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Numerical characterization of time delay signature in chaotic vertical-cavity surface-emitting lasers with optical feedback

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ABSTRACT

We have discussed numerically the characterization of time delay (TD) signature in a chaotic vertical-cavity surface-emitting laser (VCSEL) subject to optical feedback, where the results for polarization-preserved optical feedback (PPOF) and polarization-rotated optical feedback (PROF) are presented comparatively. It is found that, when the feedback strength is moderate, TD signature can be retrieved successfully by analyzing the total intensity, intensities of x-linearly polarized (x-LP) mode, and y-LP mode in a VCSEL with PPOF; while for PROF case, the intensity amplitude peaks in autocorrelation function (ACF)/delayed mutual information (DMI) of the two orthogonal LP modes are located at multiple values of TD. Furthermore, the influence of the feedback strength and time delay has been investigated. The results show that, a further increase of the feedback strength (roughly $> 40\ \mathrm{ns}^{-1}$) leads to the successful identification of the TD signature in the two LP mode intensities of VCSELs with PROF. It is also observed that the estimation of TD depends significantly on the value of the spin–flip relaxation rate (γ_s). For low relaxation rates the TD signature could be much easier to be revealed by the two LP modes than that for large values of relaxation rate in VCSELs with PROF. Finally the influence of injection current and the device linear anisotropies is investigated, and the results of TD estimation are confirmed by calculating permutation entropy (PE).

1. Introduction

Two research directions for nonlinear time-delay systems have attracted considerable attention: one focuses on their potential applications in chaos secure communications [1–3], and the other involves the parameter estimation of these systems [4–7]. These two directions are associated with each other, in other words, the concealment of intrinsic parameters in chaotic systems could enhance the security of chaos-based communications [8,9]. Recently, chaotic optical communications systems using semiconductor lasers (SLs) have been considered to be interesting [2,3,10–14]. This is due to the fact that they make it possible to generate higher bandwidth and higher unpredictability chaotic carrier [15]. However, the time delay (TD), which plays an important role in chaos generation, is vulnerable to identification via time series analysis. Previous works on TD identification in external-cavity semiconductor lasers (ECSLs) have shown that the TD can be easily extracted from the analysis of the chaotic output using straightforward techniques [7,16,17]. In the same context, it has been shown the TD signature in ECSLs can be hidden from intensity time series, on condition that the parameters are such that TD should be close to the relaxation oscillation period of the SLs subject to moderately weak feedback levels [7,17]. Nevertheless, in this parameter region, it recently has been proved that the TD signature can still be extracted by analyzing the phase or quadratures time series [18]. In our previous work, we demonstrated numerically the feedback parameters in ECSLs could be simultaneously identified using symbolic time series analysis (STSS) based method when the system structure and the corresponding time series are known [19]. On the other hand, it has been suggested to complicate the TD identification by introducing additional cavities [20]. The results have also been experimentally demonstrated [21]. More recently, a TD concealment scheme based on a phase-chaos electro-optical delay system has been proposed [22]. The report has shown the TD and a pseudorandom binary sequence (PRBS) can be mutually concealed. All of these considerations motivate further investigations to characterize the TD signature in chaos-based communication systems.

Vertical-cavity surface-emitting lasers (VCSELs), as a special type of SLs, have gained increasing attention for a wide range of applications as a result of several desirable characteristics, including circular beam profile with narrow divergence, a small threshold

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current, single longitudinal mode emission, wafer-scale testing and array capability [23]. Their complex polarization properties and polarization switching (PS) between two linearly polarized (LP) modes (x-LP and y-LP modes) have also attracted considerable attention [24–29]. There have been a great many investigations of chaos synchronization and chaotic optical communications in VCSELs [23,30–38]. However, reports of characterizing TD signature in VCSEL-based chaotic optical communication systems remain scarce. Among most previous reports [7,9,17,20–22,39], they have been trying to find an alternative way of concealing the TD, so as to maintain or enhance the security.

The purpose of this paper is to characterize the property of TD signature in VCSELs with optical feedback. To that end, from the TD estimation viewpoint, the polarization-resolved dynamics of a VCSEL subject to optical feedback have been investigated systematically, where the results of polarization-preserved optical feedback (PPOF) and polarization-rotated optical feedback (PROF) are compared carefully.

The remainder of this paper is structured as follows. In Section 2, a brief description of the SFM model with optical feedback is presented, and the TD estimators are defined. Section 3 contains the results of the numerical simulations. We study the TD estimation of the total intensity, and the intensities of x- and y-LP components for PPOF and PROF system, systematically and comparatively. Section 4 discusses the effect of the injection current and the device linear anisotropies on TD estimation, and the results of TD estimation are confirmed by permutation entropy (PE). Finally the basic conclusions are given in Section 5.

2. Theoretical model

The model of a VCSEL subject to optical feedback is based on the spin–flip model (SFM) [24,40]. The variables are E_x, E_y, N and n, here E_x and E_y are the two linearly polarized slowly varying components of the field, N is the total population inversion between the valence and conduction bands, while n accounts for the difference between population inversions for the spin-up and spin-down radiation channels. The rate equations are written by

$$\begin{aligned}\frac{dE_x}{dt} &= k(1+i\alpha)\left[(N-1)E_x+inE_y\right]-(\gamma_a+i\gamma_p)E_x+Rk_f\exp(-i\phi_F)E_y(t-\tau_f)\\ &\quad +Pk_f\exp(-i\phi_F)E_x(t-\tau_f)+F_x,\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{dE_y}{dt} &= k(1+i\alpha)\left[(N-1)E_y+inE_x\right]+(\gamma_a+i\gamma_p)E_y+Rk_f\exp(-i\phi_F)E_x(t-\tau_f)\\ &\quad +Pk_f\exp(-i\phi_F)E_y(t-\tau_f)+F_y,\end{aligned} \tag{2}$$

$$\frac{dN}{dt} = \gamma_N\left[\mu-N(1+|E_x|^2+|E_y|^2)+in(E_xE_y^*-E_yE_x^*)\right], \tag{3}$$

$$\frac{dn}{dt} = -\gamma_s n-\gamma_N\left[n(|E_x|^2+|E_y|^2)+iN(E_yE_x^*-E_xE_y^*)\right]. \tag{4}$$

