Optics Communications 284 (2011) 1094–1098

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Preparation of Greenberger–Horne–Zeilinger and W states of three atoms trapped in one cavity through cavity output process

Yan Xia [a,b,*], Jie Song [b], Pei-Min Lu [a], He-Shan Song [b]

[a] Department of Physics, Fuzhou University, Fuzhou 350002, China
[b] School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 9 July 2010
Received in revised form 17 September 2010
Accepted 9 October 2010

Keywords:
Entangled states
Three-level $\wedge$-type atoms
Linear optical elements

ABSTRACT

We propose a protocol to generate Greenberger–Horne–Zeilinger (GHZ) and W states of three atoms trapped in only one cavity. The setup involves one cavity and linear optical elements. The quantum information of each qubit is skillfully encoded on the degenerate ground states of the three different atoms, hence the entanglement between them is relatively stable against spontaneous emission. The advantages of the protocol are their robustness against detection inefficiency and asynchronous emission of the photons. We discuss the issue related to the practical implementation and show that the protocol is accessible within the current cavity QED technology and linear optical technology.

1. Introduction

Quantum entanglement is a resource for quantum information processing and quantum computing. Entangled states of two or many qubits not only give the possibility to test quantum mechanics against a local hidden variable theory [1,2], but also have practical applications in realizing quantum information processing protocols, such as quantum teleportation [3,4], quantum secret sharing [5], quantum cryptography [6], quantum secure direct communication [7,8], quantum cloning machine [9], and so on. These concepts motivated and intensive research in the generation and the manipulation of entangled states.

For tripartite systems, it has been known that there exist at least two different types of multipartite entanglement: namely, the Greenberger–Horne–Zeilinger-type (GHZ-type) entanglement [10] and the W-type entanglement [11]. These two different types of entanglement are not equivalent and cannot be converted to each other by local unitary operations combined with classical communication. As two classes of important quantum resources, great effort has been taken to studying of entangled state generation in the past years [12–24]. For example, in Refs. [12], the authors have proposed protocols to generate multi-particle entangled state in cavity. In Ref. [13], the authors have proposed a protocol to generate N distant photons GHZ state with linear optical elements. On the other hand, many works have been proposed to generate entangled states using the combination of separate cavities and optics elements [17–23]. For example, in Ref. [17], the authors have proposed protocols to generate of multi-particle entangled state with separate cavities and linear optical elements. Very recently, Su et al. [22] have presented the first experimental results generating continuous variable quadripartite GHZ entangled state of electromagnetic fields. The approach based on indistinguishability was shown to have a lot of advantages, among them the robustness is the most distinct one.

Experimentally, it is difficult to generate multi-atom entangled states with only cavity technology and the inevitable interaction between system and environment will destroy the system quantum coherence, i.e., causing decoherence. In particular, the partial quantum information of a qubit is encoded on the excited state of atoms (ions) in some protocols, which means that the entanglement of the qubits is fragile (not stable). If two- or more atoms trapped in only one multi-mode cavity, and we want to generate atoms entangled states with the help of linear optical elements, how can we do this? This problem has not been addressed.

We propose a simple protocol to generate stable maximally entangled GHZ and W states of three atoms trapped in only one multi-mode cavity with the help of linear optical elements. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over long distances. The protocol is based on the combination of the atom-cavity interaction and linear optics elements. The success of the protocol depends upon the detection of a photon leaking out of the cavity, and thus the fidelity is also not affected by the imperfection of the photon detectors.

The model we are considering consists of three different Λ-type three-level atoms (Fig. 1), with the three atoms $(1,2,3)$ are trapped in one three-mode optical cavity A, as shown in Fig. 2. All the three atoms have one degenerate excited states $|e\rangle_j$ $(j=1,2,3)$, two degenerate

* Corresponding author. Department of Physics, Fuzhou University, Fuzhou 350002, China. Tel.:+86 591 22865133.
E-mail address: xia-208@163.com (Y. Xia).

