



Evolution of an Airy beam obstructed by a transparent particle

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ABSTRACT

As comparing with a beam blocked by an opaque obstacle, a beam blocked by a transparent obstacle is most often encountered in practice, especially in optical micro-control. In present paper, the propagation of a beam blocked by a transparent obstacle is modeled. As a special case, an analytical expression for an Airy beam blocked by a transparent obstacle is derived, and the evolution of its intensity in free space is studied and discussed. Study shows that the disturbed phase of Airy beam due to the transparent obstacle may cause many phase singularities during propagation. However, with the increase of the propagation distance, a disturbed Airy beam exhibits a similar property as the self-healing of its intensity.

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1. Introduction

An Airy beam is the solution to the force-free Schrödinger equation and can exhibit a non-spreading property in free space [1]. In recent years, many attentions have been paid on the propagation of Airy beam due to their wide applications and many interesting properties [2–6]. Self-healing ability is one of the most important properties of Airy beam [5–7]. It makes them very useful in optical tweezing, micro-control and microscopy [8].

Although theoretical and experimental studies of the self-healing properties of many non-diffracting beams, such as Bessel beam and Airy beam, have been carried out, all of these are restricted to the situation that these beams are obstructed by an opaque obstacle [6–10]. In fact transparent particles are most often encountered in practice. For example, in biology, transparent particles are often needed to trap by tweezers, or there exists water drop unavoidable in the propagation path. These transparent particles not only cause the change of the amplitude but also cause the change of the phase of the incident beam. This disturbed phase of a beam will cause the change of the intensity and phase distribution during propagation. However, to the best of our knowledge, the propagation of a beam disturbed by a transparent particle has not been taken into account. In present paper, the propagation of a beam obstructed by a transparent particle is modeled. As an example, the evolution of Airy beam with a transparent particle is analytically studied.

2. Model for a beam obstructed by a transparent particle

We assume that a transparent particle is located on the initial plane where a beam passes through, as shown in Fig. 1.

Fig. 1 shows that an Airy beam expanded by an expander propagates along the z -axis. However, there is a transparent particle like a water drop on the travel path due to some reason. For simplicity, the transparent particle is assumed to be located on the initial plane in the paper. Therefore, not only the amplitude but also the phase of the incident beam will change due to the obstruction of the transparent particle. Because the size of the particle is small in practice, the change of the amplitude and phase is restricted to the small area at initial plane. Because the effects of a particle on the amplitude of a beam can be regarded as the obstruction of an opaque particle, only the change in the phase of the incident beam is considered. Therefore, the optical field of a collimated beam passing through a transparent particle can be written as

$$U_1(x_1, y_1) = A_0(x_1, y_1) \exp[i\phi(x_1 - x_0, y_1 - y_0)] W(x_1 - x_0, y_1 - y_0) \quad (1)$$

where (x_1, y_1) are the coordinates of the initial plane, (x_0, y_0) are the center coordinates of the transparent particle, $A_0(x_1, y_1)$ is the amplitude distribution of a incident beam without disturbance, ϕ is the phase caused by the transparent particle, and W is a window function which restrict the effect of the transparent particle to a certain area. Because the window function is in the exponential, the production of ϕ and W can be used to express an additional phase within some part. By using the Taylor series expansion for exponential function, Eq. (1) can be rewritten as

$$U_1(x_1, y_1) = A_0(x_1, y_1) \sum_{n=0}^{\infty} \frac{i^n}{n!} \phi^n(x_1 - x_0, y_1 - y_0) W^n(x_1 - x_0, y_1 - y_0) \quad (2)$$

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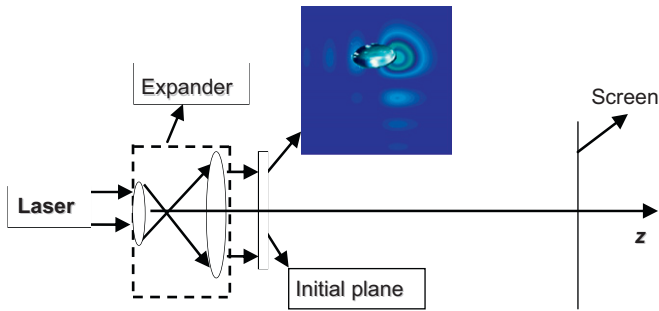


Fig. 1. Propagation geometry where Airy beam is obstructed by a transparent particle like water drop.

