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Scheme to generate three-mode continuous-variable entanglement in cavity quantum electrodynamics

Qing-Xia Mu *, Yong-Hong Ma, Ling Zhou

School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, China

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ABSTRACT

We propose a theoretical method of generating a three-mode continuous-variable entanglement in an optical cavity with an atomic cloud. The scheme uses Raman transitions between stable atomic ground states and under the input–output theory a three-mode entangled light of the output fields can be produced. The characters of the tripartite entanglement, the degree of the quadrature-phase amplitude correlations among the three modes are discussed by applying a sufficient inseparability criterion for multipartite continuous-variable entanglement, which was proposed by van Loock and Furusawa. The dependences of the correlation on the effective coupling constants are theoretically analyzed.

1. Introduction

Quantum entanglement has attracted great interest as it is considered to be the most important resource for future quantum information and quantum computation. In particular, continuous-variable (CV) entanglement is a key element in Einstein–Podolsky–Rose paradox [1] and has been widely researched due to its unconditionalness for the implementation of many quantum information processes, such as unconditional quantum teleportation [2], quantum dense coding [3], entanglement swapping [4], and quantum telecloning [5]. Recently, it seems that people are putting more effort on the generation of CV entanglement, which mainly involves parametric amplification or oscillation process [6], the Kerr effect in optical fibers [7], and atomic ensembles or a single atom with cavity QED [8–11]. For instance, Guzmán [8] proposed a method to implement unitary single mode and two-mode field squeezing from an atomic cloud in an optical cavity. We also present a scheme to generate output squeezing and entanglement of two cavity modes from a single atom [11]. However, the above work is confined to the bipartite systems.

It is shown that multipartite CV entanglement has become the key ingredient for advanced multipartite quantum teleportation networks [12], controlled dense coding [13], and quantum telecloning [14]. Therefore, much attention has been paid to the production of multipartite (especially tripartite) entanglement, such as combining squeezed states on beam splitters [13,15], or via the interaction of multiple input beams in nonlinear optical material with parametric process [16–19]. So far a few proposals have been demonstrated on the generation of bright tripartite entanglement light via atom–cavity system [20,21]. L*ü* et al. [20] proposed a scheme for achieving fully tripartite CV entanglement in a tripartite correlated emission laser consisting of the four-level Y-type atomic ensemble and a triply resonant cavity. Li et al. [21] studied theoretically the steady-state tripartite entanglement in a three-mode quantum-beat laser that operates well above threshold. On the other hand, the generation of entanglement among three bright beams of light has been demonstrated experimentally [22].

In this research, we extend the two-mode cavity system [8] to the three-mode case and explore tripartite continuous-variable entanglement, consisting of an optical cavity with an atomic cloud. The interaction of three cavity fields is formed by beating them in three distinct Raman transitions between the atomic ground states. Based on the standard criteria proposed by van Loock and Furusawa, we show that genuinely tripartite entanglement with distinct frequencies can be produced at the output. The dependences of quantum correlations of the amplitude and phase quadratures among the three cavity modes on the effective coupling constants are also discussed.

2. The model and calculations

The system consists of N identical four-level atoms, which are placed in a three-mode optical cavity. The atomic ensemble can be sodium atoms in a vapour cell, where the lower states are two hyperfine levels of $|F=1\rangle$ and $|F=2\rangle$ of $3^2S_{1/2}$, and the upper states are $|F'=1\rangle$ and $|F'=2\rangle$ of $3^2P_{1/2}$, respectively. The energy level configuration of each atom is shown in Fig. 1. The levels $|0\rangle$ and $|1\rangle$ are stable with a long lifetime. The energy of the level $|0\rangle$ is taken to be zero as the reference point. The lower lying level $|1\rangle$ and the upper levels $|2\rangle$ and $|3\rangle$ have the energy ω_1, ω_2 and ω_3 ($\hbar=1$), respectively.

* Corresponding author.
E-mail address: zhlhxn@dlut.edu.cn (L. Zhou).

