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Exact diffraction calculation from fields specified over arbitrary curved surfaces

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ABSTRACT

Calculation of the scalar diffraction field over the entire space from a given field over a surface is an important problem in computer generated holography. A straightforward approach to compute the diffraction field from field samples given on a surface is to superpose the emanated fields from each such sample. In this approach, possible mutual interactions between the fields at these samples are omitted and the calculated field may be significantly in error. In the proposed diffraction calculation algorithm, mutual interactions are taken into consideration, and thus the exact diffraction field can be calculated. The algorithm is based on posing the problem as the inverse of a problem whose formulation is straightforward. The problem is then solved by a signal decomposition approach. The computational cost of the proposed method is high, but it yields the exact scalar diffraction field over the entire space from the data on a surface.

1. Introduction

When the input field is specified over a planar surface, the calculation of monochromatic scalar optical diffraction can be accomplished in a straightforward manner by plane wave decomposition or the Rayleigh–Sommerfeld diffraction integral, or by other methods derived from these. Integration over the planar surface allows computation of the exact diffraction field over the entire space. However, if the input field is specified over a curved surface, rather than a planar surface, straightforward integration over the curved surface may not provide the exact field over the entire space. Calculation of the exact diffraction field from a curved surface requires greater care and is the subject of this work [16].

Diffraction field calculation by direct integration over the surface on which the input field is specified, is essentially a weighted superposition of the free-space diffraction kernel. However, direct integration gives the exact field only when the integrated surface field value remains unaltered by the propagation from other surface elements. If we simply ignore such mutual interactions, the calculated field will be different from the actual field. The method we set forth is based on the following observation. No such interactions exists when the input field is specified over a plane; therefore it is straightforward to express the field on an arbitrary curved surface (and indeed any region of the entire space) as a weighted superposition integral of the free-space diffraction kernel over a planar surface. In the problem we wish to solve, the field is known over a curved surface and we wish to obtain the field over a planar surface (which would then also enable us to calculate it over the entire space). Since it is not straightforward to express the field on the planar surface in terms of the field on the curved surface, we express the field on the curved surface in terms of that on the planar surface, and solve an inverse problem to obtain the field on the planar surface. The inverse problem arising from this exact formulation can be solved by employing several methods and standard algorithms, each with their pros and cons. In this paper, we propose a signal decomposition algorithm for this purpose.

Our interest in diffraction calculations from curved surfaces stems from our work on computer generated holography (CGH) and three-dimensional imaging and television [1–4,11,17,20–23]. Since the diffraction field from an arbitrarily shaped object is the field that we desire to recreate at the display end, its accurate calculation is of utmost importance.

In both computer graphics and CGH, objects are commonly modeled as a set of sample points distributed over space [8,9,14,15]. It is assumed that the characteristics of the continuous object can be sufficiently represented by these sample points. A straightforward approach to compute the diffraction field created by an object is to superpose the fields created by each sample point of the object; doing so amounts to treating each sample point as a light source. We will refer to diffraction field calculation approaches based on superposition of the fields at each sample point of the object as “source model” approaches. In these approaches, it is assumed that the value of each source is independent of the field at other points. Then, the independently computed fields from these points are superposed. The calculated field will be the same as the actual field only if the points truly act as sources (i.e., if the values of these sources are not perturbed by the superposed field generated by the other sources). However, usually there are complicating interactions. Consequently, the field calculated using the source model will not be exact or may even be significantly in error. Diffraction field calculations based on the source model have the advantage of having reasonable

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computational complexities, but they are not necessarily exact except when all the sample points are given over a planar surface.

With the term mutual interaction we refer to the fact that the field at a given input point is not independent of the field at the other input points; in other words, it is not possible to specify them independently and arbitrarily.

Ignoring the mutual interactions and straightforwardly superposing the specified input field values will not give exact results. Instead, a simultaneous calculation of the diffraction field due to the given input points is necessary. We will refer to approaches based on such simultaneous calculation of the diffraction field as "field model" approaches. The diffraction field computation method presented in this paper is based on such an approach and uses a decomposition of the field specified over an orientable manifold onto a function set obtained from the intersection of the propagating plane waves by the manifold.

The algorithm we propose can be used for both two-dimensional (2D) and three-dimensional (3D) spaces. For simplicity we will first discuss the 2D case. In the 3D case, numerical issues due to larger data sets arise. Nevertheless, as a proof of concept the extension of the proposed algorithm to the 3D case is also presented.

2. Calculation of the diffraction field using the source model

Computation of the diffraction field arising from the samples of an object or a set of given sample points over the space can be performed in several ways. One of the most commonly employed methods is to superpose the fields emitted by the sample points. As discussed in Section 1, we refer to such methods as source model methods. In the literature, there are several diffraction field computation algorithms based on the source model approach [5,8,9,13]. Implementation of source model algorithms is rather straightforward because mutual interactions are not taken into consideration.

