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Analytically vectorial structure of an apertured Laguerre–Gaussian beam in the far-field

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ABSTRACT

Based on the angular spectrum representation of an arbitrary electromagnetic beam and the method of stationary phase, an analytically vectorial structure of an apertured Laguerre–Gaussian beam in the far-field has been derived without any approximation. The analytical expressions of the energy flux of the TE term, the TM term, and the apertured Laguerre–Gaussian beam are also presented in the far-field, respectively. The energy flux distributions of the TE term, the TM term, and the apertured Laguerre–Gaussian beam are numerically demonstrated in the far-field reference plane. The influences of the *f*-parameter, the truncation parameter, the radial and angular mode numbers, and the dependent relation of angle on the energy flux distributions in the far-field of the TE term, the TM term, and the apertured Laguerre–Gaussian beam are also discussed in detail.

1. Introduction

The cylindrically symmetric higher-order modes of laser cavities with spherical mirrors are Laguerre–Gaussian beams. As apertures usually appear in the practical optical systems, the apertured Laguerre–Gaussian beams receive considerable interest. Diffraction of Laguerre–Gaussian beams by an aperture was early examined in 1972 [1]. The propagation and diffraction of apertured Laguerre–Gaussian beams have been evaluated by means of the generalized Huygens–Fresnel integral [2]. A simple analytic expression to evaluate the far-field diffraction pattern of a general high-order Laguerre–Gaussian beam from a circular aperture has been presented [3]. The relative phase shift of Laguerre–Gaussian beams through an apertured paraxial optical *ABCD* system has been investigated, and the dependence of the relative phase shift on the beam and system parameters has been numerically illustrated [4]. That the circular aperture decreases the M^2 factor of a high-order symmetrical Laguerre–Gaussian beam and enhances its brightness has been demonstrated [5]. Based on the complex Gaussian expansion of the hard-edged-aperture function, approximately analytic expressions for the output-field of standard and elegant Laguerre–Gaussian beams through apertured fractional Hankel transform systems have been derived [6]. The approximately analytical expressions of standard and elegant Laguerre–Gaussian beams through an annular apertured paraxial *ABCD* optical system have also been presented, respectively [7,8]. On the basis of the truncated second-order moments method in the cylindrical coordinate systems, closed-form expressions for the generalized M^2 factor of truncated standard and elegant Laguerre–Gaussian beams have respectively proposed [9]. The kurtosis parameter of standard and elegant Laguerre–Gaussian beams passing through apertured optical systems has been calculated [10]. Based on the relations between Laguerre–Gaussian and Hermite–Gaussian beams, the approximately analytical propagation equations of the rotational symmetrical Laguerre–Gaussian beams along with their even and odd modes through a paraxial *ABCD* optical system with rectangular hard-edged aperture have been derived [11]. By expanding the Bessel function J_0 appearing in the Fresnel–Kirchhoff integral into a finite sum of complex Gaussian functions, an analytical expression for a Laguerre–Gaussian beam diffracted through a hard-edge aperture has been derived [12].

To further examine the propagation properties of an apertured Laguerre–Gaussian beam, the vectroial structure of an apertured Laguerre–Gaussian beam is investigated in the remainder of this paper. Here the apertured Laguerre–Gaussian beam is described by the angular spectrum representation of the solutions of the Maxwell's equations. The angular spectrum representation of an arbitrary electromagnetic beam can be uniquely expressed as a sum of the TE and the TM terms [13–17]. The TE term denotes the electric field transverse to the propagation axis, and the TM term means the associated magnetic field transverse to the propagation axis. The outcome is that an apertured Laguerre–Gaussian beam is also decomposed into the TE and the TM terms. Many researches and applications are conducted in the far-field. Moreover, the TE and the TM terms are orthogonal to each other in the far-field. By means of the method of stationary phase that used in Ref. [18], therefore, the analytical expressions of the TE and the TM terms of an apertured Laguerre–Gaussian beam are to be presented in the far-field. The

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corresponding energy flux distributions of the TE term, the TM terms, and the apertured Laguerre–Gaussian beam are also to be investigated in the far-field.