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We assume the atoms are initially prepared in their ground states $|0\rangle$. The atomic transition $|1\rangle \leftrightarrow |3\rangle$ is driven by a classical field of frequency ω_{L_1} with detuning Δ, while two classical fields of frequencies ω_{L_2} and ω_{L_3} are applied to the atomic transition $|0\rangle \leftrightarrow |2\rangle$ with the detunings being $\pm\Delta'$, respectively. Meanwhile, three nondegenerate modes ν_1, ν_2 and ν_3 of the cavity are coupled with the atomic transitions $|0\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ with the detunings $\Delta - \delta$ and $\pm\Delta' - \delta'$, respectively. Then the Hamiltonian for the system can be written as

$$H = H_0 + H_{int}, \tag{1}$$

with

$$H_0 = \sum_{k=1}^{N} \sum_{i=0}^{3} \omega_i |i\rangle_k \langle i| + \sum_{j=1}^{3} \nu_j a_j^\dagger a_j, \tag{2}$$

and

$$\begin{aligned} H_{int} = \sum_{k=1}^{N} \Big[& g_1 a_1 |3\rangle_k\langle 0| + g_2 a_2 |2\rangle_k\langle 1| + g_3 a_3 |2\rangle_k\langle 1| \\ & + \Omega_1 e^{-i\left(\omega_{L_1} t - \phi_1\right)} |3\rangle_k\langle 1| + \Omega_2 e^{-i\left(\omega_{L_2} t - \phi_2\right)} |2\rangle_k\langle 0| \\ & + \Omega_3 e^{-i\left(\omega_{L_3} t - \phi_3\right)} |2\rangle_k\langle 0| \Big] + H.c. \end{aligned} \tag{3}$$

Here a_j $(a_j^\dagger)$ $(j = 1, 2, 3)$ are the annihilation (creation) operators of the cavity modes with the corresponding coupling constants g_j $(j = 1, 2, 3)$, respectively. Ω_j are the Rabi frequencies of the classical fields with the relative phases ϕ_j. We now give a description of the mechanism for pumping light. Firstly, the classical field Ω_2 pumps the atom from the ground state $|0\rangle$ to the excited state $|2\rangle$, and the photon a_2 was emitted from the transition $|2\rangle \longleftrightarrow |1\rangle$ under the action of cavity mode 2; and then the classical field Ω_1 pumps the atom from the state $|1\rangle$ to the excited state $|3\rangle$, and the photon of cavity mode 1 was emitted when the atomic transition $|3\rangle \leftrightarrow |0\rangle$ happens. In this interaction process, the effective Hamiltonian can be written as $a_1^\dagger a_2^\dagger$. Then, the classical field Ω_3 pumps the atom from the ground state $|0\rangle$ to the excited state $|2\rangle$, and under the action of cavity mode 3, the photon a_3 was emitted due to the transition $|2\rangle \longleftrightarrow |1\rangle$; and then like before, the classical field Ω_1 pumps the atom from the state $|1\rangle$ to the excited state $|3\rangle$, and the photon a_1 was emitted from transition $|3\rangle \longleftrightarrow |0\rangle$ under the cavity mode 1. The effective Hamiltonian in this process can be written as $a_1^\dagger a_3^\dagger$. In addition, while the cavity mode 2 leads to the emission of photon a_2 from the transition $|2\rangle$ to $|1\rangle$, the atom can also be excited to the state $|2\rangle$ by absorbing photon a_3. In this interaction process, the effective Hamiltonian can be written as $a_2^\dagger a_3$.

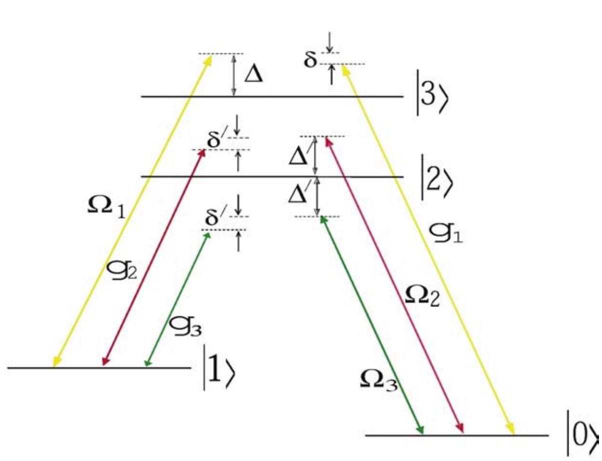

Fig. 1. Level scheme of the atomic system. The atomic transitions $|0\rangle \leftrightarrow |3\rangle$, $|1\rangle \leftrightarrow |2\rangle$ are coupled with the three nondegenerate modes ν_1, ν_2 and ν_3 of the cavity, while the atomic transitions $|1\rangle \leftrightarrow |3\rangle$ and $|0\rangle \leftrightarrow |2\rangle$ are induced by three classical fields with angular frequencies ω_1, ω_2 and ω_3.

