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An adjustable magnification method of lensless Fourier holography based on the equivalent spatial frequency

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ABSTRACT

The equivalent spatial frequency is defined according to the characteristic of lensless Fourier hologram. It is found from the theoretical analysis that the adjustable magnification without aberration can be achieved only if the relation of the equivalent spatial frequency is satisfied. This process can be regarded as numerical magnification of the real object. Meanwhile, its corresponding reconstruction algorithm is given. To testify the method, a transmission setup and a reflection setup are used to record lensless Fourier holograms of a small object and a large object, respectively. Magnification and de-magnification of the reconstructions with improved image quality demonstrate the effectiveness of the proposed method. In addition, the effects of the under-sampling problem in phase-unwrapping are minimized.

1. Introduction

Digital holography is a 3D imaging technique that records the object wavefront as a fringe pattern upon interference with a reference wave and reconstructs the object wavefront using a computer algorithm. The recording process makes it possible to get rid of the chemical wet process in the traditional holography, while the reconstruction allows direct numerical access to the complex field. It is widely used in microscopic detection, 3D display, and vibration or deformation measurement [1–3].

Compared with the recording medium of the traditional holography, the resolution of digital camera is low; the recording area is small and the bandwidth is limited. To fully utilize the space bandwidth product of the digital camera, lensless Fourier digital holographic setup has been developed and analyzed [4,5]. Moreover, the reconstruction algorithm of lensless Fourier hologram is simple because the complex field is obtained by using a single Fourier transform of the hologram. The complex field (thus, facilitating 3D imaging) will appear on the reconstruction plane (dimension, $M \times N$), where M and N are the pixel numbers of digital camera in y and x directions, respectively. In order to separate the complex field from its conjugate image, the maximum size of the complex field is limited to half of the reconstruction plane (namely, $M/2 \times N/2$). The detail of the reconstructed object has fewer pixels. There are two ways to magnify the detail section. The one is to magnify the detail digitally with the help of the digital image processing technique; however the detail will be magnified with the expansion of the pixel size. The other is to decrease the pixel size of the reconstruction plane by padding zeros around recorded hologram [6–8]. This method is widely adopted; however padding zeros will increase the computer workload in numerical reconstruction. In other cases, like in color digital holography or wavelength division multiplexing holography, the sizes of reconstructed image under different wavelength illuminations are different, therefore magnification or de-magnification of reconstructed images are necessary before superimposing to get a color image or comparing to obtain the deformation information. A flexible method to control the size of reconstructed image is needed. Li et al. have proposed an algorithm to magnify or de-magnify the object section with adjustable magnification factor; however it is complicated to complete [9].

In this paper, a numerical magnification and de-magnification of reconstructed image is proposed according to the lensless Fourier setup. After choosing the optional reconstruction wavelength or the reconstruction distance, the magnification ratio of reconstruction image can be numerically adjustable without aberration. In addition, the computer workload keeps the same during calculation of the magnification image and de-magnification one. From the experimental results, the quality of reconstructed image is greatly improved.

2. The theoretical analysis of lensless Fourier hologram

Suppose that $x-y$ is the recording plane and x_o-y_o is the object plane. The object wave $O(x,y)$ in the $x-y$ plane can be regarded as the collection of diverging spherical lights from a

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number of object points. The reference wave $R(x,y)$ is a spherical wave, whose origin is located at coordinates $R(x_r,y_r,z_o)$. The curvatures of the object and reference wavefronts are same, which is the character of the lensless Fourier setup. The interference intensity between a light generated by an object point $S(x_o,y_o,z_o)$ and the reference wave is given as:

$$I(x,y)=|R|^2+|S|^2+2|R||S|\cos\left[2\pi\left(\frac{x_o-x_r}{\lambda z_o}x+\frac{y_o-y_r}{\lambda z_o}y\right)\right] \tag{1}$$

It can be known from Eq. (1) that the spatial frequencies of interference pattern in x and y directions are given as

$$\xi=\frac{x_o-x_r}{\lambda z_o},\quad \eta=\frac{y_o-y_r}{\lambda z_o} \tag{2}$$

which are determined by the locations of object point and the reference point source, the wavelength and the recording distance z_o between the object and the recording planes. The digital hologram $I_H(x,y)$ obtained by the interference between the object wave and the reference point source is composed of a series of sine interference fringes with different spatial frequencies.

The reconstructed wavefront can be obtained by using the diffraction formula. The hologram is assumed to be illuminated by another spherical wave $C(x,y)=\exp[-j\pi/\lambda z_o(x^2+y^2)]$ diverging from the origin of x_o-y_o plane. According to Fresnel diffraction, the complex amplitude of the reconstructed image in the reconstruction plane x_i-y_i is expressed as

$$U_i(x_i,y_i,z)=\frac{\exp(jkz)}{j\lambda z}\exp\left[\frac{j\pi}{\lambda z}(x_i^2+y_i^2)\right]\times F\left\{C(x,y)I_H(x,y)\exp\left[\frac{j\pi}{\lambda z}(x^2+y^2)\right]\right\} \tag{3}$$

where F is the two-dimensional Fourier transform. To obtain a focused image, we have to make the reconstruction distance z equal to recording distance z_o. The exponential term and $C(x,y)$ in $C(x,y)I(x,y)\exp[j\pi/\lambda z(x^2+y^2)]$ of Eq. (3) are eliminated. The spherical reconstruction wave makes Eq. (3) simplified for removing the Fresnel diffraction kernel. Then, Eq. (3) can be rewritten as