2. Analytically vectorial structure in the far-field

In the Cartesian coordinate system, the Laguerre–Gaussian beam propagates toward half free space $z \geq 0$. The z-axis is taken to be the propagation axis. As Laguerre–Gaussian beams are usually treated to be linearly polarized, the Laguerre–Gaussian beam in the source plane $z=0$ is described by

$$\begin{bmatrix} E_x(\rho_0,0) \\ E_y(\rho_0,0) \end{bmatrix} = \begin{bmatrix} \left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp\left(-\frac{\rho_0^2}{w_0^2}\right)\cos(m\theta_0) \\ 0 \end{bmatrix}, \quad (1)$$

where w_0 is the Gaussian waist, and $L_n^m(.)$ is the associated Laguerre polynomial. n and m are the radial and angular mode numbers. $\rho_0 = (\rho_0, \theta_0)$, $\rho_0 = (x_0^2 + y_0^2)^{1/2}$, and $\theta_0 = \tan^{-1}(y_0/x_0)$. Here the dependence of the Laguerre–Gaussian beam on the angle θ_0 is $\cos(m\theta_0)$. A circular aperture with radius R coincides with the beam waist plane of the Laguerre–Gaussian beam. The Laguerre–Gaussian beam just behind the circular aperture is expressed as

$$\begin{bmatrix} E_x(\rho_0,0) \\ E_y(\rho_0,0) \end{bmatrix} = \begin{bmatrix} \left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp\left(-\frac{\rho_0^2}{w_0^2}\right)\cos(m\theta_0) \\ 0 \end{bmatrix}\mathrm{circ}(\zeta), \quad (2)$$

where $\zeta = \rho_0/R$. $circ(\zeta)$ is the aperture function and given by

$$circ(\zeta) = \begin{cases} 1 & 0 \leq \zeta < 1 \\ 0 & \zeta \geq 1 \end{cases}. \quad (3)$$

According to the vector angular spectrum representation of an arbitrary electromagnetic beam, the propagating electric field of the apertured Laguerre–Gaussian beam in the z-plane can be written as

$$E(\rho,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_x(p,q)\left(e_x - \frac{p}{\gamma}e_z\right)\exp[ik(px+qy+\gamma z)]dpdq, \quad (4)$$

where $\rho = (\rho, \theta)$, $\rho = (x^2+y^2)^{1/2}$, $\theta = \tan^{-1}(y/x)$, $\gamma = (1-p^2-q^2)^{1/2}$, and $k = 2\pi/\lambda$ is the wave number with λ the optical wavelength. $\boldsymbol{e}_x$ and $\boldsymbol{e}_z$ are the two unit vectors in the x- and z-directions, respectively. $A_x(p, q)$ is the x component of the vector angular spectrum and given by the Fourier transformation of the x component of initial electric field

$$\begin{aligned} A_x(p,q) &= \frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(\rho_0,0)\exp[-ik(px_0+qy_0)]dx_0dy_0 \\ &= \frac{k}{\lambda}\int_0^R \left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp\left(-\frac{\rho_0^2}{w_0^2}\right)J_m(k\rho_0 b)\cos(m\varphi)\rho_0 d\rho_0, \end{aligned} \quad (5)$$

where J_m is the mth-order Bessel function of the first kind. $b = (p^2+q^2)^{1/2}$ and $\varphi = \tan^{-1}(p/q)$. In the above integral, the following formula is used [19]:

$$J_m(k\rho_0 b) = \frac{1}{2\pi}\int_0^{2\pi}\exp\left[ik\rho_0 b\cos(\theta-\varphi) + im\left(\theta-\varphi-\frac{\pi}{2}\right)\right]d\theta. \quad (6)$$

According to the theorem of the vectorial structure of an electromagnetic beam, the propagating electric field of the apertured Laguerre–Gaussian beam can be expressed as a sum of the TE and the TM terms [13–17]**:**

$$E(\rho,z) = E_{TE}(\rho,z) + E_{TM}(\rho,z), \quad (7)$$

with $E_{TE}(\rho,z)$ and $E_{TM}(\rho,z)$ given by

$$E_{TE}(\rho,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{q}{b^2}A_x(p,q)\left(qe_x - pe_y\right)\exp[ik(px+qy+\gamma z)]dpdq, \quad (8)$$

$$E_{TM}(\rho,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{p}{\gamma b^2}A_x(p,q)\left(p\gamma e_x + q\gamma e_y - b^2 e_z\right) \times \exp[ik(px+qy+\gamma z)]dpdq, \quad (9)$$

where $\boldsymbol{e}_y$ is the unit vector in the y-direction. Similarly, the corresponding magnetic field of the apertured Laguerre–Gaussian beam can also be expressed as a sum of the TE and the TM terms [13–17]:

$$H(\rho,z) = H_{TE}(\rho,z) + H_{TM}(\rho,z), \quad (10)$$

with $H_{TE}(\rho,z)$ and $H_{TM}(\rho,z)$ given by

$$H_{TE}(\rho,z) = \eta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{q}{b^2}A_x(p,q)\left(p\gamma e_x + q\gamma e_y - b^2 e_z\right) \times \exp[ik(px+qy+\gamma z)]dpdq, \quad (11)$$

$$H_{TM}(\rho,z) = -\eta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{p}{\gamma b^2}A_x(p,q)\left(qe_x - pe_y\right) \times \exp[ik(px+qy+\gamma z)]dpdq, \quad (12)$$

where $\eta = (\varepsilon_0/\mu_0)^{1/2}$. ε_0 and μ_0 are the electric permittivity and the magnetic permeability of vacuum, respectively. Here, the TE and TM terms denote that the longitudinal components of the electric and magnetic fields are equal to zero, respectively. As the divergence condition of the electric field should be satisfied and the polarized direction of every plane wave component must be perpendicular to its own wave vector, the TE and TM terms of the apertured Laguerre–Gaussian is unique.

Inserting Eq. (5) into Eq. (8), the TE term of the propagating electric field for the apertured Laguerre–Gaussian beam yields

$$E_{TE}(\rho,z) = \frac{k}{\lambda}\int_0^R\left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp\left(-\frac{\rho_0^2}{w_0^2}\right)U(\rho_0,\rho,z)\rho_0 d\rho_0, \quad (13)$$

with $U(\rho_0,\rho,z)$ given by

$$U(\rho_0,\rho,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{q}{b^2}J_m(k\rho_0 b)\cos(m\varphi)\left(qe_x - pe_y\right) \times \exp[ik(px+qy+\gamma z)]dpdq. \quad (14)$$

In the far-field regime, the condition $k(\rho^2+z^2)^{1/2} \to \infty$ is satisfied. Accordingly, the method of stationary phase is applicable to the far-field. By means of the method of stationary phase [20], Eq. (14) can be analytically expressed as

$$U(\rho_0,\rho,z) = -\frac{i\lambda yz}{\rho^2 r^2}J_m\left(\frac{k\rho_0\rho}{r}\right)\cos(m\theta)\exp(ikr)\left(ye_x - xe_y\right), \quad (15)$$

where $r = (\rho^2+z^2)^{1/2}$. To obtain the analytical expression of the TE term of the propagating electric field, the mth-order Bessel function of the first kind and the associated Laguerre polynomial should be expanded as follows [19]:

$$J_m\left(\frac{k\rho\rho_0}{r}\right) = \sum_{l=0}^{\infty}\frac{(-1)^l(k\rho\rho_0)^{2l+m}}{2^{2l+m}l!(l+m)!r^{2l+m}}, \quad (16)$$

$$L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right) = \sum_{s=0}^{n}\frac{(n+m)!}{s!(n-s)!(m+s)!}\left(\frac{-2\rho_0^2}{w_0^2}\right)^s. \quad (17)$$