In these equations, the subscript x (y) refers to $x-$ ($y-$) LP mode. When considering PPOF, $R=0$ and $P=1$; while for PROF, $R=1$ and $P=0$. The last terms in Eqs. (1) and (2) are the spontaneous emission noises described by the following Langevin sources:

$$F_x = \sqrt{\beta_{sp}(N+n)/2}\,\chi_1+\sqrt{\beta_{sp}(N-n)/2}\,\chi_2 \tag{5}$$

$$F_y = -i\left(\sqrt{\beta_{sp}(N+n)/2}\,\chi_1-\sqrt{\beta_{sp}(N-n)/2}\,\chi_2\right) \tag{6}$$

where the spontaneous emission rate β_{sp} is equal to$10^{-6}\ \text{ns}^{-1}$, and $\chi_{1,2}$ represents independent Gaussian white noise of unitary variance and zero mean value. The internal parameters of the laser are as follows: k is the field decay rate, α is the linewidth enhancement factor, γ_N is the decay rate of N, γ_s is the spin–flip relaxation rate, $\gamma_a(\gamma_p)$ is the linear dichroism (birefringence), μ is the normalized injection current (e.g., $\mu\approx 1$at threshold, and note that the anisotropies slightly modify this value). The parameter k_f is the feedback strength, and $\phi_F=\omega_0\tau_f$ is the accumulated optical phase. Note that ω_0 is the optical frequency of the x- or y-LP mode at the solitary laser threshold without linear anisotropies, and τ_f is the external cavity delay time. These parameters were chosen in their typical ranges [24,40]: $k=300\ \text{ns}^{-1}$, $\alpha=3$, $\gamma_N=1\ \text{ns}^{-1}$, $\gamma_s=50\ \text{ns}^{-1}$, $\gamma_a=-0.1\ \text{ns}^{-1}$, $\gamma_p=6\ \text{ns}^{-1}$, $\lambda=850$ nm and $\mu=1.2$. The initial conditions were selected for the x-LP state of the solitary laser.

TD is a key parameter concerning the security of nonlinear time delayed system. The estimation of the TD has attracted a lot of attention, and many statistical methods have been designed to identify it. The well-known methods include autocorrelation function (ACF), delayed mutual information (DMI), filling factor, extreme statistics and permutation-information-theory approach [4,6–7,17,41]. In this work, ACF and DMI were mainly used to retrieve the TD due to their robustness to noise and higher computational efficiency.

The ACF $[C(\theta)]$ that measures how well time series matches its time-shifted version is defined as

$$C(\theta) = \frac{\langle (I(t+\theta)-\langle I(t)\rangle)(I(t)-\langle I(t)\rangle)\rangle}{(\langle I(t)-\langle I(t)\rangle\rangle^2\langle I(t+\theta)-\langle I(t)\rangle\rangle^2)^{1/2}}, \tag{7}$$

where $I(t)=|E(t)|^2$ is the intensity time series, and $I(t+\theta)$ is which contains a time shift θ with respect to $I(t)$. DMI is also a statistic measure based on information theory, to be defined precisely, measures the nonlinear dependence of two variables, namely $I(t)$ and $I(t+\theta)$. The definition of DMI $[M(\theta)]$ is

$$M(\theta) = \sum_{I(t),I(t+\theta)} P(I(t),I(t+\theta))\log_2\frac{P(I(t),I(t+\theta))}{P(I(t))P(I(t+\theta))}, \tag{8}$$

Note that $P(I(t))(P(I(t+\theta)))$ is the marginal probability density function and, $P(I(t),I(t+\theta))$ is the joint probability density function. These functions are suitably evaluated from time series by a simple box-counting algorithm as in our previous report [19].

3. Numerical results

In the literature [7,17,20–22], many reports related to TD concealment focused on the region where the relaxation oscillation period is close to the delay time and the feedback levels are moderately weak. They demonstrated that the TD can be concealed or suppressed in that region. However, with these laser and feedback parameters, the complexity of chaotic output is low. Furthermore, understanding the TD properties in other regions may help gain deeper insight into the fundamental mechanisms in chaos generation. For these reasons, in this section we pay attention to the characteristics of TD signature in chaotic VCSELs with moderate or strong feedback.

3.1. Bifurcation diagram manifest

To begin with, we will show the VCSEL works in a chaotic regime with the feedback parameters space of interest. Here, bifurcation diagram was employed to show the different operation regimes of the laser diode. Figs. 1 and 2 visualize the bifurcation diagrams versus feedback strength and time delay, respectively. The bifurcation diagrams were obtained by plotting all the maxima and minima of total intensity ($I_T=|E_x|^2+|E_y|^2$), x-LP intensity ($I_x=|E_x|^2$) and y-LP intensity ($I_y=|E_y|^2$) for each value of k_f or τ_f. Besides a long simulation time was consumed and we began to record time series after 50 ns, in order to make sure that all transients had died out. As shown in Fig. 1, the outputs of