If W is a hard-edge aperture, namely, $W^m(x_1, y_1) = W(x_1, y_1)$, from Eq. (2) we can derive (see Appendix A)

$$U_1(x_1, y_1) = A_0(x_1, y_1)(1 + W(x_1 - x_0, y_1 - y_0)\exp[i\varphi(x_1 - x_0, y_1 - y_0)] - 1) \quad (3)$$

With the help of Eq. (3) and the diffraction integral, the propagation of a beam obstructed by a transparent particle can be studied.

3. Analytical expression for an Airy beam obstructed by a transparent particle

The amplitude of two-dimensional exponentially truncated Airy beam at initial plane is given by

$$A_0(x_1, y_1) = u_0(x_1)u_0(y_1) \quad (4)$$

where $u_0(\chi)$ is one-dimensional finite energy Airy and is expressed as [5–7,11–16]

$$u_0(\chi) = Ai\left(\frac{\chi}{\omega_0}\right) \exp\left(a\frac{\chi}{\omega_0}\right) \quad (5)$$

here $\chi = (x_1, y_1)$, ω_0 is arbitrary transverse scale and a in the exponential function is a parameter associated with the truncation of Airy beam. The 2-D finite energy Airy beam can be generated from Gaussian beam via cubic phase [11,12]. The window function is assumed as a hard-edge circular aperture of radius R which describes the size of a transparent particle. That is

$$W(x_1, y_1) = \begin{cases} 1 & x_1^2 + y_1^2 \leq R^2 \\ 0 & x_1^2 + y_1^2 > R^2 \end{cases} \quad (6)$$

because the propagation properties of an Airy beam with different disturbed phase are different, for simplicity, the effect of the transparent particle on the phase of an Airy beam is assumed as a lens in present paper, namely

$$\varphi(x_1, y_1) = \exp\left[-\frac{ik}{2f}(x_1^2 + y_1^2)\right], \quad (7)$$

where f is the focal length, $k = 2\pi/\lambda$ is wave number and λ is wavelength. With the help of Fresnel diffraction integral,

$$U_2(x_2, y_2) = \frac{k}{2\pi iz} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U_1(x_1, y_1) \times \exp\left\{\frac{ik}{2z}[(x_2 - x_1)^2 + (y_2 - y_1)^2]\right\} dx_1 dy_1 \quad (8)$$

the optical field at observer plane can be calculated, where (x_2, y_2) is a pair of coordinates of z -plane. It is difficult to derive an analytical expression from Eq. (8) due to the hard-edge aperture. To perform the integral in Eq. (8), the expansion of the hard-edge

aperture by a Gaussian function is used [17], namely,

$$W(x_1, y_1) = \sum_{n=1}^M C_n \exp\left[-\frac{g_n}{R^2}(x_1^2 + y_1^2)\right], \quad (9)$$

where C_n and g_n are the expansion coefficients, M is the number of the expansion coefficients. In this paper we adopt the expansion coefficients in Ref. [11] ($M=16$) which can give more precise results [18,19]. Substituting Eqs. (3)–(5), (7) and (9) into (8), we can obtain the analytical expression as

$$U_2(x_2, y_2) = U_{20}(x_2)U_{20}(y_2) + \sum_{n=1}^M C_n [U_{22}(x_2, x_0)U_{22}(y_2, y_0) - U_{21}(x_2, x_0)U_{21}(y_2, y_0)] \quad (10)$$

where $U_{20}(\chi)$ is the one-dimensional optical field of an Airy beam in free space without obstruction and is given as

$$U_{20}(\chi) = Ai\left[\frac{\chi}{\omega_0} - \left(\frac{\xi}{2}\right)^2 + ia\xi\right] \exp\left[\left(a + \frac{i}{2}\xi\right)\frac{\chi}{\omega_0} + \frac{ia^2}{2}\xi - \frac{a\xi^2}{2} - \frac{i\xi^3}{12}\right] \quad (11)$$

