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Polarization and intensity properties of converging J_0-correlated Schell-model beams

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ABSTRACT

Based on the cross-spectral density matrix, closed form result for propagation equation of electromagnetic J_0-correlated Schell-model beams (EJSMBs) through a paraxial optical system is obtained and the focusing properties are studied. Both numerical calculation and physical interpretation are obtained. It is found that a tunable dark hollow area, which has potential applications in optical trapping, can be obtained by altering the coherence parameter and the focal length. It is also shown that even though the original field is unpolarized, the beams can become fully polarized in the focal region with its width being tunable by changing the coherence parameter. The relevance of this work to applications such as coherent detection in optical communication is also discussed.

1. Introduction

In the past decades, the partially coherent beams have been investigated extensively because of their wide applications in modern optics [1–4]. However, much work has been done on the characteristics and propagation of the partially coherent beams [1–6]. A typical example, i.e., the Gaussian Schell-model beams (GSMBs) has attracted much attention [6,7]. There is another type of partially coherent beam originated by a J_0-correlated Schell-model planar source, which is referred as electromagnetic J_0-correlated Schell-model beams (EJSMBs) [1]. They find that the intensity profile of the beams is analogous to the Bessel–Gaussian beams, but the degree of coherence is not shift-invariance. Later Borghi proposed a simple model of the incoherent superpositions of the suitably shifted and titled Gaussian beams to describe J_0-correlated Schell-model beams (JSMBs) and derived the M^2 factor of the JSMBs [8].

Earlier analysis of the J_0-correlated Schell-model beams is carried out within a scalar representation [8–11]. It is worthwhile to investigate the EJSMBs that are both partially polarized and partially coherent. In this paper, we extend the scalar model of J_0-correlated Schell-model beams to the vector form by using the cross-spectral density matrix. With the analytical expressions, the focusing properties of the EJSMBs, i.e., the intensity distribution and the polarizations are analyzed. We obtain a tunable dark hollow area and a fully polarized region around the focus.

2. Propagation of beam cross-spectral density matrix of the EJSMBs

The EJSMBs are generated by a planar, secondary, statistically stationary stochastic source, located in the plane $z=0$, close to the z direction and radiating into the half-space $z \geq 0$. The second-order correlation properties of the beam in the space-frequency domain may be characterized by the (electric field) cross-spectral density matrix [4,11–13]:

$$W(r_1,r_2,\omega) = \begin{pmatrix} W_{xx}(r_1,r_2,\omega) & W_{xy}(r_1,r_2,\omega) \\ W_{yx}(r_1,r_2,\omega) & W_{yy}(r_1,r_2,\omega) \end{pmatrix}, \tag{1}$$

where ω is the frequency and r_1 and r_2 are the radius vectors corresponding to two typical points in the $z=0$ plane. Each element of the cross-spectral density matrix can be expressed as [4,12–14]:

$$W_{ij}(r_1,r_2,\omega) = \langle E_i^*(r_1,\omega)E_j(r_2,\omega)\rangle \quad (i,j = x,y), \tag{2}$$

where the asterisk denotes the complex conjugate and the angular brackets represent the average over the statistical ensemble of the beams. For the case of the EJSMBs, the spectral degree of coherence is the Bessel function of the first kind, zero order. In this paper, we consider a source whose spectral density of the electric fields S_i and S_j are of Gaussian shape. Then the elements of the cross-spectral density matrix have the form [5]:

$$W_{ij}(r_1,r_2,\omega) = \sqrt{S_i(r_1,\omega)}\sqrt{S_j(r_2,\omega)}\mu_{ij}(r_2-r_1,\omega),(i,j = x,y) \tag{3}$$

$$S_i(r,\omega) = A_i^2 \exp\left[-\frac{r^2}{2\sigma_0^2}\right], \tag{4}$$

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$$\mu_{ij}(r_2-r_1,\omega) = B_{ij}J_0\left(\beta_{ij}|r_2-r_1|\right). \tag{5}$$

Where A_i and B_{ij} are amplitude factors, σ_0 is the spot-size of the Gaussian distribution and β_{ij} is real constant. These parameters are independent on positions, but in general, depend on frequency. According to the paraxial propagation formula of the partially coherent theory, the propagation of the EJSMBs through an optical system parameterized by transfer matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is characterized by [4,14]:

$$W_{ij}(r_1',r_2',z,\omega) = \left(\frac{k}{2\pi B}\right)^2 \iint W_{ij}(r_1,r_2,0,\omega) \times \exp\left\{\frac{-ik}{2B}\left[A\left(r_1^2-r_2^2\right) -2(r_1\cdot r_1'-r_2\cdot r_2') + D\left(r_1'^2-r_2'^2\right)\right]\right\}dr_1 dr_2, \tag{6}$$

where r_1' and r_2' represent two points in the object plane, and k is the wave number related to the wavelength λ by $k=2\pi/\lambda$. In focusing condition, the transfer matrix can be described as $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1-z/f & z \\ -1/f & 1 \end{pmatrix}$, where f is the focal length. After tedious integral calculation, we obtain:

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) = {} & \frac{A_iA_jB_{ij}\pi^2N_f^2}{M} \times \exp\left[-\frac{\pi^2N_f^2\left(\rho_1^2+\rho_2^2\right)}{M}\right] \\ & \times \exp\left[i\frac{\pi N_f\left(1+\Delta z/f+\pi^2N_f^2\Delta z/f\right)}{M}\left(\rho_2^2-\rho_1^2\right)\right] \\ & \times \exp\left[-\frac{2\sigma_{ij}^2(1+\Delta z/f)^2}{M}\right] \times J_0\left\{\frac{2\pi N_f\sigma_{ij}}{\sqrt{M}}\right. \\ & \left.\times\left[\rho_1^2\exp(2i\theta(z)) + \rho_2^2\exp(-2i\theta(z)) - 2\rho_1\rho_2\cos(\varphi_1-\varphi_2)\right]^{\frac{1}{2}}\right\}, \end{aligned} \tag{7}$$

and

$$M = \pi^2N_f^2(\Delta z/f)^2 + (1+\Delta z/f)^2$$

where

$$N_f = 4\sigma_0^2/\lambda f, \tag{8}$$

$$\rho = r'/2\sigma_0, \tag{9}$$

$$\sigma_{ij} = \beta_{ij}\sigma_0, \tag{10}$$

$$\Delta z = z-f, \tag{11}$$

$$\theta(z) = \arctan\left(\frac{-z}{\pi N_f\Delta z}\right), \tag{12}$$

(ρ_i, φ_i) ($i=1, 2$, unless otherwise stated) denotes polar coordinates in the plane, ρ represents normalized radius, unless otherwise stated and σ_{ij} is called the coherence parameter, demonstrating the source's coherent character. A beam source with high coherence usually has a small σ_{ij}. The diagonal elements of the cross-spectral density matrix determine the behavior of the optical intensity distribution $I(\boldsymbol{\rho}, z)$. And all the elements in the matrix will contribute to the degree of polarization $P(\boldsymbol{\rho}, z)$ at any point, as is evidence from the corresponding formula [13,15]:

$$I(\boldsymbol{\rho},z) = \mathrm{Tr}(W) = W_{xx} + W_{yy}, \tag{13}$$

$$P(\boldsymbol{\rho},z) = \sqrt{1-\frac{4\mathrm{Det}(W)}{[\mathrm{Tr}(W)]^2}}, \tag{14}$$

where Det(W) and Tr(W) denote the trace and determinant of the cross-spectral density matrix. The change of intensity and polarization properties on the focusing propagation will be discussed in detail in the following.

3. The effect of propagation parameters on the focusing properties of the partially polarized, partially coherent EJSMBs

3.1. Intensity distribution

Substituting from Eq. (7) into Eq. (13), we can determine the intensity distribution of the field at any point in the space $z\geq 0$.

$$\begin{aligned} I(\boldsymbol{\rho},z) = {} & \frac{A_x^2\pi^2N_f^2}{M} \times \exp\left[-\frac{2\pi^2N_f^2\rho^2}{M}\right]\exp\left[-\frac{2\sigma_{xx}^2(1+\Delta z/f)^2}{M}\right] \\ & \times I_0\left[\frac{4\pi N_f\sigma_{xx}(1+\Delta z/f)\rho}{M}\right] + \frac{A_y^2\pi^2N_f^2}{M} \\ & \times \exp\left[-\frac{2\pi^2N_f^2\rho^2}{M}\right]\exp\left[-\frac{2\sigma_{yy}^2(1+\Delta z/f)^2}{M}\right] \\ & \times I_0\left[\frac{4\pi N_f\sigma_{yy}(1+\Delta z/f)\rho}{M}\right], \end{aligned} \tag{15}$$

and

$$M = \pi^2N_f^2(\Delta z/f)^2 + (1+\Delta z/f)^2.$$

Letting $\boldsymbol{\rho}=0$ in Eq. (15), we can obtain the axial intensity of the focused EJSMBs. So Fig. 1 shows axial intensity distribution of focused EJSMBs with $\sigma=\sigma_{xx}=\sigma_{yy}=0.1$, 1 and 2, where $N_f=3$, and $f=1$m. Obviously, Fig. 1 indicates that the point of maximum intensity along the axis is not usually at the geometrical focal plane $z=f$, but is somewhat in front of the focal plane, which is referred as focal shift. As indicated in Fig. 1, the relative focal shift depends on the coherence parameter. It increases with σ [11]. Furthermore, letting $z=f$, we can obtain the transverse distribution of intensity at the focal plane. Fig. 2 shows the variance of the intensity distribution with the changing of the transverse position for different σ.

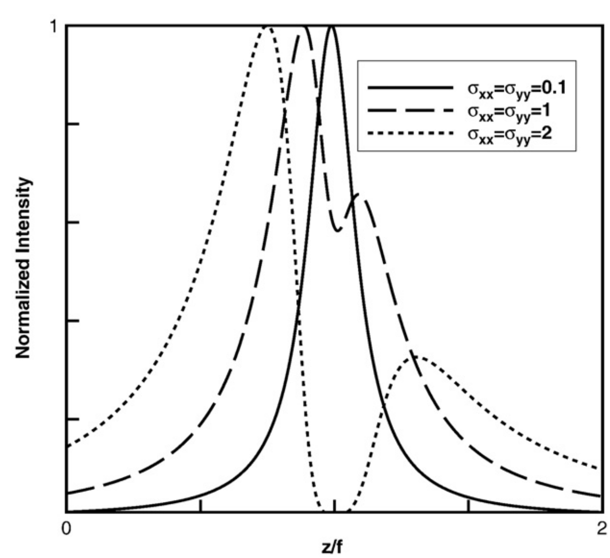

Fig. 1. Normalized axial intensity distribution of focused EJSMBs for different values of σ_{xx} and σ_{yy}. $\sigma_{xx}=\sigma_{yy}=0.1$ (solid curve), $\sigma_{xx}=\sigma_{yy}=1$ (tiny dash curve), and $\sigma_{xx}=\sigma_{yy}=2$ (large dash curve)($f=1$m).