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Entanglement and nonlocality of one- and two-mode combination squeezed state

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ABSTRACT

We investigate the entanglement and nonlocality properties of one- and two-mode combination squeezed vacuum state (OTCSS, with two-parameter λ and γ) by analyzing the logarithmic negativity and the Bell's inequality. It is found that this state exhibits larger entanglement than that of the usual two-mode squeezed vacuum state (TSVS), and that in a certain regime of λ, the violation of Bell's inequality becomes more obvious, which indicates that the nonlocality of OTCSS can be stronger than that of TSVS. As an application of OTCSS, the quantum teleportation is examined, which shows that there is a region spanned by λ and γin which the fidelity of OTCSS channel is larger than that of TSVS.

1. Introduction

Entanglement between quantum systems plays a key role in quantum information processing, such as quantum teleportation, dense coding, and quantum cloning. In recent years, various entangled states have brought considerable attention and interests of physicists because of their potential uses in quantum communication [1,2]. For instance, the two-mode squeezed state is a typical entangled state of continuous variable and exhibits quantum entanglement between the idle-mode and the signal-mode in a frequency domain manifestly. Theoretically, the two-mode squeezed state is constructed by the two-mode squeezing operator $S=exp[\lambda(a_1a_2-a_1^\dagger a_2^\dagger)]$ [3–5] acting on the two-mode vacuum state $|00\rangle$,

$$S|00\rangle = \text{sech}\lambda\, exp\left[-a_1^\dagger a_2^\dagger tanh\lambda\right]|00\rangle, \tag{1}$$

where λ is a squeezing parameter, the disentangling of S can be obtained by using SU(1,1) Lie algebra, $[a_1a_2, a_1^\dagger a_2^\dagger] = a_1^\dagger a_1 + a_2^\dagger a_2 + 1$, or by using the entangled state representation $|\eta\rangle$ [6–8], which was constructed according to the idea of Einstein, Podolsky and Rosen in their argument that quantum mechanics is incomplete [9].

Using the relation between Bosonic operators and the coordinate Q_i, momentum P_i, $Q_i = \left(a_i + a_i^\dagger\right)/\sqrt{2}, P_i = \left(a_i - a_i^\dagger\right)/\left(\sqrt{2}\text{i}\right)$, and introducing the two-mode quadrature operators of light field, $x_1 = (Q_1+Q_2)/2$, $x_2=(P_1+P_2)/2$, the variances of x_1 and x_2 in the state $S|00\rangle$ are in the standard form

$$\langle 00|S^\dagger x_2^2 S|00\rangle = \frac{1}{4}e^{-2\lambda}, \langle 00|S^\dagger x_1^2 S|00\rangle = \frac{1}{4}e^{2\lambda}, \tag{2}$$

thus we get the standard squeezing for the two quadrature: $x_1 \to \frac{1}{2}e^{\lambda}x_1$, $x_2 \to \frac{1}{2}e^{-\lambda}x_2$. On the other hand, the two-mode squeezing operator can also be recast into the form $S = exp[\text{i}\lambda(Q_1P_2 + Q_2P_1)]$. Then some interesting questions naturally rise: what is the property of the following operator

$$V = exp[-\text{i}(\lambda_1 Q_1 P_2 + \lambda_2 Q_2 P_1)], \tag{3}$$

with two parameters $\lambda_1 = \lambda e^{\gamma}, \lambda_2 = \lambda e^{-\gamma}, \lambda > 0$? What is the normally ordered expansion of V and what is the state $V|00\rangle$? What are the entanglement and nonlocality properties of $V|00\rangle$? When $\gamma = 0$, Eq. (3) just reduces to the usual two-mode squeezing operator S. Thus we can consider V as a generalized two-mode squeezing operator and $V|00\rangle$ as one- and two-mode combination squeezed vacuum state (OTCSS).

In this paper, we investigate entanglement properties and quantum nonlocality of $V|00\rangle$ in terms of logarithmic negativity and the Bell's inequality, respectively. Subsequently, we consider its application in the field of quantum teleportation by using the characteristic function formula. It is shown that this state exhibits larger entanglement than that of the usual two-mode squeezed vacuum state (TSVS); and in a certain smaller regime of λ, that the nonlocality of this state can be stronger than that of TSVS due to the presence of γ. In addition, application to quantum teleportation with OTCSS is also considered, which shows that there is a region spanned

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by λ and γ in which the fidelity of OTCSS channel is larger than that of TSVS.

