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The method of the boundary diffraction wave for impedance surfaces

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ABSTRACT

The line integral of the boundary diffraction wave theory is extended for the diffraction process of waves by the impedance surfaces with edge discontinuities. With this aim, the exact diffraction field expression of Maliuzhinets is transformed into a line integral. The method is applied to the scattering problems of waves by a spherical reflector with edge discontinuity and the diffracted fields are evaluated asymptotically. The resultant expressions of the waves are examined numerically.

1. Introduction

The theory of the boundary diffraction wave (BDW) is based on the qualitative ideas of Young [1] and is an important method for the analysis of the diffraction effects by optical devices [2–5]. According to Young, the scattered field by an obstacle consists of the interference of two sub-fields. The first one is the geometrical optics (GO) waves, which propagates unaffected by the scatterer and has a discontinuity at the geometrical shadow. The second component is the boundary diffraction field that is radiated by the edge or wedge discontinuity of the obstacle and compensates the GO wave at the transition region. For this reason, the diffracted wave has a phase shift of 180° at the shadow boundary. The ideas of Young were formulated by Maggi [6] and Rubinowicz [7] independently. They managed to separate the diffraction integral of Kirchhoff [8], which represented the total scattered field by an obstacle, into two parts as the GO and diffracted field components. The diffracted field was expressed in terms of a line integral over the edge discontinuity of the scatterer. The line integral of Maggi and Rubinowicz was valid for the plane and spherical wave incidence. Miyamoto and Wolf extended the theory of BDW for arbitrary waves [9,10]. Rubinowicz investigated in detail the method of BDW and proposed two simple models for the arbitrary incidence in the review paper [11].

However the actual form of the BDW theory, developed by the authors mentioned above, has two important defects. First of all the line integral of BDW is not uniform at the shadow boundary, because the diffracted field approaches to infinity at this region [12]. This problem was first realized and mentioned by Rubinowicz [13]. In the related work, he offered a method for the uniformization of the BDW line integral. Thus Rubinowicz was the first person, who developed a uniform theory of diffraction, although this fact is not well known in the literature [14]. We also obtained a uniform line integral representation of the BDW theory, in its actual form developed by Maggi–Rubinowicz, by using an asymptotic relation of the Fresnel function [15].

The second important defect of the actual theory is the incorrect diffraction fields that are obtained by the line integrals of Maggi and Rubinowicz. This problem occurs because the line integrals of BDW method are based on the diffraction integral of Kirchhoff. The integral of Kirchhoff yields incorrect edge diffraction contributions like the method of physical optics (PO) [16,17]. We outlined the reasons of the incorrect diffraction contributions of the PO and Kirchhoff integrals and developed a new method, which gives the exact diffracted waves by conducting surfaces [18,19]. Based on the developed method, we reintroduced the line integral of the BDW theory [20] and its uniform representation [21,22]. The recent developments about the theory of BDW can be found in Ref. [23].

The line integral of BDW theory desires a further improvement, which is related with the boundary conditions of the scatterer's surface. In reality, the obstacle absorbs a portion of the incident wave and the power of the reflected wave is less than that of the incident field. Such surfaces are modeled by the impedance boundary condition [24]. For example a metallic surface, coated with a dielectric layer, can be represented by the impedance boundary condition. The first solution of the diffraction problem of waves by an impedance half-plane was put forward by Senior with the method of Wiener–Hopf factorization [25]. In 1960, Maliuzhinets obtained a more general solution of the scattering problem for a wedge with different face impedances [26]. This solution also includes the case of the half-plane. Maliuzhinets used the mathematical theory of diffraction, developed by Sommerfeld [27]. Thus the actual line integral of the BDW theory cannot be used for the analysis of the diffracted fields by the impedance surfaces. In 2008, we investigated the effect of the impedance boundary conditions on the line integral of the diffraction fields [28]. In that study, we used a PO based solution of the

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half-plane problem, which was in harmony with the diffracted field expression of Senior [29].

The aim of this paper is to introduce a more general line integral of the BDW theory that will include the diffraction waves by edge discontinuities with different face impedances. Thus we will transform the edge diffraction field of Maliuzhinets into a line integral by using a method, developed by us for wedge diffraction problems of PO [30]. The new integral will be applied to the scattering problem of waves by a spherical reflector with edge discontinuity. The diffracted waves will be evaluated asymptotically and the resultant fields will be examined numerically.

A time factor of $exp(j\omega t)$ is taken into account and suppressed throughout the paper. ω is the angular frequency. The cylindrical and spherical coordinates are represented by (ρ,ϕ,z) and (r,θ,ϕ) respectively.

2. Theory

We take into account a semi-infinite half-plane, the geometry of which is given in Fig. 1, and is located at the plane of $S=\{(x,y,z);x\in(0,\infty),y=0,z\in(-\infty,\infty)\}$.

