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# Multi-frame blind deconvolution using sparse priors

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ABSTRACT

In this paper, we propose a method for multi-frame blind deconvolution. Two sparse priors, i.e., the natural image gradient prior and an $l_1$-norm based prior are used to regularize the latent image and point spread functions (PSFs) respectively. An alternating minimization approach is adopted to solve the resulted optimization problem. We use both gray scale blurred frames from a data set and some colored ones which are captured by a digital camera to verify the robustness of our approach. Experimental results show that the proposed method can accurately reconstruct PSFs with complex structures and the restored images are of high quality.



## 1. Introduction

In applications such as remote sensing, the obtained images are often blurred due to defocusing, atmospheric disturbance, relative motion, etc. The process can be modeled by convolving a latent image with a PSF plus some noise:

$$\mathbf{g} = \mathbf{ho} + \mathbf{n} \tag{1}$$

where **g**, **o** and **n** denote the vector forms of the blurred image, latent image and additive noise respectively, **h** stands for the convolution matrix of the PSF. In the frequency domain, Eq. (1) is converted into the following equation:

$$G(u) = H(u)O(u) + N(u) \tag{2}$$

where $G(u)$, $H(u)$, $O(u)$ and $N(u)$ are the discrete Fourier Transforms of the blurred image, PSF, latent image and additive noise respectively.

Image deconvolution aims to estimate the latent image from the blurred. It can be divided into two categories in terms of whether the PSF is known, i.e., non-blind and blind deconvolution. There are numerous approaches for both of them, such as the non-blind deconvolution methods proposed in Refs. [1–4] and the blind deconvolution methods in Refs. [5–8].

However, even for non-blind deconvolution, the problem is ill-posed. This is partly because in frequency domain, the energy of $N(u)$ concentrates in the high frequency region while $H(u)$ is low pass. The power of $N(u)$ will be amplified and results in unwanted artifacts such as noise and ringing. Furthermore, in most cases, it is impractical to obtain an accurate PSF and the negative artifacts will be more serious.

In blind image deconvolution, since we have to estimate both the latent image and PSF, the condition is even worse. There are so many pairs of **h** and **o** fitting Eq. (1) that it is difficult to determine a good solution. Obviously, the results of blind deconvolution also suffer from negative artifacts as in non-blind deconvolution.

To make the solution stable, researchers have designed various regularization methods, e.g., in Tikhonov regularization [9], the regularization term $Q(\mathbf{o}) = \sum_j \|\mathbf{f}_j\mathbf{o}\|_2^2$ (where each $\mathbf{f}_j$ denotes the convolution matrix of a certain derivative filter, $j$ is the filter index) is proposed. The advantage of Tikhonov regularization is its computational convenience, but the restored image tends to be smooth. Another famous regularization method is the total variation (TV) $Q(\mathbf{o}) = \sum_k \sqrt{(\mathbf{f}_1\mathbf{o})_k^2 + (\mathbf{f}_2\mathbf{o})_k^2}$ (where $\mathbf{f}_1$ and $\mathbf{f}_2$ are the convolution matrices of the derivative filters $f_1 = [-1,\ 1]$ and $f_2 = [-1,\ 1]^T$, $k$ denotes the pixel index) which have been successfully used in both non-blind [10–12] and blind image deconvolution [13–15]. Although the TV is capable of preserving image edges during deconvolution, it results in a nonlinear optimization problem. Fortunately, efficient methods have been proposed to solve it and obtain satisfactory results (e.g., the methods in Refs. [11,12]). In Ref. [16], the authors analyze the conditions for edge-preserving regularization and propose a series of regularization terms. Recently, many new effective approaches emerge, e.g., the authors of Ref. [17] design two regularization terms which derive from the bilateral and joint bilateral filters. They are used to modify the Richardson–Lucy (RL) algorithm and give birth to a successful progressive inter-scale and intra-scale non-blind deconvolution method. In Ref. [18], the sparse natural image gradient prior is introduced for regularization, with iteratively reweighted least squares (IRLS) method, it reaches very good result. In Ref. [19], the authors adopt a new prior based on the responses of some edge detectors to regularize the latent image. The PSF is regularized with TV. The result is achieved using an alternating