0030-4018/$ – see front matter.
doi:10.1016/j.optcom.2010.10.036

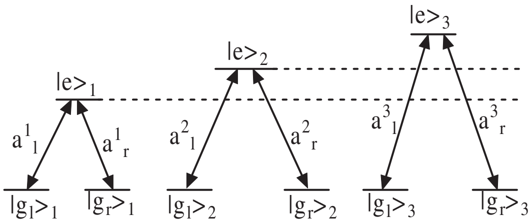

Fig. 1. Atomics level structures.

ground states $|g_l\rangle_j$ and $|g_r\rangle_j$, as shown in Fig. 1. The quantum information is encoded on the states $|g_l\rangle$ and $|g_r\rangle$. The three atoms $(1,2,3)$ transition $|e\rangle_j\to|g_l\rangle_j$ and $|e\rangle_j\to|g_r\rangle_j$ are strongly coupled with left and right circularly polarizing cavity modes F_j, respectively. The frequencies of the cavity modes are different but they are no better than each other, that is $F_1\approx F_2\approx F_3$. The atomic level structure can be achieved by Zeeman sublevels [25] and has been realized to entangle two atoms [26]. We suppose the three atoms are all initially prepared in their excited states and cavity in the vacuum state. We require here that the cavity is one-sided such that the photons leakage occurs through the side of the cavity facing the linear optical elements. The Hamiltonian governing the evolution of the atom-cavity systems can be given in the interaction picture by (setting $\hbar=1$)

$$H=\sum_{j=1,2,3}\left(\lambda_L^j a_L^j|e\rangle_{jj}\langle g_l| + \lambda_R^j a_R^j|e\rangle_{jj}\langle g_r| + h.c.\right), \tag{1}$$

where L, R denote the left- and right-circularly polarizing cavity modes F_1, F_2, and F_3, a_k^{j+}, a_k^j $(k=R,L)$ are the creation and annihilation operators of the j mode in the cavity A and λ_k^j are the coupling constants (atom 1 vs mode F_1, and atom 2 vs mode F_2, and atom 3 vs mode F_3, The frequencies of the cavity modes are different but they are no better than each others, that is $F_1\approx F_2\approx F_3$. The atoms and the cavity are prepared initially in its excited states $|eee\rangle_{123}$ and vacuum states $|00\rangle_{lr}^j$, respectively. The upper levels $|eee\rangle_{123}$ can decay to the two degenerate ground states $|g\rangle_l^j$ and $|g\rangle_r^j$ with the rates $2\gamma_l^j$ and $2\gamma_r^j$, respectively, and the cavity has a leakage rate 2κ. Hence, the master equation describing the evolution of density operator ρ (atom and cavity) is given by

$$\begin{aligned}\dot\rho=&-i\left(H_{eff}\rho-\rho H_{eff}^\dagger\right)+2\kappa\sum_{j=1,2,3}\left(a_L^j\rho a_L^{j\dagger}+a_R^j\rho a_R^{j\dagger}\right)\\&+2\sum_{j=1,2,3}\left(\gamma_l^j|g_l\rangle_{jj}\langle e|\rho|e\rangle_{jj}\langle g_l|+\gamma_r^j|g_r\rangle_{jj}\langle e|\rho|e\rangle_{jj}\langle g_r|\right),\end{aligned} \tag{2}$$

