Optics Communications 284 (2011) 1038–1044

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Accurate semi analytical model of an optical fiber having Kerr nonlinearity using a robust nonlinear unconstrained optimization method

Raja RoyChoudhury [a,*], Arundhati RoyChoudhury [b]

[a] Department of Applied Electronics and Instrumentation Engineering, Sikkim Manipal Institute of Technology, Majitar, Rangpo, East Sikkim-737 132, India
[b] Department of Physics, Sikkim Manipal Institute of Technology, Majitar, Rangpo, East Sikkim-737 132, India

ARTICLE INFO

Article history:
Received 23 December 2007
Received in revised form 6 October 2010
Accepted 6 October 2010

Keywords:
Single mode fiber
Kerr non linearity
Dispersion

ABSTRACT

This paper presents a semi analytical formulation of modal properties of a non linear optical fiber having Kerr non linearity with a three parameter approximation of fundamental modal field. The minimization of core parameter (U) which involves Kerr nonlinearity through the non-stationary expression of propagation constant, is carried out by Nelder–Mead Simplex method of non linear unconstrained minimization, suitable for problems with non-smooth functions as the method does not require any derivative information. Use of three parameters in modal approximation and implementation of Simplex methods enables our semi analytical description to be an alternative way having less computational burden for calculation of modal parameters than full numerical methods.

1. Introduction

In telecommunication and optical signal processing [1,2] nonlinear effects in optical fiber such as, stimulated Raman Scattering (SRS), stimulated Brillouin scattering (SBS), optical Kerr effect etc, find many useful applications. Optical Kerr non linearity is proved to be useful in soliton formation [3], fiber grating pair optical pulse compression [4] and regeneration [5] etc. These applications made use of optical Kerr effect induced various secondary effects, such as, self phase modulation, cross phase modulation, four wave mixing etc. Furthermore, Kerr effect includes optical parametric amplification frequency compensation [6], optical phase conjugation [7]. A proper design of single mode fiber with high nonlinearity, specific dispersion properties and low value of attenuation can render all the above-mentioned applications possible.

But, the vast field of applications of non linear optical fiber is associated with lack of analytical solutions and standardization of the modal parameters. Hence, the non linear phenomenon is generally described by various numerical methods [1,8,9]. Accurate solution of scalar wave equation for non linear fiber is one of the basic requirements and it should also permit easy computations of the modal parameters. Nevertheless, the formulation should be such that, different specifications of the optical fiber can be fitted into the formulation for characterization of the same. Moreover, the fundamental mode is required to predict, variety of parameters in the case of non linearity present in both core and the cladding or only in core [9–12].

In this paper, we present an algorithm, which incorporates a novel approximate solution of the fundamental modal field. We apply Nelder–Mead Simplex method [21] for nonlinear unconstrained minimization as it is widely used to solve parameter estimation and similar statistical problems, where the function values are uncertain, noisy or even discontinuous. Calculation is carried out in Matlab (7.1) platform. The variation of effective index with power can be obtained by our formulation. It is also shown that the calculation of dispersion characteristics of such a fiber with our proposed field yields agreement with the numerical results [9]. The related analytical expressions are also presented.

We implement variational technique as it is proved to be the most useful tool to find the solution of the scalar wave equation in linear [13–15], and non linear fiber [9,12,16,17]. However the effectiveness of the variational method depends on the right choice of the form of the trial field. Our choice of fundamental field has included three parameters, which has an advantage over those found in the literature for similar kind of investigations with single or two parameter approximations [9,12–16]. These three parameters incorporate more flexibility in the formulation of fundamental mode, to meet a particular design specification. Optimized values of these parameters, obtained by Nelder–Mead Simplex methods [21] for nonlinear unconstrained minimization using Matlab software are also presented over a wide range of normalized frequency for a particular specification of optical fiber with Kerr non linearity.

The variational formulation requires optimization for each specification of the optical fiber. Again, the optimization process requires expressions for core parameter U. The analytical expressions for U provided by us involve many fiber parameters, such as, core radius (a), refractive indices of core and cladding (n_{co} and n_{cl}), non linear

* Corresponding author.
E-mail address: ra_ch2@yahoo.co.in (R. RoyChoudhury).

0030-4018/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2010.10.022

coefficient (n_2) and wavelength ($\lambda = 2\pi/k$). Hence, any desired specification can be incorporated by varying these parameters.

Due to non linear effect, the expression of propagation constant (β) is not stationary. So, unlike the linear regime, core parameter (U) could not be directly minimized, as depicted in previous literatures [9]. For this Euler Lagrangian equation is solved which involves formulation of Lagrangian for each form of trial solution. In our novel approach, we implement Nelder–Mead Simplex method [21] of optimization for unconstraint nonlinear minimization using Matlab software. Thus we directly minimize U considering non linear effects through the expression of propagation constant (β). This unique model is much simpler, but yields good agreement with the numerical result [9].

Moreover, the evaluation of integrals presented here, is useful for all non linear effect having same order of non linearity, such as, four photon parametric mixing, stimulated Raman scattering, self phase modulation etc. [6]. These are characterized by third order of non linearity and involve same form of integral as required in the formulation for Kerr non linearity. So, the analytical results presented in this paper may be useful to characterize other non linear properties of optical fibers.

