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# Deterministic quantum dense coding with cluster state in optical systems

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ABSTRACT

We propose two optical schemes for implementing the deterministic single-particle and two-particle quantum dense coding using four-qubit cluster states. In the protocols, the photon is neuter particle, so it has longer decoherence time with the environment than other particles. It is easy to implement single-bit gate using the linear optical elements under certain conditions, so the transformations performed on the photons by Alice can be easily achieved. Here the cluster states can be exactly discriminated using the parity detector, PBS and FS-PBS. In addition, the success probabilities of the dense coding are both equal to 1.



## 1. Introduction

Quantum entanglement shared by more than two parties is essential, since it plays a significant role in the development of quantum communication networks and quantum computation. Quantum dense coding is one of the important applications of quantum entanglement and it allows two classical bits of information to be transmitted by sending only one quantum bit [1]. It has been experimentally demonstrated in both discrete and continuous quantum variable regimes [2–5]. Applying multipartite entanglement and dense coding on quantum communication networks we can transmit more classical information through passing on less quantum resource, so much more attention has been paid to quantum dense coding. Some theoretical schemes for quantum dense coding have been proposed [6–9] by using GHZ state and nonmaximally two-particle entangled states. Recently, some schemes were proposed to implement quantum dense coding with two three-level atoms via cavity QED [10,11]. In experiment, quantum dense coding has been implemented on individual atomic qubits with the use of two trapped $^{9}Be^{+}$ ions [12]. As one of the possible candidates for engineering quantum entanglement, the optical systems always attract much attention [13–15]. This is due to the fact that photons are not only travelling fast and easy to operate, but also uneasy to interact with environment in the transmission process. Mozes et al. first initiated the discussion of deterministic dense coding [16] using both numerical and analytical methods. Later someone give a mathematical proof [17] of the interesting phenomena mentioned in Ref. [16]. In 1996 K. Mattle et al. [2] had reported the realization of a quantum dense coding using polarization entangled photons. However, reliably resolving all four Bell states using linear optics alone is impossible [18], strong nonlinear interactions are needed. Recently, Schuck et al. demonstrated new possibilities of complete Bell measurement and realized an optimal dense coding protocol [19].

In this paper, we propose two schemes to implement the single-particle and two-particle quantum dense codings using four-qubit cluster states in optical systems. In the protocols, Alice sent her photon (s) to Bob after coding on it (them) by linear optical elements. The four (eight) states are completely discriminated by employing quantum nondemolition detectors (QND) parity detectors, PBS and FS-PBS when Bob received Alice's photon(s). According to the outcome of the measurement Bob can distinguish Alice's operations on her photon(s), and he can obtain the two (three) bits of classical information. The QND devices are generally based on cross-Kerr nonlinearities, and the cross-Kerr nonlinearities have become available with electromagnetically induced transparency (EIT) [20]. The photons are neuter particles, so they have longer decoherence time with the environment than other particles. In an experiment, it is easy to implement a single-bit gate using the linear optical elements. The cluster states can be exactly discriminated and the success probabilities are both equal to 1.

The paper is organized as follows. In Sec. 2, we show the scheme to implement single-photon quantum dense coding using cluster state in optical system. In Sec. 3, we present an experimentally feasible scheme to implement the two-photon quantum dense coding using cluster state in optical system. The paper ends with a conclusion.

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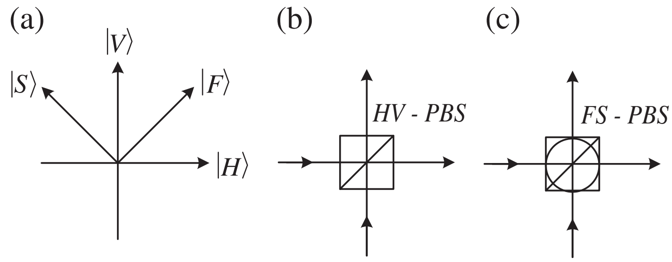


**Fig. 1.** (a) Orientations of the H/V and the F/S polarization basis used in this paper. The F/S basis is rotated $+45^{\circ}$ with respect to the H/V basis. (b) and (c) show the symbols of polarizing beam splitters in the H/V and F/S basis.

## 2. Scheme to implement single-photon quantum dense coding using cluster state

The scheme for generating the four-particle cluster states had been proposed [21] by using two single-photon states and one two-photon polarization entangled state as input resources. The photonic qubit $|0\rangle$ and $|1\rangle$ are represented by horizontal $|H\rangle$ and vertical $|V\rangle$ polarizations in our scheme. We suppose that the four-photon cluster state is given by

$$|\psi\rangle^{+} = \frac{1}{2}[(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4) + (|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 - |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4)]. \quad (1)$$