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minimization approach. In Ref. [20], a fast blind deconvolution method is designed, whose efficiency comes from the usage of shock filter and Gaussian prior. Furthermore, the GPU technique is also utilized to accelerate the computation. In Ref. [21], the authors improve the PSF estimation process of the method proposed in Ref. [20]. Although it is slower, the estimated PSF is of higher accuracy. The blurred image is restored with a TV-$l_1$ approach. In Ref. [22], a novel Bayesian model is proposed, in which the PSF, local image differences and the imaging model errors are all assumed to follow Student's-t distribution, the resulted problem is solved with a variational Bayesian inference method. Considering the topic of this paper, some of the denoted single image blind deconvolution methods can be extended straightforwardly for solving multi-frame blind deconvolution problem.

In multi-frame blind deconvolution, different blurred versions of the same target are provided. Although it suffers from the same problems as in single image blind deconvolution, multiple frames contain more information, with proper regularization and optimization methods, we can expect better result. In early researches such as Refs. [23,24], the authors mainly focus on noise priors and design some inverse filters. In Ref. [25], the authors give an analysis on some constrained algorithms and discuss in what cases using multiple blurred frames is better than one. In recent years, due to the successful applications of TV or Tikhonov regularization, some effective multi-frame blind deconvolution approaches are proposed, e.g., the author of Ref. [26] adopts a well designed iterative algorithm to solve the TV based cost function. In Ref. [27], the efficiency of the TV regularization method comes from the application of a splitting technique. In Ref. [28], a method with Tikhonov regularization in frequency domain is proposed, denoising and sharpening techniques are also used to improve the restored images. There are also some other methods which have been proven suitable for certain applications [29–31].

In recent studies, sparse priors become a focus of image processing [18,32–34]. In this paper, we adopt the sparse natural image gradient prior to regularize the latent image [18]. Moreover, due to the sparse energy distribution of the PSF, we use another $l_1$-norm based prior to regularize it. Then we formulate the multi-frame blind deconvolution problem under Bayesian probabilistic framework. During optimization, we use an alternating minimization approach to solve the resulted problem. The latent image and PSFs are alternately optimized until convergence. Experimental results on real world blurred images show that the proposed method is insensitive to misalignments between the blurred frames and can accurately reconstruct PSFs with complex structures. The quality of the restored image is comparable with that of some state of the art methods. Since most modern digital cameras can work in continuous shooting mode, it is easy to obtain multiple blurred frames of the same scene under bad shooting environments (e.g., shooting in low-light environment, the long exposure time will result in blurred images), in such cases, our method will be a good choice for image restoration.

The arrangement of the paper is as follow. In Section 2, we make a description of the adopted sparse priors and formulate the problem under Bayesian framework. In Section 3, we demonstrate the optimization approach. In Section 4, we test our approach on both gray scale and color blurred frames. Finally, we make a conclusion in Section 5.

## 2. Problem formulation

Suppose that we have obtained $m$ blurred frames of the same object $\mathbf{o}$ which are denoted by $\mathbf{g}_i$ ($i=1, 2,\ldots, m$). If the corresponding PSFs are $\mathbf{h}_i$ ($i=1, 2,\ldots, m$), we obtain a series of equations:

$$\mathbf{g}_i = \mathbf{h}_i\mathbf{o} + \mathbf{n}_i \tag{3}$$

where $\mathbf{n}_i$ denotes the additive noise for the frame $\mathbf{g}_i$.

In Bayesian probabilistic framework, the solution of each equation in Eq. (3) is the maximum a posteriori (MAP) estimation of the conditional likelihood$P(\mathbf{o},\mathbf{h}_i|\mathbf{g}_i)$, i.e.,

$$(\mathbf{o},\mathbf{h}_i) = \arg\max_{(\mathbf{o},\mathbf{h}_i)} P(\mathbf{o},\mathbf{h}_i|\mathbf{g}_i) = \arg\max_{(\mathbf{o},\mathbf{h}_i)} P(\mathbf{g}_i|\mathbf{o},\mathbf{h}_i) * P(\mathbf{o}) * P(\mathbf{h}_i) \tag{4}$$