$$U_i(x_i,y_i,z_o)=\frac{\exp(jkz_o)}{j\lambda z_o}\exp\left[\frac{j\pi}{\lambda z_o}(x_i^2+y_i^2)\right]F\{I_H(x,y)\} \tag{4}$$

where $F\{I_H(x,y)\}=F\{|R|^2\}+F\{|O|^2\}+F\{R^*O\}+F\{RO^*\}$ is the Fourier transform of the hologram and $O(x,y)$ is the object wave expressed in terms of Fresnel diffraction, like

$$O(x,y)=\frac{\exp(jkz_o)}{j\lambda z_o}\exp\left[\frac{j\pi}{\lambda z_o}(x^2+y^2)\right]\times F\left\{O(x_o,y_o)\exp\left[\frac{j\pi}{\lambda z_o}(x_o^2+y_o^2)\right]\right\} \tag{5}$$

It is seen that the reconstruction requires a single Fourier transform of the hologram. If the distance between the reference point source and the object is large enough, it is possible to extract the real image $F\{RO^*\}$. Since the object wave $O(x,y)$ expressed in terms of Fresnel diffraction contains another Fourier transform, the pixel size of reconstructed image is same as that of the hologram.

3. The theory of magnification of the reconstructed image

From the spatial frequency expression of the hologram given in Eq. (2), it is known that the spatial frequency is proportional to the wavelength, the distance between the object and the reference point and the recording distance. If we use another wavelength laser λ' instead of λ as the light source, to keep the same spatial frequencies the distance between the object point and reference point source will be varied with the wavelength. The relation becomes:

$$\xi=\frac{x_o-x_r}{\lambda z_o}=\xi'=\frac{x'_o-x'_r}{\lambda' z_o},\quad \eta=\frac{y_o-y_r}{\lambda z_o}=\eta'=\frac{y'_o-y'_r}{\lambda' z_o} \tag{6}$$

We define ξ' and η' as equivalent spatial frequencies. Meanwhile, if the recording distance z_o is replaced by another value z'_o, the similar equivalent spatial frequencies become

$$\xi=\frac{x_o-x_r}{\lambda z_o}=\xi'=\frac{x'_o-x'_r}{\lambda z'_o},\quad \eta=\frac{y_o-y_r}{\lambda z_o}=\eta'=\frac{y'_o-y'_r}{\lambda z'_o} \tag{7}$$

In the above equations, the position of reference point source is fixed. When we consider another recording distance or wavelength, the equivalent object size is enlarged or shrank. Thus, corresponding reconstructed image is magnified or de-magnified by using another wavelength or the reconstruction distance.

Suppose that, the dimension of the recorded object is $L_{oy}\times L_{ox}$, the recording distance is z_o, the wavelength is λ, the dimension of the hologram is $M\Delta_y\times N\Delta_x$, Δ_y and Δ_x is CCD pixel size in y and x direction, respectively. The reconstructed images only cover a part of the reconstruction plane $M\Delta_y\times N\Delta_x$. According to Eq. (7), when we suppose z'_o instead of the real recording distance z_o, the equivalent size of object is as $l'_{oy}\times l'_{ox}$:

$$l'_{ox}=\frac{z_o}{z'_o}N\Delta_x,\quad l'_{oy}=\frac{z_o}{z'_o}M\Delta_y \tag{8}$$

The size of the reconstructed images is increased or decreased by the ratio of z_o/z'_o. Hence, the magnification factor of the reconstructed image is adjusted by choosing the optional ratio.

As an example, if we expect the object $L_{oy}\times L_{ox}$ is displayed with the size $M\Delta_y\times N\Delta_x$, the ratio between recording distance and the equivalent reconstruction distance should be

$$\frac{z_o}{z'_o}=\frac{L_{ox}}{N\Delta_x},\quad \frac{z_o}{z'_o}=\frac{L_{oy}}{M\Delta_y} \tag{9}$$

The magnification process is shown in Fig. 1, in which the arrow with solid lines represents the recorded object and the arrows with dashed lines are the reconstructed real images. When we use different recording distances in the diffraction calculation, the equivalent size of the object will be changed so as the reconstructed image.

Meanwhile, we can change the wavelength of the reconstructed wave from λ to λ', the equivalent size of the object will be changed, according to Eq. (6), as

$$l'_{ox}=\frac{\lambda}{\lambda'}N\Delta_x,\quad l'_{oy}=\frac{\lambda}{\lambda}M\Delta_y \tag{10}$$

It is obvious that the size recorded is magnified or de-magnified with the ratio λ/λ'.

To display the object with the designed size $M\Delta_y\times N\Delta_x$, Eq. (10) should be satisfied. The diagram of magnification by changing the wavelength is given in Fig. 2, in which the reconstructed image

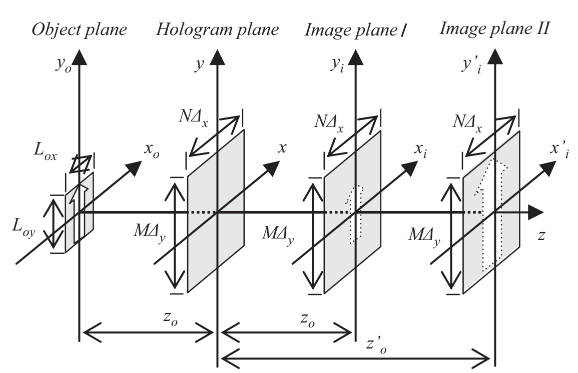

Fig. 1. Magnification of the reconstruction image by changing the reconstruction distance.