Consider that photon 1 belongs to Alice, photon 2 belongs to Bob and photons 3 and 4 belong to Charlie. In order to realize a single-photon quantum dense coding, one of the four local operations $\{I, \sigma_1^z, \sigma_1^x, i\sigma_1^y\}$ is performed on photons 1 by Alice. Here $I$ is the identity operator and $\sigma_j^i$ are three Pauli operators of the $j$th photon. Then we introduce how to realize the operations using optical elements. First we introduce half-wave plate (HWP), phase modulator (PM) and QND parity detector *PD* [22]. After photon incident on HWP with its major axis angle $\theta = \pi/4$ to horizontal direction, the operation $\sigma^x$ is implemented, that is $|H\rangle \leftrightarrow |V\rangle$. And when angle $\theta = \pi/8$ the Hadamard transformation is implemented, that is $|H\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. And PM can realize operation $\sigma^z$ on incident photon, that is $|H\rangle \rightarrow |H\rangle$ and $|V\rangle \rightarrow -|V\rangle$. When a photon passes through the PM and then come across the HWP, the operation $i\sigma^y$ is performed. *PD* is used to distinguish the even/odd polarization parity inputs. If Alice performs one of four local operations $\{I, \sigma_1^z, \sigma_1^x, i\sigma_1^y\}$ on photon 1, the state of four photons, which is given in Eq. (1), evolves into the following four states

$$|\psi\rangle^{\pm} = \frac{1}{2}[(|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4) \pm (|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 - |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4)]. \quad (2)$$

$$|\phi\rangle^{\pm} = \frac{1}{2}[(|V\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |V\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4) \pm (|H\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 - |H\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4)]. \quad (3)$$

The polarization conventions [23] that will be used in this paper are shown in Fig. 1. The horizontal and vertical polarizations of photon are presented by $|H\rangle$ and $|V\rangle$, respectively, and measurement is made on the basis of $|F\rangle$ and $|S\rangle$. PBS oriented in the HV basis always transmits H-polarized photons and reflects V-polarized photons. PBS oriented in the FS basis always transmits F-polarized photons and reflects S-polarized photons $|H\rangle = \frac{1}{\sqrt{2}}(|F\rangle - |S\rangle)$, $|V\rangle = \frac{1}{\sqrt{2}}(|F\rangle + |S\rangle)$. After coding on photon 1, Alice sends it to Bob. In order to determine which operation Alice applied Bob has to need Charlie's helps. The setup is depicted in Fig. 2. Suppose that four photons 1, 2, 3 and 4 enter modes A, B, C and D of our cluster state analyzer, respectively. At first, Charlie performs Hadamard transformations on particles 3 and 4, then the states of the four particles evolve into

$$|\psi\rangle^{\pm} = \frac{1}{2}[(|H\rangle_1|H\rangle_2|H\rangle_3 \pm |V\rangle_1|V\rangle_2|V\rangle_3)|H\rangle_4 + (|H\rangle_1|H\rangle_2|V\rangle_3 \pm |V\rangle_1|V\rangle_2|H\rangle_3)|V\rangle_4], \quad (4)$$

$$|\phi\rangle^{\pm} = \frac{1}{2}[(|V\rangle_1|H\rangle_2|H\rangle_3 \pm |H\rangle_1|V\rangle_2|V\rangle_3)|H\rangle_4 + (|V\rangle_1|H\rangle_2|V\rangle_3 \pm |H\rangle_1|V\rangle_2|H\rangle_3)|V\rangle_4]. \quad (5)$$

Charlie sends his photon 4 to pass through a PBS and detects it. We suppose that photon 4 is in the state of $|H\rangle_a$. Then the states of photons 1, 2 and 3 collapse into

$$|\psi\rangle^{\pm} = \frac{1}{2}(|H\rangle_1|H\rangle_2|H\rangle_3 \pm |V\rangle_1|V\rangle_2|V\rangle_3) \quad (6)$$

$$|\phi\rangle^{\pm} = \frac{1}{2}(|V\rangle_1|H\rangle_2|H\rangle_3 \pm |H\rangle_1|V\rangle_2|V\rangle_3) \quad (7)$$

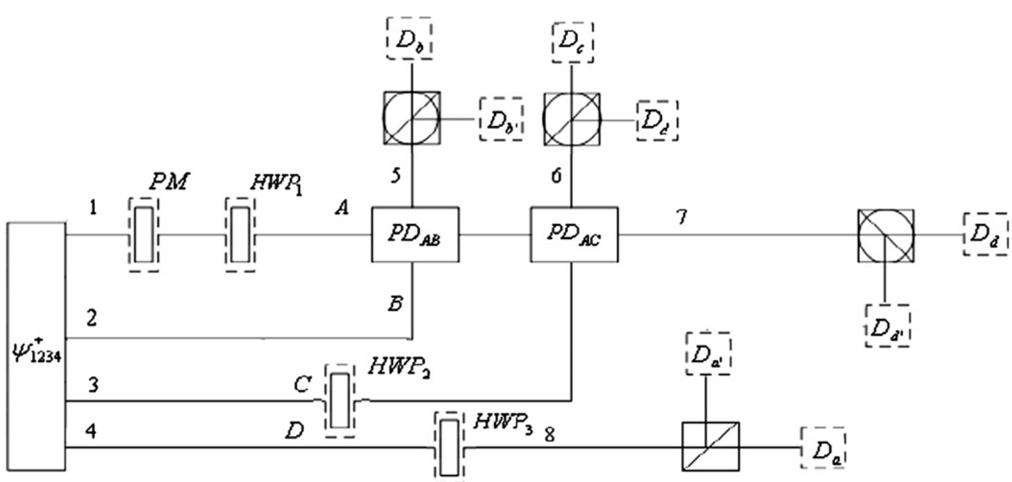


**Fig. 2.** The setup to implement single-photon quantum dense coding using cluster states. We keep down *PM* and withdraw $HWP_1$ to implement the operation $\sigma_1^z$. The other operations are similar, we choose the *PM* or *HWP* according to the requirement. Four photons enter the analyzer from input ports A, B, C and D. $HWP_2$ and $HWP_3$ perform the Hadamard transformation. $PD_{AB}$ and $PD_{AC}$ are QND parity detectors. $D_i (i = a, a', b, b', c, c', d, d')$ are photon detectors.