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Cooperative scattering effect between Stokes and anti-Stokes field stimulated by a stream of atoms

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ABSTRACT

This paper is devoted to the study of the cooperative two-photon scattering processes between two resonator modes stimulated by an excited atomic beam. It has been proved that these collective scattering phenomena between the Stokes and anti-Stokes resonator modes are taking place due to the energy transfer between these fields. The statistical properties of the Stokes and anti-Stokes photons have been described using the photon correlation functions. The numerical solution of this equation describes the statistical transformation of n-Stokes photons into anti-Stokes photons.

1. Introduction

The cooperative Raman processes in Λ-type three level atoms, in which the Stokes (or anti-Stokes) photons are emitted (or absorbed) by interacting with an external pump field, were analyzed in [1]. G.S. Agarwal and R.R. Puri [2], analyzed cavity quantum electrodynamics models focused on the quantum dynamics of various Raman coupling in ideal cavities. Recently, the applications of cavity quantum electrodynamics with an ensemble of Λ-type atoms have attracted a considerable interest. The combination of Raman or double Raman interactions inside optical resonators give us more opportunities to generate the squeezed light [3], entangled states [4], and the optical Fock states [5], as well as to achieve an interfacing between collective atomic excitations and single photons [6]. S.K.Y. Lee and C.K. Law [7] studied the collective quantum dynamics of photons and atoms driven by Raman transitions inside a low Q cavity. The investigation of the authors of the present paper of the cooperative effects between the Stokes and anti-Stokes fields inside the cavity with a higher Q cavity led to interesting results [8]. It is necessary to mention here that the correlation between the Stokes and anti-Stokes photons was amply studied by many scientists.

The present paper focuses on the study of the interaction of an atomic stream prepared in an excited (or ground) state (see Fig. 1) in the interaction with two cavity modes in the scattering regime, as presented in Fig. 2. As the lifetime of the atom in the cavity is considered smaller than the cooperative scattering time between the Stokes and anti-Stokes fields, we have eliminated the atomic variables and we have focused our attention on the transformation of the Stokes field into anti-Stokes field and vice versa, as the function of a prepared atomic inversion and field states. This nonlinear process takes place due to the atomic polarization that serves as a source of ignition and generation of processes of such non-stationary field transformation. Taking this process into account, that takes place with the absorption (emission) of new quanta, we have introduced the bi-boson operators describing such a quantum transformation between the scattering photons into the two cavity modes. The master equation that describes this process makes it possible to predict the behavior of quantum fluctuations in a non-stationary process of absorption and radiation of the Stokes and anti-Stokes photons. The stationary and non-stationary solutions of this master equation reveal a strong quantum correlation between the Stokes and anti-Stokes fields. This correlation function provides us with an opportunity to manipulate the quantum statistics of photons generated in the scattering field as a function of atomic inversion and cavity parameters, such as detuning, losses, and quality factor of the cavity.

2. Hamiltonian and master equation

Let us consider a stream of two-level atoms passing through a cavity, as presented in Fig. 1. We can analyse the virtual state $|i\rangle$ of a single atom situated between the excited $|e\rangle$ and the ground $|g\rangle$ states as is experimentally demonstrated in two-photon micromaser model [9]. Assuming the cavity modes to be off-resonance with the transition though the intermediary state $|i\rangle$, the two-photon

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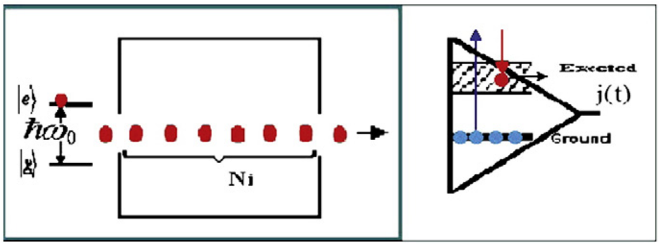

Fig. 1. Atomic pump of scattering process with transformation of n-Stokes photons in anti-Stokes field and detection possibilities.

processes between the states $|e\rangle$ and $|g\rangle$ in the cavity can be described by the equation for the density matrix

$$\frac{\partial\hat{\rho}(t)}{\partial t}=\frac{i}{\hbar}[\hat{\rho}(t),H]+\Lambda_a\hat{\rho}+\Lambda_b\hat{\rho}. \tag{1}$$

