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Teleportation of entangled states without Bell-state measurement via a two-photon process

A.D. dSouza, W.B. Cardoso *, A.T. Avelar, B. Baseia

Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia, GO, Brazil

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ABSTRACT

In this letter we propose a scheme using a two-photon process to teleport an entangled field state of a bimodal cavity to another one without Bell-state measurement. The quantum information is stored in a zero- and two-photon entangled state. This scheme requires two three-level atoms in a ladder configuration, two bimodal cavities, and selective atomic detectors. The fidelity and success probability do not depend on the coefficients of the state to be teleported. For convenient choices of interaction times, the teleportation occurs with fidelity close to the unity.

Quantum entanglement [1] is the cornerstone of exotic phenomena in quantum mechanics. It radically differs from ingredients of classical physics and plays an important role to demonstrate fundamental aspects of the theory. Entangled states constitute useful resources to perform tasks that cannot be realized by classical states such as superdense code [2], entangled-based quantum cryptography [3,4], and quantum teleportation [5]. In particular, quantum teleportation provides a mechanism to transfer, from a system to another, the quantum information contained in the state of one or more qubits using a quantum channel (entangled state) plus a classical channel to transfer an additional classical information required to reconstruct the teleported state. Besides being useful for quantum communication via quantum computers, quantum teleportation is fundamental for universal quantum computation [6].

Quantum teleportation has been experimentally proved in various physical contexts, such as in traveling waves [7], optical continuous-variables [8], nuclear magnetic resonance [9], photons in waveguides [10], trapped ions [11], etc. Nonetheless, in the important scenario of microwave cavity QED it remains as a challenge yet. In the theoretical realm, Davidovich et al. [12] proposed a scheme to teleport an unknown atomic state between two high-Q cavities initially prepared in entangled photon number states. An alternative scheme proposed by Cirac and Parkins [13] employed two additional atomic levels of one of a correlated pair to teleport atomic states. Other proposals can also be found in [14–17].

In Ref. [18] Zheng proposed a scheme for approximately and conditionally teleporting an unknown atomic state in a cavity QED, a procedure known as "teleportation without Bell-state measurement". In it, only one particle of the entangled pair should be detected, which projects the other particle in a known state and simplifies the reconstruction of the teleported state. So, the use of a single atomic detection and an appropriate atom–field interaction allows one to distinguish a specific Bell-state among four possibilities. From the experimental point of view, the simplicity of the apparatus is achieved at the expense of a reduction of the success probability. After Ref. [18] various schemes of teleportation without Bell-state measurement were proposed [19–21]. In Ref. [19] a one-photon process described by the Jaynes–Cumming model was used to teleport entangled states from a bimodal cavity to another. In [20] the scheme was extended for teleportation of GHZ-states.

Here we propose a scheme using a two-photon process to teleport an entangled bimodal cavity-field state consisting of a zero- and two-photon from a cavity to another without Bell-state measurement. It is worth mentioning that the two-photon process has been demonstrated in [22] for a microwave QED cavity and offers some advantages in comparison with the one-photon process, as the reduction of interaction times due to the increasing of the atom–field coupling strength and the lower decoherence induced by stray fields [23]. In addition, two-photon process can be easily obtained with Rydberg atoms with principal quantum number $n>89$, which can be state-sensitively detected using tunneling field ionization with quantum efficiencies above 80% and an ionization efficiency above 98% [24]. To describe the two-photon process we will use the two-photon Jaynes–Cummings model in the full microscopical Hamiltonian approach (FMHA) as explained in Ref. [25]. Different from the effective Hamiltonian approach, the FMHA is also valid for small average photon number. In Ref. [26] the reader will find a more detailed discussion about the validity of the effective Hamiltonian approach

* Corresponding author.
E-mail address: wesleybcardoso@gmail.com (W.B. Cardoso).

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and its connection with the FMHA, concerned with quantum teleportation of atomic states.

We will consider a three-level atom that interacts with a single mode of a cavity field. In the absence of a driven field acting upon the atom, the model describing the atom–field interaction is given by the FMHA. In the interaction picture the Hamiltonian reads [25]

$$H_{\mathrm{FMHA}} = \hbar g_1\left(a|e\rangle\langle f|e^{-i\delta t} + a^{\dagger}|f\rangle\langle e|e^{i\delta t}\right) + \hbar g_2\left(a|f\rangle\langle g|e^{i\delta t} + a^{\dagger}|g\rangle\langle f|e^{-i\delta t}\right), \quad (1)$$

where g_1 and g_2 stand for the one-photon coupling constant with respect to the transitions $|e\rangle \leftrightarrow |f\rangle$ and $|f\rangle \leftrightarrow |g\rangle$, respectively. The detuning δ is given by

$$\delta = \Omega - \left(\omega_e - \omega_f\right) = \left(\omega_f - \omega_g\right) - \Omega, \quad (2)$$

where Ω is the cavity-field frequency and ω_e, ω_f, and ω_g are the frequencies associated with the atomic levels $|e\rangle$, $|f\rangle$, and $|g\rangle$, respectively. In what follows we present a brief review of our work in [26].

The state that describes the combined atom–field system is written as

$$|\psi(t)\rangle = \sum_n \left[C_{e,n}(t)|e,n\rangle + C_{f,n}(t)|f,n\rangle + C_{g,n}(t)|g,n\rangle\right], \quad (3)$$

where the $|k,n\rangle$, with $k = e,f,g$, indicate the atom in the state $|k\rangle$ and the field in the Fock state $|n\rangle$. The coefficients $C_{k,n}(t)$ stand for the corresponding probability amplitudes.

