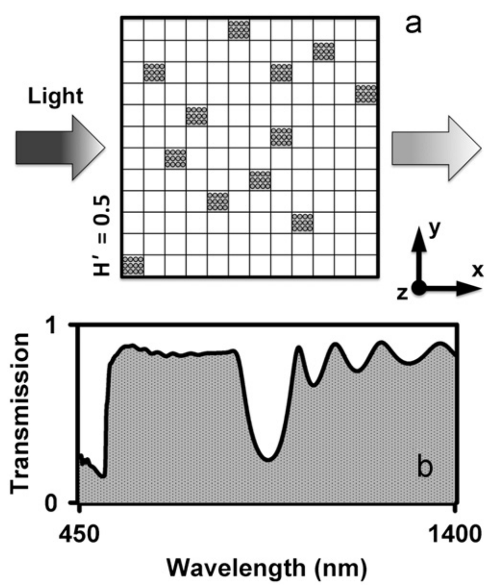

Fig. 1. (a) Schematic of a non-homogeneous photonic structure with clusters of 12 pillars in SiO_2 embedded in a TiO_2 matrix, corresponding to a Shannon index of 0.5. (b) Example of transmission spectrum of an ideal photonic crystal ($H' = 1$). In this study, we have considered the average transmission in the spectral range 450–1400 nm (gray area below the curve).

The ideal two-dimensional photonic crystal presents an homogeneus distribution of the scattering elements [9]. This means that, dividing the crystal area in regular space units, every unit cell contains the same number of pillars. The ideal photonic crystal used in this experiment is composed by 12×12 photonic crystal cells and each unit cell own one pillar. Starting from this regular structure, and keeping constant the number of pillars throughout the crystals, less regular media have been synthesized by concentrating more and more pillars in certain unit cells of the original crystal [19]. Concentrating the diffractive elements in unit cells we produced clusters of increasing size, i.e. aggregations of a different number of pillars. Crystal 2 presents clusters of size two. Cristal 3 has clusters of three pillars. Crystal 4 has clusters of four pillars and so on. We have repeated the pillars aggregation since to reach the Crystal 16. Such a method produces crystal of different structural homogeneity, from the most homogeneus (ideal photonic crystal) to the least (the crystal with highest cluster size, e.g. Crystal 16).

Using the Shannon–Wiener index, it is possible to define with a statistical measurement the homogeneity of the photonic structures. The Shannon–Wiener H' index is a diversity index used in statistics

$$H' = -\sum_{j=1}^{s} p_j \log p_j \quad (1)$$

where p_j is the proportion of the j-fold species and s is the number of the species. This index is widely used in statistics as an evenness measurement and in physics in the field of information theory [1]. Dividing H' by $\log(s)$ we have normalized the index constraining it within the range (0, 1). We used the normalized Shannon index as a measurement of the homogeneity of the transmission medium:

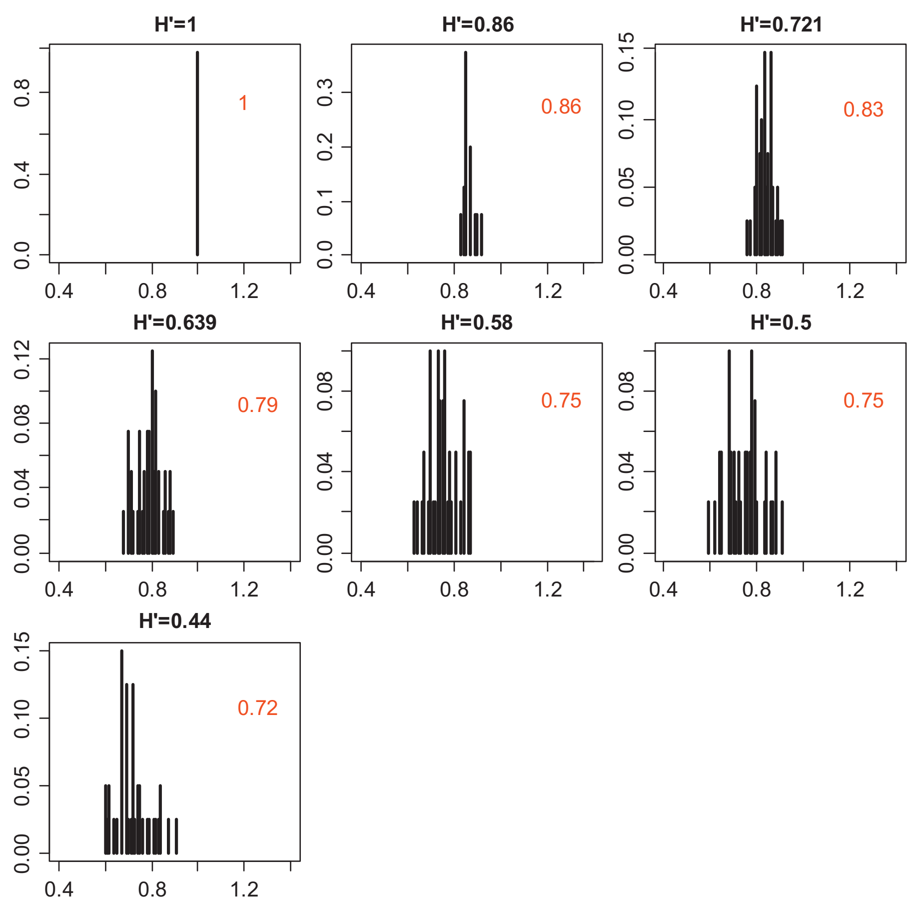

Fig. 2. Normalized transmission distribution for each cristal analyzed. The outcomes have been normalized by the Crystal 1 average transmission. The chart main legend indicates the Shannon index value for that crystal. Each distribution is the result of 50 replicates.