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New beat length for writing periodic structures using Bessel beams

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ABSTRACT

With the ultimate aim of exploiting the self-focusing behaviour to create periodic structures, we have investigated the behaviour of Bessel–Gauss beams in Kerr-like nonlinear media and have identified that a previously proposed nonlinear beat length is inaccurate with increasing power. By studying the behaviour of the beam we suggest a correction; providing a much better description of the beat length. This correction is tested against results from numerical simulations confirming the improved accuracy. Within the, scalar, nonparaxial limit we show that this modified beat length is valid for beam powers surpassing the paraxial self-focusing threshold. From this modified beat length equation, the appropriate experimental variables may be chosen to create accurate periodic structures by direct laser writing in a single exposure.

1. Introduction

The integration of optical designs by direct laser writing within materials such as heavy metal oxide (HMO) glass is often plagued by nonlinear beam reshaping due to Kerr-like self focusing [1]. This is particularly striking in the case of Gaussian beams where one can observe a significant variation in effective focal depth with beam power. By numerically studying the propagation of Bessel–Gauss beams we report on their potential application for writing periodic structures in a single shot, exploiting the natural self-focusing effect to our advantage.

As previously reported [2] a Bessel–Gauss beam propagating in a Kerr-like nonlinear medium exhibits periodic modulation of the axial field intensity along the optical axis. As the power is increased the modulation depth grows, permitting the central lobe intensity to periodically exceed the threshold for material modification without adverse effects on the beam as a whole. This could enable periodic structures to be built in a single exposure, using optics with a far lower numerical aperture than required to build the structure point by point. Combining this with raster scanning; volume Bragg-like structures, similar to those described in [3], could be formed.

As yet, one problem remains unanswered. As the beam power is increased not only does the modulation deepen, its associated beat length increasingly begins to deviate from the low power limit shown in [2] coupling together the power of the beam and the period of any generated structure. Although the variation of beat length with power is clearly observable in previous numerical results [2] it does not appear to have received significant attention. Accounting for this variation with increasing power would allow accurate control of the size and separation of these structures.

2. Bessel beams

The Bessel beam is a well known exact, diffraction free solution to the scalar, linear, isotropic and homogeneous Helmholtz wave equation. In a linear medium these propagate as

$$A(r,z) = J_0(k_r r)\exp(ik_z z), \tag{1}$$

where k_r and k_z are the radial and longitudinal wavenumbers respectively and a time convention of $\exp(-i\omega t)$ has been assumed. The construction of a Bessel beam may be considered as the summation of an infinite set of plane waves with their optical axes aligned on the surface of a cone. The inner cone angle, θ, denotes the angle any one of these plane waves makes with the principle optical axis of the beam. One such plane wave is shown in Fig. 1. The longitudinal and transverse wavenumbers are related by

$$k_z = \sqrt{k^2 - k_r^2} = k\cos\theta. \tag{2}$$

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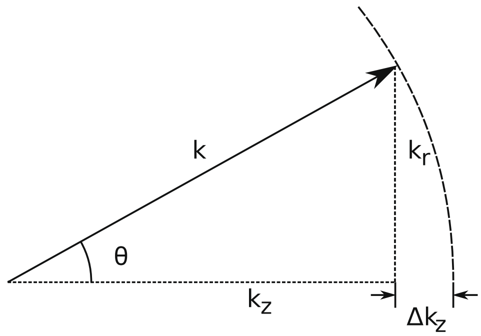

Fig. 1. Representation in *k*-space.

In the paraxial limit this reduces to

$$k_z = k - k_r^2/(2k). \tag{3}$$

Such Bessel beams are unphysical as the power is technically unbounded. A commonly used physical approximation is the Bessel–Gauss beam. Here a Gaussian envelope is used to ensure the beam is of finite power. Using this Gaussian window comes at a cost; the Bessel–Gauss beam is no longer diffraction free. By considering the shadow cast by the windowing function we may write this diffraction-free length as

$$z_{\mathrm{BD}} = \frac{w_0}{\tan\theta}, \tag{4}$$

where w_0 is the $1/e$ radius of the Gaussian window.

