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Controllable Kerr nonlinearity with vanishing absorption in a four-level inverted-Y atomic system

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ABSTRACT

The giant enhancement of Kerr nonlinearity in a four-level inverted-Y atomic system is investigated theoretically. Compared with that generated in a generic three-level system, the Kerr nonlinearity can be enhanced by several orders of magnitude with vanishing linear absorption. The physical mechanism leading to the giant enhancement of Kerr nonlinearity is also discussed.

1. Introduction

One of the most intriguing phenomenon in atom-light interactions is represented by electromagnetically induced transparency (EIT) [1–3]. Its potential applications range from lasing without inversion (LWI) [4–8] and controlled group velocity [9–11] to enhanced nonlinear optics [12]. The generic EIT consists of three-level atoms, then people extended their interest to multi-level atomic systems. Recently the enhancement of third-order Kerr nonlinearity with reduced absorption in multi-level atomic systems have attracted much interest because of its applications in nonlinear and quantum optics [13–23]. Schmidt and Imamoglu investigated the giant Kerr nonlinearity with vanishing absorption in a four-level N-configuration system in which the ideal EIT regime is disturbed by introducing an additional off-resonant level [13]. A quasi-three-level scheme is proposed to enhance the third-order nonlinearity with vanishing absorption [15]. Joshi and Xiao studied a four-level inverted-Y atomic system, and they found that the cross-Kerr nonlinearity in such a system could produce a phase shift of order π and might be used for realizing polarization quantum phase gates [17]. Niu et al. investigated the possibility of giant enhancement of the Kerr nonlinearity in the double-dark-resonance atomic system, they showed that the interacting double-dark resonances give rise to an order of magnitude increase of the Kerr nonlinearity [18]. Recently the giant cross-Kerr nonlinearity was studied in carbon nanotube quantum dots with spin-orbit coupling [23].

In this paper, motivated by the work [17], we investigate the Kerr nonlinearity in self-phase modulation (SPM) in a four-level inverted-Y atomic system. As pointed in Ref. [17], the inverted-Y system can provide large cross-Kerr effect between the probe and signal fields, and is very straightforward to implement in a scheme involving hyperfine levels of ^{87}Rb atoms [24]. The large cross-phase modulation (XPM) can be used for quantum phase gate [25], all-optical switching [26], deterministic optical quantum computation [27] and so on. Here we notice that another third-order nonlinearity effect, i.e., the Kerr nonlinearity in SPM is not discussed in the scheme. Because the Kerr nonlinearity in SPM can be used for generation of optical solitons [28,29] and so on, it is significative to investigate the Kerr nonlinearity in SPM in the inverted-Y system. In this atomic system, we show that the Kerr nonlinearity can be enhanced by several orders of magnitude with vanishing absorption within the right transparency window by modulating the intensity and the detuning of the coherent-control field. In addition, we also discuss the physical mechanism leading to the giant enhancement of Kerr nonlinearity.

2. Model and equations

The atomic-level scheme considered here is the same as that in Ref. [17] and is shown in Fig. 1. A weak-probe field E_p with Rabi frequency $g = \left(\vec{E_p}\cdot\vec{d_{21}}\right)/2\hbar$ whose central frequency ω_p is close to the frequency of the atomic transition $|1\rangle \leftrightarrow |2\rangle$. The transition $|0\rangle \leftrightarrow |2\rangle$ interacts with a strong coupling field (frequency ω_s) having Rabi frequency $\Omega = \left(\vec{E_s}\cdot\vec{d_{20}}\right)/2\hbar$, and a coherent-control field (frequency ω_c) with Rabi frequency $G = \left(\vec{E_c}\cdot\vec{d_{32}}\right)/2\hbar$ is applied to the transition $|2\rangle \leftrightarrow |3\rangle$. The higher excited state $|3\rangle$ decays with a rate $2\gamma_3$ to the lower excited state $|2\rangle$ which decays to the ground state $|1\rangle$ with a rate $2\gamma_2$, and $2\gamma_1$ denotes the population relaxation decay rate from $|2\rangle$ to $|0\rangle$. Here the lower three levels form a Λ-type configuration, and the levels $|1\rangle$, $|2\rangle$ and $|3\rangle$ form a ladder-type system.

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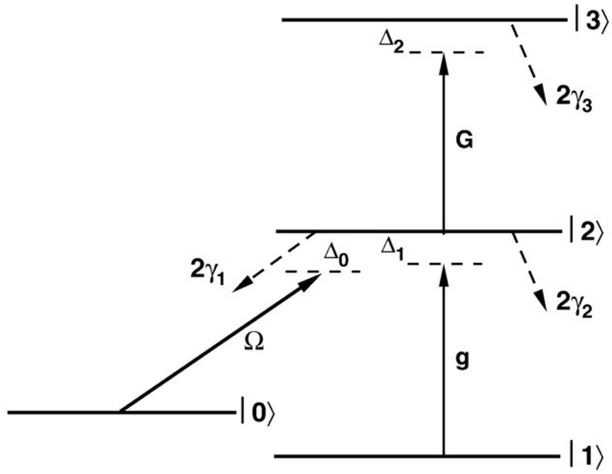

Fig. 1. Schematic diagram of the four-level inverted-Y system considered.

Under the rotating-wave approximation, the density matrix equation of motion can be written as

$$\begin{aligned}
\dot{\rho}_{00} &= 2\gamma_1\rho_{22} - 2\gamma_0\rho_{00} + i\Omega(\rho_{02} - \rho_{20}),\\
\dot{\rho}_{11} &= 2\gamma_2\rho_{22} + 2\gamma_0\rho_{00} + ig(\rho_{12} - \rho_{21}),\\
\dot{\rho}_{33} &= -2\gamma_3\rho_{33} - iG(\rho_{23} - \rho_{32}),\\
\dot{\rho}_{10} &= -[\gamma_0 - i(\Delta_1 - \Delta_0)]\rho_{10} + i\Omega\rho_{12} - ig\rho_{20},\\
\dot{\rho}_{02} &= -(\gamma_1 + \gamma_2 - i\Delta_0)\rho_{02} + ig\rho_{01} + iG\rho_{03} + \Omega(\rho_{00} - \rho_{22}),\\
\dot{\rho}_{03} &= -[\gamma_0 + \gamma_3 - i(\Delta_0 + \Delta_2)]\rho_{03} + iG\rho_{02} - i\Omega\rho_{23},\\
\dot{\rho}_{23} &= -(\gamma_1 + \gamma_2 + \gamma_3 - i\Delta_2)\rho_{23} - ig\rho_{13} - i\Omega\rho_{03} + iG(\rho_{22} - \rho_{33}),\\
\dot{\rho}_{12} &= -(\gamma_1 + \gamma_2 - i\Delta_1)\rho_{12} + i\Omega\rho_{10} + iG\rho_{13} + ig(\rho_{11} - \rho_{33}),\\
\dot{\rho}_{13} &= -[\gamma_3 - i(\Delta_1 + \Delta_2)]\rho_{13} + iG\rho_{12} - ig\rho_{23},
\end{aligned} \tag{1}$$

the above equations are constrained by $\sum_i \rho_{ii} = 1$ and $\rho_{ij} = \rho^*_{ji}$, $(i, j = 0, 1, 2, 3$, and $i \neq j)$, $2\gamma_0$ is related to the nonradiative relaxation rate of state $|0\rangle$, and Δ_0, Δ_1, and Δ_2 are the detunings of the coupling, the probe and the coherent-control fields respectively.

