Optics Communications 284 (2011) 297–300

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Preparation of arbitrary correlated states of photons in multiple spatially separated cavities

Zi-Hong Chen *, Zhen-Biao Yang, Chang-Geng Liao, Cheng-Li Luo

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 30002, China

ARTICLE INFO

Article history:
Received 12 May 2010
Received in revised form 19 August 2010
Accepted 15 September 2010

Keywords:
Correlated field states
Cascade M-level atom
Resonant interaction

ABSTRACT

We proposed a scheme for generating arbitrary multiple spatially separated correlated field states. It is based on the resonant interaction of a cascade $(M+1)$ level atoms and M separated cavities. Finally, we discuss the experimental feasibility of the scheme.

1. Introduction

Entanglement is one of the most peculiar features of quantum mechanics. Entangled quantum states come in many flavours, such as Bell [1], Einstein–Podolsky–Rosen [2], GHZ [3], or W states [4]. It can be prepared in trapped ions [5–7], cavity QED [8,9], superconducting circuits [10], semiconductor quantum dots [11], linear optical systems [12,13] and impurity spins in solids [14–16].

In recent years, much attention has been paid to the generation of so-called correlated or anticorrelated states of light in cavity QED. For example, Deb et al. [17] has proposed a scheme for producing two-mode anticorrelated states. The drawback of this method is that it required one of the two modes to be initially prepared in the Fock state $|N\rangle$. Although Martinis et al. have successfully generated Fock states with up to 15 photons by improving the lifetime of the high Q microwave resonator [18], it is difficult to achieve the preparation of the Fock state $|N\rangle$ in experiments. More recently, Ikram et al. [19] presented a method for preparing arbitrary anticorrelated field states in the two cavities. Subsequently, Zheng [20] and Zhong et al. [21] proposed a scheme for generating arbitrary anticorrelated states of a two-mode field, respectively. As for the preparation of correlated field states, Zheng and Guo [22] proposed a scheme for the generation of arbitrary multi-mode quantum states of a field with equal photon numbers in these modes. It is based on the injection of a sequence of $(M+1)$ level atoms into a cavity involving M modes. However, there has been no report about multi-mode cavity with "strong coupling" experimentally yet, thus it would be difficult to implement the scheme of ref. [22]. On the other hand, the correlation between multi-separated systems can not only be used to test local hidden variable theories against quantum mechanics [1], but also be useful in fields regarding quantum information, such as quantum cryptography [23], quantum computation [24], and quantum teleportation [25]. However, apparently there is no method of generating arbitrary correlated states in three or more cavities thus far. In this paper, we propose a scheme to prepare arbitrary correlated states of photons in multiple spatially separated cavities. It extends the idea of refs. [22,26], generating desire states by injecting a sequence of $(M+1)$ level atoms into M single-mode resonator and detecting them in the ground state. The distinct advantage of the scheme is that it does not require multi-mode cavities in the process. So it might be more experimentally feasible than that of ref. [22].

2. Generation of correlated states in multi-separated cavities

At first, we consider a system in which N $(M+1)$ level atoms with the ladder configuration are sent in excited state through M single-mode cavities successively as shown in Fig. 1. We set that (assuming $\hbar=1$)

$$\omega_{ci} = \omega_{M+1-i}-\omega_{M-i}. \tag{1}$$

Here ω_{ci} is the frequency of the ith cavity, while ω_{M+1-i} and ω_{M-i} is the energy of the atom and the corresponding energy level $|e_{M+1-i}\rangle$

* Corresponding author.
E-mail address: zi-hong_chen@hotmail.com (Z.-H. Chen).

0030-4018/$ – see front matter
doi:10.1016/j.optcom.2010.09.042

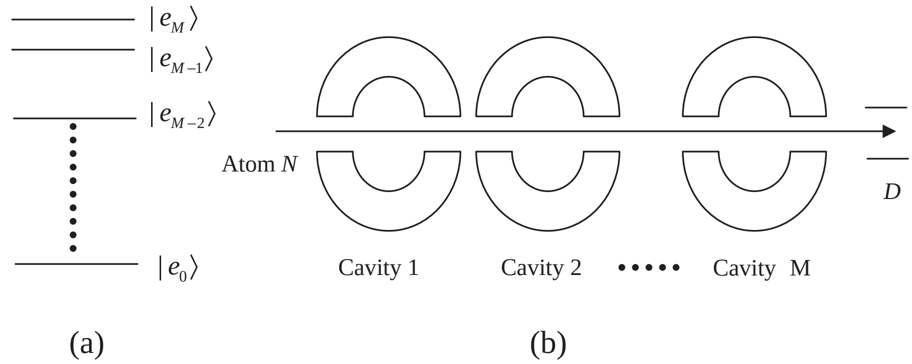

Fig. 1. (a) The schematic drawing of structure of atomic energy level. (b) The atoms in $\frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|e_M\rangle + \varepsilon_N|e_0\rangle)$ state one by one through these cavity 1,2…M, and then detected by detector.

and $|e_{M-i}\rangle$. If the frequency difference between any two cavities is so large that only the transitions between the adjacent levels are allowed and each one is only coupled with the corresponding cavity, the Hamiltonian for such a system is given by

$$H = \sum_{i=1}^{M} \omega_i A_i^+ A_i + \sum_{i=1}^{M} \omega_i a_i^+ a_i + \left[\sum_{i=1}^{M} g_i a_i A_{M+1-i}^+ A_{M-i} + HC\right] \tag{2}$$

where ω_i is the energy of the atom in the ith level ($\omega_0 = 0$), a_i^+ and a_i are the creation and annihilation operators for the ith cavity with frequency ω_{ci}, respectively, A_i^+ and A_i are the creation and annihilation operators of the atom in the level i, g_i is the atom–cavity mode coupling constant, and HC means Hermitian conjugate.