It is reasonable to assume that the solution of the multi-frame blind deconvolution is

$$\begin{aligned}(\mathbf{o},\mathbf{h}_1,\mathbf{h}_2,\cdots,\mathbf{h}_m) &= \arg\max_{(\mathbf{o},\mathbf{h}_1,\mathbf{h}_2,\cdots,\mathbf{h}_m)} \prod_{i=1}^{m} P(\mathbf{o},\mathbf{h}_i|\mathbf{g}_i) \\ &= \arg\max_{(\mathbf{o},\mathbf{h}_1,\mathbf{h}_2,\cdots,\mathbf{h}_m)} \prod_{i=1}^{m} [P(\mathbf{g}_i|\mathbf{o},\mathbf{h}_i) * P(\mathbf{o}) * P(\mathbf{h}_i)]\end{aligned} \tag{5}$$

With negative natural logarithmic operation, Eq. (5) is converted into Eq. (6)

$$(\mathbf{o},\mathbf{h}_1,\mathbf{h}_2,\cdots,\mathbf{h}_m) = \arg\min_{(\mathbf{o},\mathbf{h}_1,\mathbf{h}_2,\cdots,\mathbf{h}_m)} \left[-\sum_{i=1}^{m} \log P(\mathbf{g}_i|\mathbf{o},\mathbf{h}_i) - m\log P(\mathbf{o}) - \sum_{i=1}^{m} \log P(\mathbf{h}_i)\right] \tag{6}$$

There are three terms $-\sum_{i=1}^{m}\log P(\mathbf{g}_i|\mathbf{o},\mathbf{h}_i)$, $-\log P(\mathbf{o})$ and $-\sum_{i=1}^{m}\log P(\mathbf{h}_i)$ that need to be modeled in Eq. (6).

Suppose that $P(\mathbf{g}_1|\mathbf{o},\mathbf{h}_1)$, $P(\mathbf{g}_2|\mathbf{o},\mathbf{h}_2)$, ..., $P(\mathbf{g}_m|\mathbf{o},\mathbf{h}_m)$ are Gaussian and identically distributed, then

$$-\sum_{i=1}^{m}\log P(\mathbf{g}_i|\mathbf{o},\mathbf{h}_i) \propto \sum_{i=1}^{m}\left\|\mathbf{h}_i\mathbf{o}-\mathbf{g}_i\right\|_2^2 \tag{7}$$

where $||\bullet||_2$denotes the $l_2$-norm.

To model the term $-\log P(\mathbf{o})$, we use the natural image gradient prior which assumes that the gradients of a natural image follow a sparse probabilistic distribution. We take an example in Fig. 1 to demonstrate it.

Fig. 1(a) shows a natural image. Fig. 1(b) is the probabilistic distribution of its gradients along the horizontal direction. The authors of Ref. [18] propose to model this distribution with a function$y=\exp(-c_1|x|^p-c_2)$, where $c_1>0$ is a scale factor, $0<p<1$, $c_2$ is a constant ($c_2$ can be ignored because it does not impact the optimization). In Fig. 1(c), the red curve denotes the natural logarithm of the probabilistic distribution which is shown in Fig. 1(b), the blue curve denotes the natural logarithm of the function used to model it, i.e., $\log(y)=-0.6|x|^{0.5}-2.5$.

If we assume that the distributions of the gradients along different directions are identical, then the term $-\log P(\mathbf{o})$ can be modeled by

$$-\log[P(\mathbf{o})] = c_1\sum_{j=1}^{n}\left\|\mathbf{f}_j\mathbf{o}\right\|_p^p + n^*N^*c_2 \tag{8}$$

where $\mathbf{f}_j$ is the convolution matrix of a derivative filter such as $f_1=[-1,\ 1]$, $n$ is the total number of filters used and $N$ denotes the total number of pixels in $\mathbf{o}$. The definition for $||\bullet||_p$ is the same as $l_p$-norm, i.e., $\mathbf{x}_p=\left(\sum_k|x_k|^p\right)^{1/p}$ (where $x_k$ denotes the $k$-th element of $\mathbf{x}$), but we should pay attention that, since $0<p<1$, it is a quasi-norm, we use the sign $||\bullet||_p$ here only for simplicity.

The most obvious feature of the PSF is its sparse energy distribution, e.g., in the matrix of a motion caused PSF, most of the elements are zero. From the theory of sparse representation [35], we know