Optics Communications 283 (2010) 5016–5020

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Nonlinear TM modes in cylindrical liquid crystal waveguide

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ARTICLE INFO

Article history:
Received 10 January 2010
Received in revised form 25 May 2010
Accepted 20 July 2010

Keywords:
Nonlinear optics
Waveguide
Liquid crystal
Nematics

ABSTRACT

We consider a nonlinear cylindrical fiber with a nematic liquid crystal (LC) core having initially the escaped configuration and subject to the action of a normal mode propagating electromagnetic field of arbitrary intensity. We derive a set of coupled equations governing the nonlinear dynamics of the electromagnetic field and the confined LC. We solve numerically these coupled equations and find simultaneously the distorted textures of the nematic inside the cylinder and the Transverse Magnetic modes in the guide. We analyze the dependence of these solutions on the electromagnetic field intensity by assuming consistently soft boundary conditions. We have found a dramatic correlation in the spatial distribution of the nematic's configuration and the Transverse Magnetic modes. Thus, the director adopts configurations completely guided by the specific mode involved. We show that the cut-off frequencies and dispersion relations can be tuned by varying the intensity of the electromagnetic propagating field.

1. Introduction

The giant optical nonlinear response – a factor of 6–10 orders of magnitude larger [1,2] than that of doped glasses – makes feasible to consider the deformation provoked in the nematic fiber by the same electromagnetic field which is propagated. Several systems for which a nematic is submitted to a large intensity optical field whose configuration is not anchored to waveguide boundary conditions, give rise spatial patterns and solitons [3–5]. The basic mechanism which governs these time independent patterns is the balance between the nonlinear refraction (self-focussing) and the spatial diffraction of the nematic. A study of these experiments using separation of scales [6,7] shows that the field amplitude at the center of a Gaussian beam (inner solution), follows a nonlocal non linear Schrödinger equation which is able to describe the undulation and filamentation observed in the experiments.

Laser beam propagation in azobenzene liquid crystals in wave-guiding configuration has been considered in [8]. They found that spatial solitons can be formed at microwatt power levels of a He–Ne laser beam and they analyzed several well-known processes of nonlinear propagation such as undulation of solitons, their interaction and merging. *Cis-trans* isomerization of azobenzene molecules and related change in the LC order parameter is the underlying mechanism of optical nonlinearity that makes possible formation of solitons. In [9], it is shown that optical reorientation nonlinearity in twisted nematic liquid crystalline waveguides is large enough to observe spatial solitons with milliwatts of light power. It is important to stress that in general nematicons [7] and other spatial solitons found in nonlinear systems are coherent structures formed in regions of the system where both orientational and optical fields have lost influence from the boundary conditions. In this sense, all these balanced and robust profiles of energy, called solitons, are asymptotic solutions which are not to be forced by strict boundary conditions but they have to satisfy only certain mean-field matching conditions. Indeed, as long as the confining cell of the liquid crystal turns to be larger, the bias-free confinement is more notorious [10]. In this manuscript we are interested instead in analyze the role played by the boundary conditions within the optical-orientational non linear coupling of a liquid crystal cylindrical waveguide.

There are some pioneering studies in the literature [11–13] where Transverse Magnetic (TM) nonlinear modes in nematic liquid crystal waveguides of different geometries are considered. For a slab waveguide [11] and starting from Maxwell's equations and from the torque equation for the nematic director, it was derived a set of nonlinear differential equations and it was exactly solved by a numerical technique based on the continuation method for the case of planar initial alignment of the nematic director. The director reorientation induced by the guided light itself gives rise to such strong nonlinear effects as self-confinement and intrinsic optical bistability. For a cylindrical fiber [12] an iterative numerical scheme was used to determine the propagation constant as a function of optical power. This nonlinear problem is solved by using an iterative scheme in which the nematic configuration in the absence of optical fields is obtained. Then this is substituted into the wave equation to find the first approach to the optical field. The procedure is repeated to find the following approaches. In our work, we instead solve

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0030-4018/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2010.07.049

simultaneously the orientational and electromagnetic boundary value problems parametrized by the optical field intensity.