Our paper is arranged as follows. In Section 2, we derive the normal ordering form of one- and two-mode combination squeezing operator by using the technique of integration within an ordered product (IWOP) of operators. In Section 3, using the Weyl ordering form of single-mode Wigner operator and the order-invariance of Weyl ordered operators under similar transformations, we derive analytically the Wigner function of $V|00\rangle$. Sections 4 and 5 are devoted to investigating the entanglement properties and the nonlocal properties OTCSS by using the Bell's inequality and the logarithmic negativity, respectively. An application to quantum teleportation with OTCSS is involved in Section 6. We end with the main conclusions of our work.

2. The normal ordering form of V and fluctuations in $V|00\rangle$

In order to know $V|00\rangle$, we need to derive the normal ordering form of the unitary operator V by virtue of the IWOP technique [10–12]. Using the Baker–Hausdorff formula,

$$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}[A,[A,[A,B]]] + \cdots, \tag{4}$$

and noticing that

$$i[\lambda_1 Q_1 P_2 + \lambda_2 Q_2 P_1, Q_1] = \lambda_2 Q_2, \tag{5}$$

$$i[\lambda_1 Q_1 P_2 + \lambda_2 Q_2 P_1, Q_2] = \lambda_1 Q_1, \tag{6}$$

$$i[\lambda_1 Q_1 P_2 + \lambda_2 Q_2 P_1, P_1] = -\lambda_1 P_2, \tag{7}$$

$$i[\lambda_1 Q_1 P_2 + \lambda_2 Q_2 P_1, P_2] = -\lambda_2 P_1, \tag{8}$$

we have

$$V^{-1} Q_1 V = Q_1 \cosh\lambda + Q_2 e^{-\gamma} \sinh\lambda, \tag{9}$$

$$V^{-1} Q_2 V = Q_2 \cosh\lambda + Q_1 e^{\gamma} \sinh\lambda, \tag{10}$$

$$V^{-1} P_1 V = P_1 \cosh\lambda - P_2 e^{\gamma} \sinh\lambda, \tag{11}$$

$$V^{-1} P_2 V = P_2 \cosh\lambda - P_1 e^{-\gamma} \sinh\lambda. \tag{12}$$

Thus, in order to keep the eigenvalues invariant under the V transformation, i.e.,

$$V^{-1} Q_k V |q_1 q_2\rangle' = q_k |q_1 q_2\rangle', (k = 1,2), \tag{13}$$

the base vector must be changed to

$$|q_1 q_2\rangle' = V^{-1}|q_1 q_2\rangle = \left|\Lambda^{-1}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}\right\rangle, \Lambda = \begin{pmatrix} \cosh\lambda & e^{-\gamma}\sinh\lambda \\ e^{\gamma}\sinh\lambda & \cosh\lambda \end{pmatrix}, \tag{14}$$

where $|q_1 q_2\rangle = |q_1\rangle \otimes |q_2\rangle$, and $|q_k\rangle$ is the coordinate eigenstate,

$$|q_k\rangle = \pi^{-1/4} \exp\left[-\frac{1}{2}q^2 + \sqrt{2} q a^\dagger - \frac{1}{2}a^{\dagger 2}\right]|0\rangle. \tag{15}$$

Using the completeness relation $\int_{-\infty}^{\infty} dq_1 dq_2 |q_1, q_2\rangle\langle q_1, q_2| = 1$, we have

$$V^{-1} = \int_{-\infty}^{\infty} dq_1 dq_2 \left|\Lambda^{-1}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}\right\rangle \langle q_1, q_2|, \tag{16}$$

which leads to

$$V = \int_{-\infty}^{\infty} dq_1 dq_2 \left|\Lambda\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}\right\rangle \langle q_1, q_2|. \tag{17}$$