The Hamiltonian that expresses the interaction of atoms with the Stokes and anti-Stokes modes of the cavity electromagnetic field (EMF) can be represented through the atomic and field operators

$$\begin{aligned}\hat{H}=&\sum_{j=1}^{N}\hbar\omega_0\hat{R}_{zj}+\hbar\omega_a\hat{a}^{\dagger}\hat{a}+\hbar\omega_b\hat{b}^{\dagger}\hat{b}\\&+i\sum_{j=1}^{N}G(k_a,k_b)\left\{\hat{R}_j^{-}(t)\hat{J}^{+}-\hat{J}^{-}\hat{R}_j^{+}(t)\right\},\end{aligned} \tag{2}$$

where the last term represents the interaction Hamiltonian part, $\hat{R}_{zj}$ is the population inversion of the atom j; $\hat{R}_j^{+}$ and $\hat{R}_j^{-}$ are operators that describe the excitation and lowering operators between the $|g\rangle$-ground and the $|e\rangle$-excited states of the atomic subsystem, respectively (see Fig. 2). The operator $\hat{a}^{\dagger}(\hat{a})$ is the creation (annihilation) of the Stokes photons and $\hat{b}^{\dagger}\left(\hat{b}\right)$ is the creation (annihilation) of the anti-Stokes field operators. The interaction constant $G(k_a,k_b)\approx 2d_{ei}d_{gi}g_a g_s/(\hbar\delta)$ describes the effective nonlinear coupling of an atom with cavity modes k_a and k_b with the energies $\hbar\omega_a$, $\hbar\omega_b$ and the polarizations e_{λ_s} e_{λ_a}, respectively [10]; d_{ei} and d_{gi} are the transition dipoles between the $|i\rangle$ virtual, the $|e\rangle$-excited and $|g\rangle$-ground states respectively; $g_i=\sqrt{2\pi\hbar\omega_i/V}\left(\mathbf{e}_{\lambda_i},\mathbf{n}_d\right), i=s,a$ is the coupling constant with i-cavity modes, V is the cavity volume. In order to describe the scattering processes, we introduced in the Hamiltonian (Eq. (2)) the collective operators for the Stokes and anti-Stokes modes, $\hat{J}^{+}=\hat{a}\hat{b}^{\dagger}$ and $\hat{J}^{-}=\hat{b}\hat{a}^{\dagger}$, $\delta=(E_i-E_e)/\hbar-\omega_s$, where E_g, E_e and E_i are the energies of ground, excited and virtual states of the atom. The operator $\hat{J}^{+}=\hat{a}\hat{b}^{\dagger}$ describes the simultaneous process of the creation of the anti-Stokes and annihilation of the Stokes photons. The reverse process is described by operator $\hat{J}^{-}=\hat{b}\hat{a}^{\dagger}$. As within a short period of time, during interaction, the total number of photons in the cavity is conserved, we can introduce the photons inversion operator between the Stokes and anti-Stokes photons: $\hat{J}_z=\left(\hat{b}^{\dagger}\hat{b}-\hat{a}^{\dagger}\hat{a}\right)/2$ with energy

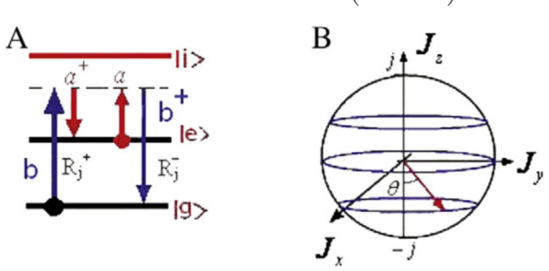

Fig. 2. Photon representation. Collective excitation of atom with absorption of Stokes photon and generation of anti-Stokes quanta, and vice versa (A). The geometric representation of the symmetry of such generation field in a phase space of the bimodal cavity (B).

difference between the modes $\hbar\tilde{\omega}=\hbar\omega_b-\hbar\omega_a$. The photon losses from the cavity are described by the second term of Eq. (1) as follows

$$\Lambda_a\hat{\rho}=\frac{k_b}{2}\left[\hat{b}\hat{\rho}(t),\hat{b}^{\dagger}\right]+\frac{k_a}{2}\left[\hat{a}\hat{\rho}(t),\hat{a}^{\dagger}\right]+h.c.$$

Here k_a and k_b are the rates of photon losses from the cavity.

Let us assume that the mean number of atoms in the cavity is N. The damping of atomic polarization can be described by the third term of Eq. (1) and we have [11–15]

$$\Lambda_b\hat{\rho}=\sum_{j=1}^{N}\left\{\frac{\gamma_{\parallel}}{2}\left(\left[\hat{R}_j^{+},\hat{\rho}\hat{R}_j^{-}\right]+\left[\hat{R}_j^{+}\rho,\hat{R}_j^{-}\right]\right)+\left(\gamma_{\perp}-\frac{\gamma_{\parallel}}{2}\right)\left(\left[\hat{R}_{jz},\hat{\rho}\hat{R}_{jz}\right]+\left[\hat{R}_{jz}\hat{\rho},\hat{R}_{jz}\right]\right)\right\}.$$

In this case the two-photon coherence de-phasing $\gamma_{\perp}$, according to [13], can be presented through the sum of two damping mechanisms of polarization $\gamma_{\perp}=\gamma_{\perp f}+\gamma_{\perp d}$, in which $\gamma_{\perp d}$ and $\gamma_{\perp f}$ correspond to the polarization damping connected with scattering processes and the mean value of flight time $\gamma_{\perp f}^{-1}$ of the atom through the cavity respectively [17]. $T_{\parallel}=\gamma_{\parallel}^{-1}$ is the effective longitudinal relaxation time, which depends on the mechanism of the creation of atomic inversion in multi-photon excitation (see, for example, [13–17]).