Depending on the distribution of the sample points over the space, the effect of mutual interactions can be significant. A simple example will help illustrate the issues involved (Fig. 1). We consider three points P_1, P_2, P_3 which create the field, $\psi(\mathbf{x})$, over the entire space. Here the coordinate vector $\mathbf{x}$ denotes $[x \quad z]^T$. According to the source model approach, the field over the reference line is computed by superposing the fields created by the field samples at $P_1=\mathbf{x}_1$, $P_2=\mathbf{x}_2$, $P_3=\mathbf{x}_3$ (which are assumed as sources). The diffraction field emanating from these sample points is usually calculated by using the kernel of the Rayleigh–Sommerfeld (RS) diffraction integral. More specifically, the field $f(\mathbf{x})$ over the line $z=0$ is expressed in terms of the strengths $\psi(\mathbf{x}_1)$, $\psi(\mathbf{x}_2)$, $\psi(\mathbf{x}_3)$ of the sources as

$$f(\mathbf{x}) = \psi(\mathbf{x}_1)h(\mathbf{x}-\mathbf{x}_1) + \psi(\mathbf{x}_2)h(\mathbf{x}-\mathbf{x}_2) + \psi(\mathbf{x}_3)h(\mathbf{x}-\mathbf{x}_3) \tag{1}$$

where $h(\mathbf{x})$ denotes the 2D version (i.e., $y=0$ case) of the RS diffraction kernel due to propagating waves,

$$h(\mathbf{x}) = \frac{1}{j\lambda}\frac{\exp\left(j\frac{2\pi}{\lambda}\sqrt{x^2+y^2+z^2}\right)}{\sqrt{x^2+y^2+z^2}}\cos\theta, \tag{2}$$

where $\cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$ and λ is the optical wavelength. The above expression gives the field on the reference plane arising from these three points. Quite often, the $\cos\theta$ term is ignored; omitting this term may result in significant errors if θ is not small. It is instructive to note that the Rayleigh–Sommerfeld kernel is the impulse response for field computations where the wave propagates out from a planar surface; this is different than a spherically symmetric propagation out from a simple free-standing point source. Using the expression we have obtained for the field on the reference line, we may now calculate back the field values at P_1, P_2, P_3 again by using the RS diffraction integral. In general it turns out that the values obtained are not equal to the original values specified at P_1, P_2, P_3. Thus even with such a simple scenario, it is possible to see the effect of the interactions between the specified source points. For instance, the deviation between the calculated field and the initially specified field at P_1 is found as

$$\Delta\psi(\mathbf{x}_1) = \psi(\mathbf{x}_2)\frac{\cos\theta_{1,2}}{|\mathbf{r}_{1,2}|}\exp\left(j\frac{2\pi}{\lambda}|\mathbf{r}_{1,2}|\right) + \psi(\mathbf{x}_3)\frac{\cos\theta_{1,3}}{|\mathbf{r}_{1,3}|}\exp\left(j\frac{2\pi}{\lambda}|\mathbf{r}_{1,3}|\right) \tag{3}$$

where $\mathbf{r}_{1,2}$ is the vector between points P_1 and P_2, and $\theta_{1,2}$ denotes the angle between the vector $\mathbf{r}_{1,2}$ and the z-axis as shown in Fig. 1. Similarly, the vector $\mathbf{r}_{1,3}$ denotes the difference between the position vectors $\mathbf{x}_1$ and $\mathbf{x}_3$, and $\theta_{1,3}$ is the angle between the vector $\mathbf{r}_{1,3}$ and the z-axis. This deviation is exactly the additional field imposed on P_1 by the sources at P_2 and P_3 under the RS model. A similar deviation can be shown also for P_2 or P_3. These deviations from the initially specified fields at each sample point depend on the initial field values on the other sample points and their mutual positions in space. As a result of these interactions between the fields emanating from the sources, the source model approach may not provide the exact field over the entire space.

For a discrete set of points, there will be no mutual interactions among the sample points if the following condition is satisfied:

$$\psi(\mathbf{x}_i) = \sum_{\substack{j \\ j\neq i}} \psi\left(\mathbf{x}_j\right)h\left(\mathbf{x}_i-\mathbf{x}_j\right) = 0, \ \forall i \tag{4}$$

where $\psi(\mathbf{x}_j)h(\mathbf{x}_i-\mathbf{x}_j)$ is the field generated at location $\mathbf{x}_i$ by point j. This is satisfied for the classical case of diffraction computation from the points which lie on a plane.

By the way, above observations are still valid if other diffraction models, (for example, the Fresnel–Kirchoff approximation) are utilized instead of the RS formulation.

In the source model, the computation of the continuous diffraction field over the entire space can be expressed as an integral over a surface S_a as

$$\psi(\mathbf{x}) = \int_{S_a} \psi(\mathbf{x})h(\mathbf{x}-\mathbf{x})dS \tag{5}$$

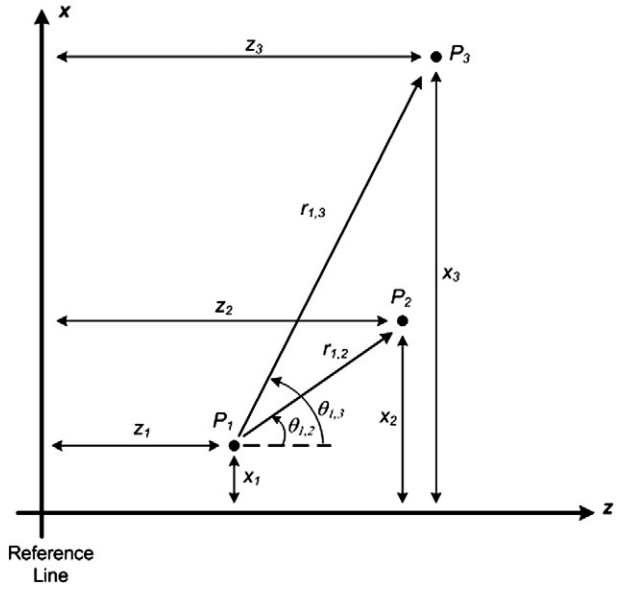

Fig. 1. Illustration of possible mutual couplings in the source model approach for a 2D space. The x-axis is taken as the reference line.