2. Theory

2.1. Basic formulations

The refractive index profile in a Kerr law media is given by [1],

$$n(r) = n_0(r) + n_{nl}I \tag{1}$$

where $n_{\rm nl}$ is the non linear coefficient (m^2/watt), I is the local power density related to intensity of optical field E as,

$$I = \frac{n_0 c\varepsilon_0 |E|^2}{2} \tag{2}$$

where, n_0 is linear refractive index, c is velocity of light in vacuum, ε_0 is permittivity of vacuum ($8.8542 \times 10^{-12}\ F/m$).

More generally, the refractive index profile is expressed as,

$$n^2(r) = n_0^2(r) + f(\alpha\psi^2) \tag{3}$$

where, ψ is the magnitude of the electric field and $\alpha = n_0^2 n_{nl} c\varepsilon_0$.

In Kerr law media [9],

$$f(\alpha\psi^2) = \alpha\psi^2. \tag{4}$$

When Kerr non linearity exists in both core and cladding, Eq. (3) reduces to,

$$n^2 = n_{co}^2 + \alpha\psi^2 \quad \text{for}\, R \le 1 \tag{5.1}$$

$$n^2 = n_{cl}^2 + \alpha\psi^2 \quad \text{for}\, R > 1. \tag{5.2}$$

R is the normalized radial variable ($= r/a$), a being the core radius, n_{co} and n_{cl} are the refractive indices of core axis and cladding respectively. Similarly, if non linearity is confined only to the core, refractive index profile will be described as,

$$n^2 = n_{co}^2 + \alpha\psi^2 \quad \text{for}\, R \le 1 \tag{6.1}$$

$$n^2 = n_{cl}^2 \quad \text{for}\, R > 1. \tag{6.2}$$

In a planar waveguide the transverse electric field ψ satisfies the scalar wave equation as given below [9],

$$\frac{d^2\psi}{dx^2} + k^2 n_0^2 \psi + k^2 f\left(\alpha\psi^2\right)\psi = \beta^2\psi \tag{7}$$

where, k is the wave number and β denotes the propagation constant. If we consider a weakly guiding optical fiber with circular symmetry, the scalar wave equation becomes [9],

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + k^2 n_0^2 \psi + k^2 f\left(\alpha\psi^2\right)\psi = \beta^2\psi. \tag{8}$$

In order to obtain an unconstrained optimization, we present a novel form of fundamental modal field approximation. To begin with, we adopt a similar approach as presented in [9]. Then, to increase the robustness of the algorithm, we introduce optimization.

The proposed field has the following structure,

$$\psi_1 = \frac{A}{a\sqrt{\langle \phi^2 \rangle}}\phi_1, \quad \text{for}\, R \le R_0 \tag{9.1}$$

$$\psi_2 = \frac{A}{a\sqrt{\langle \phi^2 \rangle}}\phi_2 \quad \text{for}\, R > R_0 \tag{9.2}$$

such that,

$$\langle \psi^2 \rangle = I = A^2$$

where,

$$\langle \psi^2 \rangle = \int_0^{R_0} |\psi_1|^2 R dR + \int_{R_0}^{\infty} |\psi_2|^2 R dR \tag{9.3}$$

$$\phi_1 = \frac{\sin(\alpha R / R_0)}{R} \tag{9.4}$$

$$\phi_2 = (\sin(\alpha)/R) e^{\mu(1-(R/R_0))}\sqrt{R_0/R} \tag{9.5}$$

and,

$$\langle \phi^2 \rangle = \int_0^{R_0} |\phi_1|^2 R dR + \int_{R_0}^{\infty} |\phi_2|^2 R dR. \tag{9.6}$$

Modal power [9] can be formulated as,

$$P = A^2 \pi c\varepsilon_0 \beta / k. \tag{10}$$

General expression of propagation constant β for non linear optical fiber having Kerr non linearity is given by [9],

$$\beta^2 = \frac{\left[-\langle \psi'^2 \rangle + k^2 n_0^2 \int_0^{\infty} \psi^2 R dR + k^2 \alpha \int_0^{\infty} \psi^4 R dR\right]}{\langle \psi^2 \rangle} \tag{11.1}$$

where,

$$\langle \psi'^2 \rangle = \int_0^{R_0} \left|\frac{d\psi_1}{dR}\right|^2 R dR + \int_{R_0}^{\infty} \left|\frac{d\psi_2}{dR}\right|^2 R dR. \tag{11.2}$$

Once propagation constant β is evaluated using Eqs. (11.1) and (11.2), the core parameter U can be obtained as,

$$U^2 = a^2\left(k^2 n^2 - \beta^2\right). \tag{12}$$

Here, $n = n_{co}$.

An accurate solution of scalar wave equation for nonlinear optical fiber can be obtained by forming the Lagrangian and solving Euler–Lagrangian equation, with a judicious choice of trial field [9]. Again, as per variational principle a trial field which minimizes U^2 through