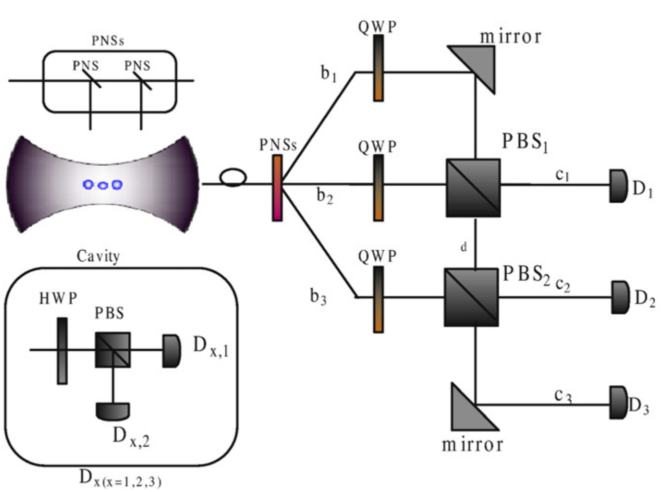

Fig. 2. Experimental setup for generation of three atoms GHZ state. The three atoms are trapped in only one cavity. *PBS* is polarizing beam splitter, *HWP* is half-wave plate, *QWP* is quarter-wave plate, *PNS* is photon number splitter (*PNS*: 50/50) and *D* is detector.

where

$$H_{eff}=H-i\kappa\sum_{j=1,2,3}\left[a_L^{j\dagger}a_L^j+a_R^{j\dagger}a_R^j-i\left(\gamma_l^j+\gamma_r^j\right)|e\rangle_{jj}\langle e|\right]. \tag{3}$$

If the whole state of the initial state is given by $|eee\rangle_{123}|000\rangle_{M_1M2M3}$, and after a enough long time t, the whole state of the system will become (assumed pure state for simplicity)

$$|\psi(t)\rangle_j=\prod_{j=1,2,3}\left(\frac{x_j|e\rangle_j|0_l\rangle_j|0_r\rangle_j+y_j|g_l\rangle_j|1_l0_r\rangle_j+z_j|g_r\rangle_j|0_l1_r\rangle_j}{\sqrt{|x|^2+|y|^2+|z|^2}}\right), \tag{4}$$

where

$$\begin{aligned}x_j=e^{-\frac{\gamma_L^j+\gamma_R^j+\kappa}{2}t}\Bigg[&\cos\left(t\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)\\&+\frac{\kappa-\gamma_l^j-\gamma_r^j}{2\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}}\sin\left(t\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)\Bigg],\end{aligned} \tag{5}$$

$$y_j=-e^{-\frac{\gamma_L^j+\gamma_R^j+\kappa}{2}t}\frac{\sin\left(t\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)}{\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}}\lambda_L^j, \tag{6}$$

$$z_j=-e^{-\frac{\gamma_L^j+\gamma_R^j+\kappa}{2}t}\frac{\sin\left(t\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)}{\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}}\lambda_R^j. \tag{7}$$

After the transformation the atom–cavity interaction is frozen since $H_{eff_j}|\psi(t)\rangle_j=0$. Now we wait for the photodetectors to click. We suppose the evolution time of every subsystems (atom 1 and cavity mode F_1, atom 2 and cavity mode F_2 and atom 3 and cavity mode F_3) to be τ_j $(\tau_1=\tau_2=\tau_3)$. So, in such an interval of time, one can obtain the state $|\psi_\tau\rangle_j$ given by Eq. (4) with the probability

$$\begin{aligned}P_j=e^{-(\gamma_L^j+\gamma_R^j+\kappa)\tau_j}\Bigg\{&\Bigg[\cos\left(\tau_j\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)\\&+\frac{\kappa-\gamma_l^j-\gamma_r^j}{2\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}}\sin\left(\tau_j\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)\Bigg]^2\\&+\frac{\sin^2\left(\tau_j\sqrt{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\right)}{\lambda_L^{j^2}+\lambda_R^{j^2}-\frac{\left(\kappa-\gamma_l^j-\gamma_r^j\right)^2}{4}}\left(\lambda_L^{j^2}+\lambda_R^{j^2}\right)\Bigg\}.\end{aligned} \tag{8}$$

Thus, the joint state of the three atoms and three modes can be given by $P_1=\prod P_j$. Because the vacuum state has no contribution to the click of the photon-detectors, so the term $|e\rangle_j|0_l\rangle_j|0_r\rangle_j$ in Eq. (4) can