An arbitrary incident field of $u_i(P)$ is illuminating the screen. P is the observation point. β and ϕ_0 are the angles of scattering and incidence. The diffracted fields by a general edge contour can be given by

$$u_d(P)=\frac{1}{2\pi}\oint_C \vec{W}\cdot d\vec{l} \tag{1}$$

where $\vec{W}$ is the potential function that can be defined by the equation of

$$\vec{W}=u_i(Q_e)f(l)\frac{\exp(-jkR_E)}{R_E}\vec{e}_l \tag{2}$$

for Q_e is a point on the diffraction edge C. l is the variable on the edge contour. $f(l)$ is a function to be determined. R_E is the distance between the diffraction and observation points. $\vec{e}_l$ is the unit vector along the edge contour. The effect of the boundary condition is included in $f(l)$. Eq. (1) can be rewritten as

$$u_d(P)=\frac{1}{2\pi}\int_{-\infty}^{\infty} u_i(z')f(\beta)\frac{\exp(-jkR_E)}{R_E}dz' \tag{3}$$

for the half-plane. Note that the edge contour of the half-screen is at $z'\in(-\infty,\infty)$. R_E is equal to $\sqrt{\rho^2+(z-z')^2}$. Now we propose that the half-plane is illuminated by the plane wave of

$$u_i(P)=u_0\exp\left[jk(x\cos\phi_0+y\sin\phi_0)\right] \tag{4}$$

where u_0 is the complex amplitude. The line integral of BDW becomes

$$u_d(P)=\frac{u_0}{2\pi}\int_{-\infty}^{\infty} f(\beta)\frac{\exp(-jkR_E)}{R_E}dz' \tag{5}$$

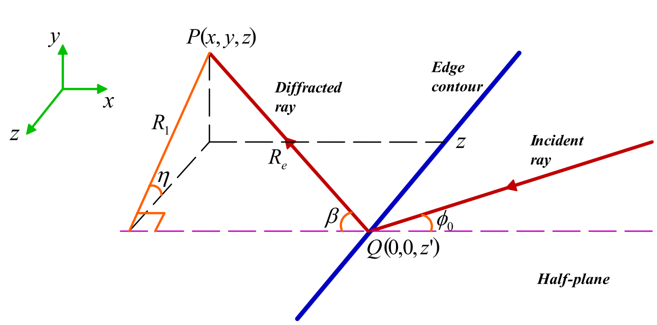

Fig. 1. Geometry of the half-screen.

in this case. The integral, in Eq. (5), can be evaluated directly by using the method of the stationary phase [16,18], since its limits vary between ± infinity. The stationary phase point is evaluated by equating the first derivative of the phase function to zero. Its value is found to be $z_s=z$ for our case. As a result the diffracted field can be evaluated as

$$u_d=u_0\frac{\exp(-j\pi/4)}{\sqrt{2\pi}}f(\pi-\phi)\frac{\exp(-jk\rho)}{\sqrt{k\rho}}, \tag{6}$$

because the stationary phase value of β is $\pi-\phi$ at $z_s=z$ according to Fig. 1. The BDW line integral can be constructed by determining the value of f for various types of diffraction problems with different boundary conditions. In this paper, we will construct the line integral for the impedance surfaces. Maliuzhinets obtained the diffracted waves by an impedance half plane as

$$u_d=\frac{\exp(-j\pi/4)}{\sqrt{2\pi}}\frac{M(\phi,\phi_0,\eta_\mp)}{\cos\phi+\cos\phi_0}\frac{\exp(-jk\rho)}{\sqrt{k\rho}} \tag{7}$$

for the incident wave, given by Eq. (4) [26]. $\eta_\mp$ can be defined by

$$\eta_\mp=\sin^{-1}\frac{Z_0}{Z_\mp} \tag{8}$$

where Z_+ and Z_- are the impedances of the upper and lower surfaces of the half-screen. Z_0 is the impedance of the free space. The function M can be introduced as

$$M(\phi,\phi_0,\theta_\mp)=\frac{\sin\frac{\phi_0}{2}}{\psi(\pi-\phi_0)}\left[\psi(-\phi)\left(\sin\frac{\phi}{2}-\cos\frac{\phi_0}{2}\right)+\psi(2\pi-\phi)\left(\sin\frac{\phi}{2}+\cos\frac{\phi_0}{2}\right)\right] \tag{9}$$

[26,31] for $\psi(x)$ can be defined by the expression of

$$\psi(x)=\psi_\pi\left(x+\frac{3\pi}{2}-\eta_+\right)\psi_\pi\left(x+\frac{\pi}{2}+\eta_+\right)\psi_\pi\left(x-\frac{\pi}{2}-\eta_-\right)\psi_\pi\left(x-\frac{3\pi}{2}+\eta_-\right) \tag{10}$$

where $\psi_\pi(x)$ is the Maliuzhinets function, which can be written as

$$\psi_\pi(x)=\exp\left(-\frac{1}{8\pi}\int_0^x\frac{\pi\sin v-2\sqrt{2}\pi\sin\frac{v}{2}+2v}{\cos v}dv\right). \tag{11}$$

The function $f(\pi-\phi)$ can be determined as

$$f(\pi-\phi)=\frac{M(\phi,\phi_0,\eta_\mp)}{\cos\phi+\cos\phi_0} \tag{12}$$

when Eq. (6) is equated to Eq. (7). Thus $f(\beta)$ reads

$$f(\beta)=\frac{M(\pi-\beta,\phi_0,\eta_\mp)}{\cos\phi_0-\cos\beta}, \tag{13}$$

since β is $\pi-\phi$. The line integral of the BDW theory reads

$$u_d(P)=\frac{u_0}{2\pi}\int_{-\infty}^{\infty}\frac{M(\pi-\beta,\phi_0,\eta_\mp)}{\cos\phi_0-\cos\beta}\frac{\exp(-jkR_E)}{R_E}dz' \tag{14}$$

for the impedance half-plane.

We can generalize the line integral by taking into consideration the geometry, given in Fig. 2. $\vec{n}_e$ is a unit vector that is perpendicular to the edge point at Q_E. $\vec{s}_i$ and $\vec{s}_d$ are the unit vectors, in the directions of the incident and diffracted rays. α is the angle of incidence. The relations of

$$\cos\alpha=\vec{s}_i\cdot\vec{n}_e \tag{15}$$