3. Results and discussions

It is known that the response of the atomic medium to the probe field is governed by its polarization $P = \varepsilon_0(E_p\chi + E^*_p\chi^*)/2$, where χ is the susceptibility of the atomic medium. By performing a quantum average of the dipole moment over an ensemble of N atoms, $P = N(\mu_{21}\rho_{12} + \mu_{12}\rho_{21})$. To derive the equations for the linear and nonlinear susceptibilities, we need to give the steady state solutions for the density matrix Eq. (1). In the present approach, an iterative method is used to achieve increasingly accurate approximations to the matrix elements[16,18,20]. The density matrix elements can be written as $\rho_{mn} = \rho^{(0)}_{mn} + \rho^{(1)}_{mn} + \rho^{(2)}_{mn} + \rho^{(3)}_{mn} + \cdots$, where each successive approximation is calculated using the matrix elements of one order less than the one being calculated. Under the condition that the probe field is very small as compared with those of the coupling and the coherent-control fields, the zeroth-order solution will be $\rho^{(0)}_{11} = 1$ and other elements are equal to zero. Under the weak-probe approximation, we can get the matrix elements in the first order. For simplicity, we set $\gamma_1 = \gamma_2 = \gamma_3 = \gamma$ and $\Delta_0 = \Delta_2 = 0$ during calculation.

$$\begin{aligned}
\rho^{(1)}_{21} &= \frac{g\left\{\Omega(\Delta_1 - i)\rho^{(0)}_{02} + \Delta_1\left[G\rho^{(0)}_{32} + (\Delta_1 - i)\left(\rho^{(0)}_{11} - \rho^{(0)}_{22}\right)\right]\right\}}{\Omega^2(\Delta_1 - i) + \Delta_1(2 + G^2 - \Delta_1^2 + 3i\Delta_1)}\\
&= -\frac{g\Delta_1}{Q}\left\{-\Omega^2\left(1 + \Delta_1^2\right) + \Delta_1\left[G^2(i - \Delta_1) + (2i + \Delta_1)\left(1 + \Delta_1^2\right)\right]\right\},
\end{aligned} \tag{2}$$

$$\rho^{(1)}_{10} = -\frac{g\Omega}{Q}\left\{\Omega^2\left(1 + \Delta_1^2\right) + \Delta_1\left[G^2(i + \Delta_1) + (2i - \Delta_1)\left(1 + \Delta_1^2\right)\right]\right\}, \tag{3}$$

$$\rho^{(1)}_{13} = \frac{\Omega G\Delta_1}{Q}\left[\Omega^2(i - \Delta_1) + \Delta_1\left(\Delta_1^2 - 2 - G^2 - 3i\Delta_1\right)\right]. \tag{4}$$

Here all parameters are reduced to dimensionless ones by scaling with γ. To obtain the third-order in ρ_{21} we need to know ρ_{02}, ρ_{32} and $\rho_{11} - \rho_{22}$ to the second order:

$$\rho^{(2)}_{02} = g\left[\frac{\rho^{(1)}_{12} - \rho^{(1)}_{21}}{2\Omega} + \frac{\Omega G\left(\rho^{(1)}_{13} - \rho^{(1)}_{31}\right) + \left(3 + \Omega^2\right)\left(\rho^{(1)}_{10} - \rho^{(1)}_{01}\right)}{2i(6 + 2\Omega^2 + 3G^2)}\right], \tag{5}$$

$$\begin{aligned}
\rho^{(2)}_{32} = g\Bigg[&\frac{i\left(\rho^{(1)}_{13} + \rho^{(1)}_{31}\right) - 2G\left(\rho^{(1)}_{12} - \rho^{(1)}_{21}\right)}{2(3 + \Omega^2 + G^2)}\\
&+ \frac{\left(2 + G^2\right)\left(\rho^{(1)}_{13} - \rho^{(1)}_{31}\right) + \Omega G\left(\rho^{(1)}_{10} - \rho^{(1)}_{01}\right)}{2i(6 + 2\Omega^2 + 3G^2)}\Bigg],
\end{aligned} \tag{6}$$

$$\rho^{(2)}_{11} - \rho^{(2)}_{22} = -\frac{g}{2i\Omega^2}\left[\left(2 + 3\Omega^2 + G^2\right)\left(\rho^{(1)}_{12} - \rho^{(1)}_{21}\right) - i\Omega\left(\rho^{(1)}_{10} + \rho^{(1)}_{01}\right)\right]. \tag{7}$$