The insertion of Eqs. (1) and (3) in the time dependent Schrödinger equation furnishes the coupled first-order differential equations for the probability amplitudes

$$\begin{aligned} \frac{dC_{e,n}(t)}{dt} &= -ig_1 C_{f,n+1}(t)\sqrt{n+1}e^{-i\delta t}, \\ \frac{dC_{f,n+1}(t)}{dt} &= -ig_1 C_{e,n}(t)\sqrt{n+1}e^{i\delta t} - ig_2 C_{g,n+2}(t)\sqrt{n+2}e^{i\delta t}, \\ \frac{dC_{g,n+2}(t)}{dt} &= -ig_2 C_{f,n+1}(t)\sqrt{n+2}e^{-i\delta t}. \end{aligned} \quad (4)$$

As usually, we consider the entire atom–field system as decoupled at the initial time $t = 0$,

$$\begin{aligned} C_{e,n}(0) &= C_e C_n(0), \\ C_{b,n+1}(0) &= C_f C_{n+1}(0), \\ C_{c,n+2}(0) &= C_g C_{n+2}(0), \end{aligned} \quad (5)$$

where the $C_n(0)$ stand for the amplitudes of the arbitrary initial field state and the C_a are atomic amplitudes of the (normalized) initial atomic state

$$|\chi\rangle = C_e|e\rangle + C_f|f\rangle + C_g|g\rangle. \quad (6)$$

From the solution of these coupled differential equations with the initial conditions in Eq. (5) we get the time dependent coefficients as

$$C_{e,n}(t) = \left[\frac{g_1^2(n+1)}{\Lambda_n \alpha_n^2}\gamma_n(t) + 1\right]C_e C_n - i\frac{g_1\sqrt{n+1}}{\Lambda_n}\sin(\Lambda_n t)e^{-i\frac{\delta t}{2}}C_f C_{n+1} + \left[\frac{g_1 g_2\sqrt{(n+1)(n+2)}}{\Lambda_n \alpha_n^2}\gamma_n(t)\right]C_g C_{n+2}, \quad (7)$$

$$C_{f,n+1}(t) = -i\frac{g_1\sqrt{n+1}}{\Lambda_n}\sin(\Lambda_n t)e^{i\frac{\delta t}{2}}C_e C_n + \left(\cos(\Lambda_n t) - \frac{i\delta}{2\Lambda_n}\sin(\Lambda_n t)\right)e^{i\frac{\delta t}{2}}C_f C_{n+1} - i\frac{g_2\sqrt{n+2}}{\Lambda_n}\sin(\Lambda_n t)e^{i\frac{\delta t}{2}}C_g C_{n+2}, \quad (8)$$

$$C_{g,n+2}(t) = \frac{g_1 g_2\sqrt{(n+1)(n+2)}}{\Lambda_n \alpha_n^2}\gamma_n(t)C_e C_n - i\frac{g_2\sqrt{n+2}}{\Lambda_n}\sin(\Lambda_n t)e^{-i\frac{\delta t}{2}}C_f C_{n+1} + \left[\frac{g_2^2(n+2)}{\Lambda_n \alpha_n^2}\gamma_n(t) + 1\right]C_g C_{n+2}, \quad (9)$$

where

$$\gamma_n(t) = \left[\Lambda_n\cos(\Lambda_n t) + i\frac{\delta}{2}\sin(\Lambda_n t) - \Lambda_n e^{i\frac{\delta t}{2}}\right]e^{-i\frac{\delta t}{2}}, \quad (10)$$

$$\Lambda_n = \sqrt{\frac{\delta^2}{4} + \alpha_n^2}, \quad (11)$$

$$\alpha_n = \sqrt{g_1^2(n+1) + g_2^2(n+2)}, \quad (12)$$

Λ_n being the Rabi frequency. The substitutions $n \rightarrow n-1$ in Eq. (8) and $n \rightarrow n-2$ in Eq. (9) allow one to obtain the $C_{f,n}(t)$ and $C_{g,n}(t)$, respectively.

Next, we first consider an entangled state of zero- and two-photons previously prepared in modes "3" and "4" of the cavity C_2, as follows

$$|\phi(0)\rangle_{34} = \alpha|0,2\rangle_{34} + \beta|2,0\rangle_{34}, \quad (13)$$

where α and β are unknown coefficients, with $|\alpha|^2 + |\beta|^2 = 1$. For details see Ref. [27], where we have recently shown how to generate the EPR and W entangled states of zero- and two-photons. The nonlocal channel is constructed by two three-level atoms (designed by subindex a and b) in a ladder configuration (Fig. 1) and the modes "1" and "2" of the cavity C_1. This nonlocal channel is prepared with the atoms previously prepared in their excited states ($|e\rangle_{a,b}$) and the cavity-field modes in the vacuum state ($|0,0\rangle_{12}$). Then, the atom a is sent to interact only with the mode 1 and soon after the atom b is led

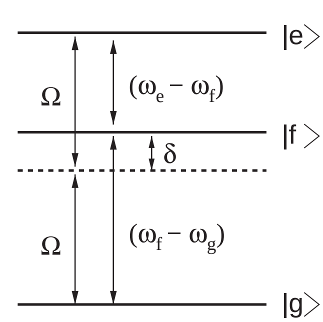

Fig. 1. Schematic diagram of the three-level atom interacting with a single mode of a cavity field.