$U_{22}(\chi, \chi_0)$ and $U_{21}(\chi, \chi_0)$ are given as

$$U_{22}(\chi, \chi_0) = \sqrt{\frac{1}{2iA_2\omega_0^2\xi}} Ai\left[\frac{1}{16A_2^2\omega_0^4} + \frac{B_2}{2A_2\omega_0}\right] \times \exp\left[\frac{i\chi^2}{2\omega_0^2\xi} - Q_2\chi_0^2 + \frac{1}{96A_2^3\omega_0^6} + \frac{B_2}{8A_2^2\omega_0^3}\right] \quad (12)$$

$$U_{21}(\chi, \chi_0) = \sqrt{\frac{1}{2iA_1\omega_0^2\xi}} Ai\left[\frac{1}{16A_1^2\omega_0^4} + \frac{B_1}{2A_1\omega_0}\right] \times \exp\left[\frac{i\chi^2}{2\omega_0^2\xi} - Q_1\chi_0^2 + \frac{1}{96A_1^3\omega_0^6} + \frac{B_1}{8A_1^2\omega_0^3}\right] \quad (13)$$

here $\xi = z/(k\omega_0^2)$ is the normalized distance, $B_1 = a/\omega_0 - 2Q_1(\chi - \chi_0) + 2A_1\chi$, $A_1 = Q_1 - i/(2\xi\omega_0^2)$, $Q_1 = g_n/R^2$, $B_2 = a/\omega_0 - 2Q_2(\chi - \chi_0) + 2A_2\chi$, $A_2 = Q_2 - i/(2\xi\omega_0^2)$ and $Q_2 = ik/(2f) + g_n/R^2$.

From Eqs. (9)–(13) we can see that when we set $f \rightarrow \infty$, Eq. (10) represents the optical field of an Airy beam in free space. If we set $f \rightarrow 0$, Eq. (10) gives the optical field of an Airy beam with an opaque obstacle which can be used to study the self-healing properties. When we assume $R \rightarrow \infty$ and $z = f$, with the help of the Fourier transform, Eq. (10) can be represented by

$$U_2(x_2, y_2) = \frac{1}{2\pi i \xi} \exp\left\{-\frac{ik}{2f}(x_0^2 + y_0^2 - x_2^2 - y_2^2)\right. \\ \left. + \frac{1}{3}\left[a + \frac{ik\omega_0}{f}(x_0 - x_2)\right]^3 + \frac{1}{3}\left[a + \frac{ik\omega_0}{f}(y_0 - y_2)\right]^3\right\} \quad (14)$$

Eq. (14) is the off-axis focused beam of an Airy beam at the focal plane. Because the propagation of an Airy beam and its self-healing properties in free space are studied widely, only the case that the receiver plane near the focal plane of the transparent particle is studied in present paper.

4. Numerical calculation and analysis

In the following calculation we set $a=0.2$, $\omega_0=0.01$ m and $\lambda=1.06$ μ m. The normalized intensity is defined as the intensity divided by the maximum of the intensity for an Airy beam without particle under the same conditions. Because the Airy beams disturbed by a transparent particle at the source plane adopted in this paper are x - y symmetric, only one-dimensional intensity distributions are studied in the following analysis.

To see the validity of the expansion for Gaussian function to express the hard-edge aperture, the comparison between the

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