Assume that the state of the system is

$$|\psi_a^N\rangle|\psi_f^{N-1}\rangle = \frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|e_M\rangle + \varepsilon_N|e_0\rangle)\sum_{n=0}^{N-1} C_n^{N-1}|\underbrace{n,n,\dots n}_{M}\rangle \tag{3}$$

before the Nth atom is injected into the system. Here $|e_0\rangle$ and $|e_M\rangle$ denotes the ground state and the highest excited state of the atom, respectively, M cavities are in the state $|\psi_f^{N-1}\rangle = \sum_{n=0}^{N-1} C_n^{N-1}|\underbrace{n,n,\dots n}_{M}\rangle$, the Nth atom is in the superposition state $|\psi_a^N\rangle = \frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|e_M\rangle + \varepsilon_N|e_0\rangle)$, ε_N is a complex parameter. If the interaction time between the Kth cavity and the Nth atom is t_{kN}, the system evolves into the state:

$$|\psi_{af}^N\rangle = \frac{1}{k_N}|e_0\rangle\left\{\sum_{n=0}^{N-1} C_n^{N-1}\left[\frac{\varepsilon_N}{\sqrt{1+|\varepsilon_N|^2}}|\underbrace{n,n,\dots n}_{M}\rangle + \frac{1}{\sqrt{1+|\varepsilon_N|^2}}(-i)^M\prod_{i=1}^{M}\sin(\sqrt{n+1}g_i t_{iN})|\underbrace{n+1,n+1,\dots n+1}_{M}\rangle\right]\right\} \tag{4}$$

after sending the Nth atom across M cavities and finding the atom in the $|e_0\rangle$. Where $\frac{1}{k_N}$ is the normalization constant. If the desired state is

$$|\psi_d\rangle = \sum_{n=0}^{N} C_n|\underbrace{n,n,\dots n}_{M}\rangle. \tag{5}$$

Compared with Eq. (4), we have

$$\begin{cases} d_0 = \frac{1}{k_N}\frac{\varepsilon_N}{\sqrt{1+|\varepsilon_N|^2}}C_0^{N-1}, \\ d_n = \frac{1}{k_N}\left[\frac{\varepsilon_N}{\sqrt{1+|\varepsilon_N|^2}}C_n^{N-1} + \frac{1}{\sqrt{1+|\varepsilon_N|^2}}C_{n-1}^{N-1}(-i)^M\prod_{i=1}^{M}\sin(\sqrt{n}g_i t_{iN})\right], \\ d_N = \frac{1}{k_N}C_{N-1}^{N-1}\frac{1}{\sqrt{1+|\varepsilon_N|^2}}(-i)^M\prod_{i=1}^{M}\sin\left(\sqrt{N}g_i t_{iN}\right). \end{cases} \tag{6}$$

We can solve this equation set and obtain the coefficients C_n^{N-1} of the state $|\psi_f^{N-1}\rangle$ and parameter ε_N. Taking $|\psi_f^{N-1}\rangle$ as a new desired state and doing the same calculations we obtain the parameter ε_{N-1} for the $(N-1)$th atom and the coefficients C_n^{N-2} for the state $|\psi_f^{N-2}\rangle$. We repeat the procedure until we reach the vacuum state for the cavity fields. Then we obtain a string of parameters $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N$ which define the initial states of a sequence of atoms we have to inject into the cavity in order to obtain the desired state $|\psi_f^N\rangle$ from the vacuum state.

The probability of finding all of the atoms in the lowest state is:

$$P_N = \prod_{j=1}^{N} P_j \tag{7}$$

where

$$P_j = \left\{\frac{\varepsilon_j^2}{1+\varepsilon_j^2} + \frac{\sum_{n=0}^{j-1}\left[C_n^{j-1}\sin^M\left(\sqrt{j+1}gt_j\right)\right]^2}{1+\varepsilon_j^2}\right\}. \tag{8}$$

Here we assume $g = g_1 = g_2 \dots = g_M$ and $t_j = t_{1j} = t_{2j} \dots = t_{Mj}$ for simplicity. In practice, the detectors have a non-zero probability of making an error and the efficiency of detector is 0.8–1 according to ref. [27]. So the actual probability is about

$$P_N = \prod_{j=1}^{N} P_j \times 0.8^N \tag{9}$$

Besides the generation of the correlated states, the scheme can also be used to prepare multiparticle GHZ states under certain conditions.