Most of the optical calculations in waveguides have been done by assuming hard anchoring boundary conditions for the nematic director. This is inconsistent with the high intensity of the propagating TM mode since in the cylinder wall the electric force can be stronger than the surface elastic force as has been shown before for this geometry [14]. Moreover, when liquid crystals are confined to small cavities, its effect is found to be significant, particularly when elastic energies imposed by the confining volume compete with molecular anchoring energies [15]. Hence we cannot ignore surface elastic terms compared with both bulk elastic terms and electric bulk contributions.

Our aim is to analyze the behavior of a nematic confined within a cylinder in which a high intensity TM mode is propagating by assuming soft anchoring boundary conditions and discern how its propagating parameters, transverse field distribution and nematic configuration depend on the mode intensity. The outline of this paper is as follows. First, we write Maxwell's equations for our system and their corresponding boundary conditions. After this, we establish the equation governing the nematic configurations within a cylinder by assuming arbitrary anchoring conditions under the action of a high intensity TM mode which propagates along the fiber. Then, we solve numerically the coupled nematic-electromagnetic field system, present and analyze our results. Finally, we address our conclusions and summarize our results.

2. Transverse magnetic field

We assume homeotropic anchoring of the nematic LC molecules at the cylinder wall. This means that the easy direction for the molecular orientation is the direction perpendicular to the wall at every point on it. However, if we have an arbitrary anchoring boundary condition, the nematic is not necessarily perpendicular at the wall. Furthermore, we restrict our model to consider the escaped radial configuration for which the director forms a certain angle relative θ to the cylinder axis. For infinite circular cylinders the symmetry implies that θ only depends on the radial distance r and the director is given by $\hat{\mathbf{n}} = sin\theta(r)\mathbf{e}_r + cos\theta(r)\mathbf{e}_z$ where $\mathbf{e}_r$ and $\mathbf{e}_z$ are the cylindrical unit vectors along the r and z directions, respectively (see Fig. 1). Now we must express the element of the dielectric tensor ϵ in terms of the perpendicular and parallel dielectric constants of the LC, $\epsilon_\perp$ and $\epsilon_\parallel$, respectively. At a point where the director forms an angle with the z-axis, the dielectric tensor in the proper coordinate system of the LC has an uniaxial form $\epsilon_{ij} = \epsilon_\perp \delta_{ij} + \epsilon_a n_i n_j$ where $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$ is the dielectric anisotropy.

As usual, TM_{lm} and TE_{lm} propagating modes are considered in studying waveguides, however, as shown in [12], for TE_{lm} modes the anisotropy and inhomogeneity of the core does not enter into Maxwell's equations. For these modes the resulting equation is equivalent to that of the well known problem of isotropic and homogeneous cylindrical waveguide [16]. Here, we only focus on TM_{lm} modes for which the amplitudes of the transverse fields are azimuthally symmetric, so that, the first parameter is taken as $l = 0$. Thus the modes in this paper will be labeled as TM_{0m}. To find the equations governing the propagation of electromagnetic waves through the nematic fiber we assume monochromatic electric $\mathcal{E}_r$, $\mathcal{E}_z$ and magnetic $\mathcal{H}_\phi$ fields propagating along the cylinder (z-axis) of the form:

$$\left(\mathcal{E}_r, \mathcal{E}_z, \mathcal{H}_\phi\right) = \left(e_r, e_z, h_\phi\right)\mathrm{Exp}[i(\beta z - \omega t)] \tag{1}$$

where the dimensionless field components are $(e_r, e_z, h_\phi) = (G_r(r,k_0), iG_z(r,k_0), F_\phi(r,k_0)/c)exp(-if)/E_0$ and E_0 is a normalization amplitude. Here we have explicitly separated the phase f and real valued amplitudes $G_z(r,k_0), F_\phi(r,k_0)$ of the mode components to simplify the resulting equations. Inserting these expressions into Maxwell's equations and separating real and imaginary parts we find [12,17]

$$\frac{dG_z}{dx} = -k_0 R \frac{\epsilon_{rr} - p^2}{\epsilon_{rr}} F_\phi, \tag{2}$$

$$G_z = \frac{1}{k_0 R \epsilon_\parallel \epsilon_\perp} \frac{\epsilon_{rr}}{x} \frac{d\left(xF_\phi\right)}{dx}, \tag{3}$$

$$G_r = \frac{pF_\phi}{\epsilon_{rr}} - \frac{i\epsilon_{rz}}{\epsilon_{rr}} G_z, \tag{4}$$

$$\frac{df}{dx} = pk_0 R \epsilon_{rz} / \epsilon_{rr}, \tag{5}$$

where $x \equiv r/R$, R is the cylinder radius and $p \equiv \beta/k_0$, being β the propagation constant. Notice that Eqs. (2) and (3) define a self-adjoint equation for F_ϕ so that their eigenvalues p are real, whereas Eq. (5) provides a phase proportional to the only non diagonal entry of ϵ.

To solve exactly the TM_{0m} modes we shall assume that the nematic cylinder is surrounded by an infinite homogeneous and isotropic cladding of dielectric constant ϵ_c. In this way the electromagnetic fields should satisfy the boundary conditions:

$$h_\phi(x = 0, k_0) = 0, \tag{6}$$

$$h_\phi(x = 1, k_0) = h^c_\phi(x = 1, k_0), e_z(x = 1, k_0) = e^c_z(x = 1, k_0), \tag{7}$$

where $h^c_\phi(x,k_0)$ and $e^c_z(x,k_0)$ are the magnetic and electric fields in the cladding whose explicit expressions are given by

$$h^c_\phi(x, k_0) = AK_1\left(xk_0 R\sqrt{p^2 - \epsilon_c}\right), \tag{8}$$

$$e^c_z(x, k_0) = -Ak_0 R\sqrt{p^2 - \epsilon_c} K_0\left(xk_0 R\sqrt{p^2 - \epsilon_c}\right), \tag{9}$$

where $K_n(x)$ is the modified Bessel function of order n [18] and A is an undetermined constant. Note that, first Eq. (6) can be derived by realizing that a Frobenius series of the solution of Eqs. (2) and (3) has a vanishing independent term. Second, Eq. (7) account for the continuity of the tangential magnetic F_ϕ and electric G_z fields at the border of the nematic cylinder. Then, inserting these definitions into Eq. (7), it turns out to be

$$G_z(1) + F_\phi(1)\epsilon_\parallel \epsilon_\perp k_0 R\sqrt{p^2 - \epsilon_c} \frac{K_0\left(k_0 R\sqrt{p^2 - \epsilon_c}\right)}{K_1\left(k_0 R\sqrt{p^2 - \epsilon_c}\right)} = 0, \quad F_\phi(0) = 0, \tag{10}$$

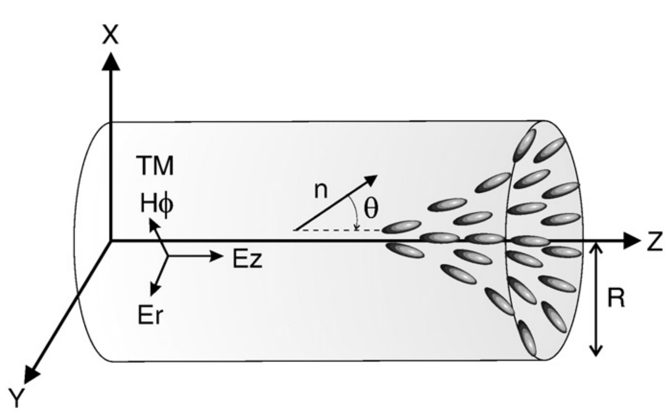

Fig. 1. Schematics of a cylindrical fiber infiltrated by a nematic liquid crystal and subject to the action of a propagating electromagnetic field.