Actually, one can check (17) by $V^{-1}V = VV^{-1} = 1$. Further using the vacuum projector $|00\rangle\langle 00| = :\exp[-a^\dagger a - b^\dagger b]:$ (: : denoting normal ordering), as well as the IWOP technique, we can put V into the normal ordering form [13],

$$\begin{aligned} V = {} & \frac{2}{\sqrt{L}} \exp\left\{\frac{1}{L}\left[\left(b^{\dagger 2} - a^{\dagger 2}\right)\sinh^2\lambda \sinh 2\gamma + 2a^\dagger b^\dagger \sinh 2\lambda \cosh\gamma\right]\right\} \\ & : \exp\left\{\frac{4}{L}\left[\left(a^\dagger a + b^\dagger b\right)\cosh\lambda + \left(b^\dagger a - a^\dagger b\right)\sinh\lambda \sinh\gamma\right] - a^\dagger a - b^\dagger b\right\} : \\ & \exp\left\{\frac{1}{L}\left[\left(b^2 - a^2\right)\sinh^2\lambda \sinh 2\gamma - 2a^\dagger b^\dagger \sinh 2\lambda \cosh\gamma\right]\right\}, \end{aligned} \tag{18}$$

where $L = 4(1 + \sinh^2\gamma \tanh^2\lambda)\cosh^2\lambda$. Eq. (18) is just the normal ordering form of V. It is obviously to see that when $\gamma = 0$, Eq. (18) just reduces to the usual two-mode squeezing operator. Operating V on the two-mode vacuum state $|00\rangle$, we obtain the squeezed vacuum state,

$$V|00\rangle = \frac{2}{\sqrt{L}} \exp\left\{\frac{1}{L}\left[\left(b^{\dagger 2} - a^{\dagger 2}\right)\sinh^2\lambda \sinh 2\gamma + 2a^\dagger b^\dagger \sinh 2\lambda \cosh\gamma\right]\right\}|00\rangle. \tag{19}$$

On the other hand, by using the transformations Eqs. (9)–(12), one can derive the variances of x_1 and x_2 in the state $V|00\rangle$ [13]

$$\left\langle (\Delta x_1)^2 \right\rangle = \frac{1}{4}\left(\cosh 2\lambda + 2\sinh^2\lambda \sinh^2\gamma + \sinh 2\lambda \cosh\gamma\right), \tag{20}$$

$$\left\langle (\Delta x_2)^2 \right\rangle = \frac{1}{4}\left(\cosh 2\lambda + 2\sinh^2\lambda \sinh^2\gamma - \sinh 2\lambda \cosh\gamma\right), \tag{21}$$

which indicate that the variances are not only dependent on parameter λ, but also on parameter γ. When $\gamma = 0$, Eqs. (20) and (21) reduce to $\left\langle (\Delta x_1)^2 \right\rangle = \frac{1}{4}e^{2\lambda}$, and $\left\langle (\Delta x_2)^2 \right\rangle = \frac{1}{4}e^{-2\lambda}$, corresponding to the usual TSVS. In particular, by modulating the two parameters (λ and γ), we can realize that

$$\left\langle (\Delta x_1)^2 \right\rangle > \frac{1}{4}e^{2\lambda}, \left\langle (\Delta x_2)^2 \right\rangle < \frac{1}{4}e^{-2\lambda}, \tag{22}$$

whose condition is given by

$$0 < \tanh\lambda < \frac{1}{1 + \cosh\gamma}, \lambda > 0, \tag{23}$$

which mean that the OTCSS may exhibit stronger squeezing in one quadrature than that of the TSVS while exhibiting weaker squeezing in another quadrature when the condition (23) is satisfied. Then, can the OTCSS exhibits stronger nonlocality or more observable violation of Bell's inequality? In the following, we pay our attention to these two aspects.

3. Wigner function of $V|00\rangle$

Wigner distribution functions [14–16] of quantum states are widely studied in quantum statistics and quantum optics. Now we derive the expression of the Wigner function of $V|00\rangle$. Here we take a new method to do it. Recalling that in Ref. [17–19] we have introduced the Weyl ordering form of single-mode Wigner operator

$$\Delta_1(q_1, p_1) = \vdots\, \delta(q_1 - Q_1)\delta(p_1 - P_1) \,\vdots, \tag{24}$$