With the above procedure, the linear absorption and the refractive part of the third-order susceptibility can be written as

$$\mathrm{Im}\left[\chi^{(1)}\right] = \frac{2N|\mu_{21}|^2}{\hbar\varepsilon_0 Q}\left[\Delta_1^2\left(2 + G^2 + 2\Delta_1^2\right)\right], \tag{8}$$

$$\begin{aligned}
\mathrm{Re}\left[\chi^{(3)}\right] = \frac{2N|\mu_{21}|^4}{3\hbar^3\varepsilon_0 g^3}\Bigg(&A\,\mathrm{Im}\left[\rho^{(1)}_{21}\right] + B\,\mathrm{Im}\left[\rho^{(1)}_{01}\right] + C\,\mathrm{Re}\left[\rho^{(1)}_{01}\right]\\
&+ D\,\mathrm{Im}\left[\rho^{(1)}_{31}\right] + F\,\mathrm{Re}\left[\rho^{(1)}_{31}\right]\Bigg),
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
Q = {}&\Omega^4\left(1 + \Delta_1^4\right) - 2\Omega^2\Delta_1^2\left(1 - G^2 + \Delta_1^2\right)\\
&+ \Delta_1^2\left[4 + G^4 + 5\Delta_1^2 + \Delta_1^4 - 2G^2\left(\Delta_1^2 - 2\right)\right],
\end{aligned}$$

$$\begin{aligned}
A = {}&-g^2\left(6 + 2\Omega^2 + 3G^2\right)\Delta_1\Big\{\left(6 + 5G^2 + G^4\right)\Delta_1^2\left(G^2 - 1 - \Delta_1^2\right)\\
&+ 3\Omega^6\left(1 + \Delta_1^2\right) + \Omega^2\Delta_1^2\left[5G^4 - 4G^2\left(\Delta_1^2 - 4\right) - 11\left(1 + \Delta_1^2\right)\right]\\
&+ \Omega^4\left[9 + 6\Delta_1^2 - 3\Delta_1^4 + G^2\left(1 + 7\Delta_1^2\right)\right]\Big\}\Big/\Omega^2\left[2\Omega^4 + \Omega^2\left(12 + 5G^2\right)\right.\\
&\left. + 3\left(6 + 5G^2 + G^4\right)\right]\Big\{\Omega^4\left(1 + \Delta_1^2\right) - 2\Omega^2\Delta_1^2\left(1 - G^2 + \Delta_1^2\right)\\
&+ \Delta_1^2\left[4 + G^4 + 5\Delta_1^2 + \Delta_1^4 - 2G^2\left(\Delta_1^2 - 2\right)\right]\Big\},
\end{aligned} \tag{10}$$

$$\begin{aligned}
B = {}&g^2\Omega\Big\{\Omega^4\left(1 + \Delta_1^4\right) + \Omega^2\left[3 + 2\left(1 + G^2\right)\Delta_1^2 - \Delta_1^4\right]\\
&+ \Delta_1^2\left[G^4 - G^2\left(\Delta_1^2 - 5\right) - 3\left(1 + \Delta_1^2\right)\right]\Big\}\Big/\Big\{2\Omega^6\left(1 + \Delta_1^2\right)\\
&+ 2\Omega^2\Delta_1^2\left[4G^4 - 2 - \Delta_1^2 + \Delta_1^4 + G^2\left(7 - 5\Delta_1^2\right)\right]\\
&+ 3\left(2 + G^2\right)\Delta_1^2\left[4 + G^4 + 5\Delta_1^2 + \Delta_1^4 - 2G^2\left(\Delta_1^2 - 2\right)\right]\\
&+ \Omega^4\left[6 + 2\Delta_1^2 - 4\Delta_1^4 + G^2\left(3 + 7\Delta_1^2\right)\right]\Big\},
\end{aligned} \tag{11}$$