Let us eliminate the operators of the atomic subsystem from master Eq. (1). Assuming that in the cavity the two-photon coherence de-phasing, $\gamma_{\perp}$ and $\gamma_{\parallel}$ is longer than the generation rate of the anti-Stokes photons, we can adiabatically eliminate the atomic variables from this equation. Taking into account that the atomic system enters in the resonator in excited state (see Fig. 1), we can eliminate the atomic operators $\hat{R}_l^{+}$ and $\hat{R}_l^{-}$ from the Heisenberg equation of a mean value of an arbitrary field operator $\hat{O}(t)$

$$\frac{d}{dt}\langle\hat{O}(t)\rangle=i\tilde{\omega}\left\langle\left[\hat{J}_z(t),\hat{O}(t)\right]\right\rangle-\frac{1}{\hbar}\sum_{l=1}^{N}G(k_a,k_b)\left\{\left\langle\hat{R}_l^{-}(t)\left[\hat{J}^{+}(t),\hat{O}(t)\right]-\left[\hat{J}^{-}(t),\hat{O}(t)\right]\hat{R}_l^{+}\right\rangle(t)\right\}. \tag{3}$$

According to the master Eq. (1), the Heisenberg equations for operators $\hat{R}_l^{+}$, $\hat{R}_l^{-}$ and $\hat{R}_z$ can be presented in the following way

$$\frac{d\hat{R}_{j,s}^{\pm}(t)}{dt}=i(\pm\omega_0-\gamma_{\perp})\hat{R}_{j,s}^{\pm}\pm\frac{2}{\hbar}G(k_a,k_b)\hat{R}_z^{j}(t)\hat{J}^{\pm}(t);$$

$$\frac{d\hat{R}_{zj}}{dt}=-\gamma_{\parallel}\left(\hat{R}_{zj}-1/2\right)-\frac{2G(k_a,k_b)}{\hbar}\left\{\hat{R}_l^{-}(t)\hat{J}^{+}(t)+\hat{J}^{-}(t)\hat{R}_l^{+}(t)\right\}.$$

The solutions of the former equation can be given by the free $\hat{R}_{j,f}^{\pm}(t)$ and source $\hat{R}_{j,s}^{\pm}(t)$ parts of the atomic operators: $\hat{R}_j^{\pm}(t)=\hat{R}_{j,f}^{\pm}(t)+\hat{R}_{j,s}^{\pm}(t)$, where

$$\begin{aligned}&\hat{R}_{j,f}^{\pm}(t)=\hat{R}_j^{\pm}(0)exp[\pm i\omega_0 t-\gamma_{\perp}t];\\&\hat{R}_{j,s}^{\pm}(t)=\pm\frac{2}{\hbar}G(k_a,k_b)\int_0^t d\tau\ exp[\pm i\omega_0\tau-\gamma_{\perp}\tau]\hat{R}_z^{j}(t-\tau)\hat{J}^{\pm}(t-\tau).\end{aligned} \tag{4}$$

Fig. 2A and B show atomic transitions with the absorption of the Stokes photon and the generation of the anti-Stokes quantum and vice versa, the symmetry of those transitions, according to bimodal representation of such generation field, can be reduced to *SU(2)* algebra representation (see Fig. 2B). In order to obtain an equation for the EMF of the cavity, it is necessary to eliminate the free parts of atomic operators. It is easy to understand that the free part of these operators is simply eliminated, when the atomic system is in an excited state. Moreover, when atoms are in an excited state it is necessary to permute the operators $\hat{R}_l^{+}$ and $\hat{R}_l^{-}$ according to the definition of the anti-normal correlation product in Eq. (3), so that the free part of these operators gives zero contributions $\hat{R}_{lf}^{+}(t)|e>=\hat{R}_l^{+}(0)|e>exp[-i\omega_0 t]=0$; $<e\left|\hat{R}^{-}(t)=<e\right|\hat{R}_{l,f}^{-}(0)exp[i\omega_0 t]=0$. If $\hat{O}(t)$ is an arbitrary operator of the cavity field subsystem, the mean values of the products are