We assume, without loss of generality, that all the cavity modes coupling frequencies g_j and the laser Rabi frequencies Ω_j are real numbers. By taking $\phi_1 = \phi_3 = -\phi_2 = -\frac{\pi}{2}$, the associated Hamiltonian in the interaction picture reads

$$\begin{aligned} H_I = \sum_{k=1}^{N} \Big[& g_1 a_1 e^{-i(\Delta-\delta)t} |3\rangle_k\langle 0| + g_2 a_2 e^{-i\left(\Delta'-\delta'\right)t} |2\rangle_k\langle 1| \\ & + g_3 a_3 e^{i\left(\Delta'+\delta'\right)t} |2\rangle_k\langle 1| - i\Omega_1 e^{-i\Delta t} |3\rangle_k\langle 1| \\ & + i\Omega_2 e^{-i\Delta' t} |2\rangle_k\langle 0| - i\Omega_3 e^{i\Delta' t} |2\rangle_k\langle 0| \Big] + H.c. \end{aligned} \tag{4}$$

We consider the dispersive detunings Δ, Δ' are sufficiently large, i.e., $\{|\Delta|, |\Delta'|\} \gg \{g_j, \Omega_j, |\delta|, |\delta'|\}$. Under the large detuning condition, the population of atoms will not change. If the atoms are initially not populated in the upper states $|2\rangle$ and $|3\rangle$ they will not populate in these states during the interaction. Then, we eliminate adiabatically the levels $|2\rangle$ and $|3\rangle$ and obtain the effective Hamiltonian [23]

$$\begin{aligned} H_{eff} = & \left[-\delta + \frac{g_1^2}{\Delta}\left(\frac{N}{2} - J_Z\right)\right] a_1^\dagger a_1 + \left[-\delta' + \frac{g_2^2}{\Delta'}\left(\frac{N}{2} + J_Z\right)\right] a_2^\dagger a_2 \\ & + \left[-\delta' - \frac{g_3^2}{\Delta'}\left(\frac{N}{2} + J_Z\right)\right] a_3^\dagger a_3 + \frac{\Omega_1^2}{\Delta}\left(\frac{N}{2} + J_Z\right) \\ & + \left(\frac{\Omega_2^2}{\Delta'} - \frac{\Omega_3^2}{\Delta'}\right)\left(\frac{N}{2} - J_Z\right) + \left(i\lambda_1 a_1 + i\lambda_2 a_2^\dagger + i\lambda_3 a_3^\dagger\right)\sqrt{N} J^+ + H.c., \end{aligned} \tag{5}$$

where

$$J_z = \frac{1}{2}\sum_{k=1}^{N} \left(|1\rangle_k\langle 1| - |0\rangle_k\langle 0|\right), \; J_+ = \sum_{k=1}^{N} \left(|1\rangle_k\langle 0|\right), \tag{6}$$

are the collective atomic spin operators. $\lambda_1 = \Omega_1 g_1/\Delta$, $\lambda_2 = \Omega_2 g_2/\Delta'$ and $\lambda_3 = \Omega_3 g_3/\Delta'$ are the effective coupling constants of the atom ensemble to the cavity modes.

In the Holstein–Primakoff representation [24], the collective atomic operators may be transformed into harmonic-oscillator annihilation and creation operators b and $b^\dagger$ of a single bosonic mode via $J_+ = b^\dagger\sqrt{N - b^\dagger b}$ and $J_z = b^\dagger b - \frac{N}{2}$. We consider that only the low-lying collective excitations in the atoms, that is to say the mean number of atoms transferred to the states $|1\rangle$ is much smaller than the total number of atoms, i.e., $\langle b^\dagger b\rangle \ll N$. Then the collective atomic operators can be approximated as $J_+ = \sqrt{N} b^\dagger$, and $J_z = -N/2$ [25,26]. Substituting these expressions into Hamiltonian (5) and choosing $\delta = \frac{N g_1^2}{\Delta_1}$, $\delta' = 0$, we find the effective Hamiltonian H_{eff} can be reduced to the form

$$H_{eff} = \left(i\lambda_1 a_1 + i\lambda_2 a_2^\dagger + i\lambda_3 a_3^\dagger\right)\sqrt{N} b^\dagger + H.c, \tag{7}$$

where the constant energy terms have been omitted. In the first term of this effective Hamiltonian, the bosonic mode of the collective atoms is coupled to the cavity mode a_1 through the linear-mixing process and via this process the possibility of exchanging the quantum information between bosonic mode b and the cavity mode a_1. Meanwhile the bosonic mode b is also entangled with the cavity mode a_2 (a_3) via the nondegenerate parametric interaction in the second (third) term, which can lead to the generation of continuous-variable entanglement between the bosonic mode b and cavity mode a_2 (a_3). Thus, mediated by the bosonic mode of the collective atoms, the entanglement of the three cavity modes can be generated inside