3. Numerical model

The propagation of the, potentially large cone angle, scalar beam in a Kerr-like medium may be described by the nonparaxial, nonlinear, Schrödinger equation (NNSE):

$$\kappa\frac{\partial^2 u}{\partial\tilde{z}^2} + i\frac{\partial u}{\partial\tilde{z}} + \nabla_\perp^2 u + |u|^2 u = 0, \tag{5}$$

where $\nabla_\perp^2$ is the transverse Laplacian, the form of which depends on the coordinate system. A scaling has been introduced appropriate to a Bessel–Gauss beam such that $\tilde{r} = rk_r$, $\tilde{z} = z/L_D$ and $L_D = 2k/k_r^2$ is the Rayleigh length of the isolated central Bessel lobe [4]. The constant $\kappa = (k_r/k)^{-2}$ is a measure of the nonparaxiality of the beam. Finally, the field is scaled as $u(\tilde{r},\tilde{z}) = \sqrt{kn_2/n_0L_d}A(\tilde{r},\tilde{z})$ where $A(\tilde{r},\tilde{z})$ is the unscaled field and the forward phase $\exp(ikz)$ has been factored out.

To recover the nonlinear Schrödinger equation (NSE) from Eq. (5) we must neglect the first term. Commonly, this is achieved by assuming the slowly varying envelope approximation (SVEA). It is worth remembering that we do not need the explicit condition that $\kappa \to 0$ for the NSE to be valid. We may model beams with modest nonparaxiality providing that we do not inadvertently violate the slowly varying envelope approximation.

For weak nonlinearity we do not expect the numerical results derived with the NNSE to manifestly differ from those derived from the NSE. By construction, providing the forward phase has been factored out, the Bessel–Gauss beam varies slowly with respect to $\tilde{z}$, this holds true for small amounts of nonlinearity.

As the nonlinear influence on the beam increases, with increasing power, the self focusing begins to dominate the behaviour of the beam. As this occurs the beam is clearly no longer varying slowly and consequently the SVEA is violated. Above a threshold power; catastrophic self focusing is predicted by the NSE [5]. It is know that the full nonlinear scalar Helmholtz or NNSE do not permit such catastrophic self focusing [6]. We test our assumptions for the beat length using both the NNSE and the NSE.

Applying the above scaling to the inner cone angle, θ in Fig. 1 gives the relation:

$$\sin\theta = k_{\tilde{r}}\sqrt{2\kappa}, \tag{6}$$

from which the usual small angle approximations can be taken.

Clearly, including the first term in Eq. (5) places a constraint on the relationship between k and k_r. Any solution obtained from the NNSE will be valid only for this value of κ and the associated inner cone angle, θ. If, however, this term is neglected; any solution obtained by the NSE will be valid for any combination of k and k_r. Put plainly, solutions obtained from the NSE, for a particular scaled transverse spatial frequency $k_{\tilde{r}}$, will be valid for any inner cone angle θ. This is provided, of course, that we do not inadvertently violate the SVEA.

As the Bessel beam field evolves in a Kerr-like nonlinear medium; the nonlinear interaction, which can be thought of as degenerate four wave mixing [2] or alternatively self-diffraction [4], generates a field propagating predominantly along the optical axis. From trivial geometrical arguments the interference between this new field and the Bessel beam, shown as Δk_z on Fig. 1, leads to a beat length:

$$z_b^0 = \frac{2\pi}{k - k_z}. \tag{7}$$

Fig. 2a shows the field intensity distribution as this plane wave beats with a linear Bessel–Gauss beam. This we shall consider our linear approximation.

4. Numerical simulation

The assumptions made above were tested by modelling the Bessel–Gauss beams with the Hankel-based Adaptive Radial Propagator (HARP). The Hankel transforms were implemented using the quasi discrete Hankel transform [7]. A symmetrised split step operator was used for the paraxial results and a finite difference operator, based on the NNSE, was used for the nonparaxial results [8]. The nonparaxial results were obtained for an inner cone angle of $\theta = 30^\circ$, corresponding to a central lobe FWHM of 0.36λ.

The numerical results in Fig. 2b shows the intensity distribution for slices across the beam path. The modulation of the Bessel–Gauss beam is clearly present. By comparing these numerical results with the linear approximation, Fig. 2a several deficiencies in the linear approximation are apparent. Energy is concentrated near the axis in the nonlinear numerical case to a far greater extent than the trivial linear case of Fig. 2a. In addition the modulation length varies, reducing for lobes closer to the central point. For the central lobe this is pronounced; eight periods in Fig. 2b as opposed to six in Fig. 2a. If this intensity modulation were to be used for writing periodic structure such unpredictable control over the spacing the points would be unacceptable; a correction to this beat length is required.

5. Power dependent beat length

Two features are not taken into account in the linear approximation. The first deficiency arises because we have incorrectly assumed that the generated field has the form of a plane wave, whereas it is Gaussian-like [2]. This accounts for the concentration of power towards the central lobes. Additionally we have neglected to take into account the Kerr-induced phase retardation on the generated field. This increases the optical path for the generated field resulting in a reduction in the modulation length which scales as average intensity.

We propose a new nonlinear beat length based on a correction to Eq. (7) [2]: