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Performance of phase compensated coherent free space optical communications through non-Kolmogorov turbulence

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The bit-error-rate (BER) performance of coherent free-space optical (FSO) links employing phase compensation techniques is investigated in weak non-Kolmogorov turbulence. Assuming that the amplitude fading and phase fluctuation follow lognormal model and Gaussian distribution respectively and using the expression of non-Kolmogorov turbulence in terms of Zernike polynomials, the signal-to-noise ratio (SNR) at the coherent receiver is analyzed and as a special case, a new closed-form expression using chi-square distribution is obtained. Thus, the influence of different compensation modes and normalized receiver diameter on BER performance is evaluated and an optimum normalized receiver diameter is suggested to achieve the minimum BER. Moreover, the impact of outer scale L_0 and the exponent value α in non-Kolmogorov spectrum is studied with the optimum diameter, which reveals that the BER has an obvious decrease with larger values of L_0 and α.

1. Introduction

Free-space optical (FSO) communication is an attractive alternative to radio-frequency (RF) links for long-range communication, by virtue of the lower transmitted power required to obtain a given bit-error-rate (BER), small volume, and absence of government regulations restricting the usage of bandwidth [1]. One of the most significant factors that limit the performance of FSO systems is the effect of atmospheric turbulence [2]. To describe atmospheric turbulence, the Kolmogorov power spectrum density model has been widely accepted and applied for many years [1,3]. However, under the condition that the atmosphere is extremely stable, Kolmogorov turbulence may not develop. Moreover, recent experiments indicate that turbulence in the upper troposphere and stratosphere deviates from predictions of the Kolmogorov spectrum [4,5]. It is very important, therefore, to find more general models to describe atmospheric turbulence when the power spectrum of turbulence exhibit non-Kolmogorov properties. In non-Kolmogorov turbulence, the power spectrum is assumed to obey an arbitrary power law and it uses a generalized power exponent α instead of the constant value 11/3 in Kolmogorov spectrum. This parameter α can vary from 3 to 5 in the three-layer model [6,7]. However, it is more often ranged in 3 to 4 [8,9].

The effects of turbulent atmosphere mainly result in amplitude and phase fluctuation to optical signals, both of which will significantly deteriorate the link performance. To detect the weak signal after its propagation in atmospheric turbulence, coherent detection is one of the most promising techniques [10,11]. Comparing to traditional intensity modulation and direct detection (IM/DD) scheme, coherent detection is a more complicated detection method but has the ability to enable higher receiver sensitivity and overcome the thermal noise effect. However, in such coherent systems, wave front distortion of the optical signal can be severe enough to invalidate the coherent detection. Phase compensation is a technique proposed and commonly used to overcome this degradation: it can correct wave front in real time by detecting the beacon signal, which traverses the same path as it is being compensated for [12,13]. This compensation technique is, however, not perfect. Moreover, the atmospheric compensation experiment carried out by Lincoln laboratory in MIT revealed that the adaptive optics usually provide good correction for weak scintillation conditions, but there is a significant degradation in correction as scintillation increases to strong regimes [14]. In theoretical analysis, Noll provides analytical expressions for compensated phase aberrations by Zernike polynomials [15] and Boreman extends Zernike expansions into non-Kolmogorov turbulence [16], where Zernike coefficient variances of the wave front have a general exponent value and thus provide a way to describe the residual phase error in phase compensated systems under non-Kolmogorov turbulence.

In this paper, we mainly concern the BER performance of phase compensated coherent links through non-Kolmogorov atmospheric turbulence. Since the homodyne detection featured by its best detection sensitivity requires the implementation of an optical phase locked loop (OPLL) which is extremely expensive to realize

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[17,18], only the heterodyne detection method is considered [19]. It is assumed that the amplitude fading follows lognormal model and phase fluctuation obeys Gaussian distribution [20]. By analyzing the signal fading after phase compensation, the signal-to-noise ratio (SNR) at the coherent receiver is studied so as to evaluate the BER performance of the links. An optimum receiver diameter is found to minimize the BER according to different parameters of both the phase compensation and non-Kolmogorov turbulence spectrum. Furthermore, the impact of these parameters is examined and discussed.

2. Zernike polynomials and residual phase variance

In coherent FSO systems, phase compensation techniques can be utilized in order to alleviate phase fluctuation induced by atmospheric turbulence. Zernike polynomials are usually used to express the residual phase variances after correction in theoretical analysis. Zernike polynomials are a series of polynomials defined on a unit circle. Each polynomial represents a type of correction mode (jth mode) and the statistical aberration of phase can be calculated analytically for any number of independent corrections. We are interested in the residual phase error after the first J modes are corrected. Firstly, using the definition by Noll [15], the Wiener spectrum of phase $\Phi_\varphi(\mathbf{k})$ is related to the phase structure function $D_\phi(r)$, given by

$$D_\phi(r) = 2\int dk\Phi_\varphi(\mathbf{k})[1-\cos(2\pi\mathbf{k}\times\mathbf{r})], \tag{1}$$

where $D_\phi(r)$ is expressed in terms of Fried parameter r_0 as $D_\phi(r)=6.88(r/r_0)^{5/3}$ in Kolmogorov turbulence, while the phase structure function in non-Kolmogorov turbulence can be found in [21]. Considering the outer scale L_0, the phase spectrum is written in an analogue manner to that in Kolmogorov turbulence as

$$\Phi_\varphi(\zeta,\alpha) = \frac{A_\alpha\left[\zeta^2+\left(\pi\frac{D}{L_0}\right)^2\right]^{-\alpha/2}}{\tilde{r}_0^{\alpha-2}}, \tag{2}$$

where $\zeta=2\pi k$, k is the spatial wave number, α is the power exponent value in non-Kolmogorov spectrum usually ranging from 3 to 4, D represents the aperture diameter of the receiver, $\tilde{r}_0$ is a quantity analogous to r_0 and it reduces to r_0 when $\alpha=11/3$. A_α is a constant and maintains the relation between the phase variance and the first correction mode. With infinite outer scale L_0, the phase spectrum is simplified as

$$\Phi_\varphi(k,\alpha) = \frac{A_\alpha k^{-\alpha}}{\tilde{r}_0^{\alpha-2}}. \tag{3}$$

The constant A_α is assigned the value so as to normalize the piston-subtracted wave-front error to 1.0299 over a pupil diameter $D=\tilde{r}_0$, according to [15]. It has the approximate form as [16].

$$A_\alpha = 1.0299\Big/\left\{2\pi^{\beta-1}\int_0^\infty \zeta^{-(\alpha-1)}\left[1-\frac{4J_1^2(\zeta)}{\zeta^2}\right]d\zeta\right\}, \tag{4}$$

where $\zeta=2\pi k$. If α is within 3 to 4, A_α can be written in a closed-form as

$$A_\alpha = \frac{2^\alpha\left[\Gamma\left(\frac{\alpha+2}{2}\right)\right]^2\Gamma\left(\frac{\alpha+4}{2}\right)\Gamma\left(\frac{\alpha}{2}\right)\sin\left(\pi\frac{\alpha-2}{2}\right)}{0.2574\pi^\alpha\Gamma(\alpha+1)}. \tag{5}$$

The residual mean-squared wave-front error Δ_J after J modes correction is defined as

$$\Delta_J = \langle\varphi^2\rangle-\sum_{j=1}^{J}\langle|a_j|^2\rangle, \tag{6}$$

where $\langle\cdots\rangle$; denotes the ensemble average and a_j is the corresponding coefficient for the compensation mode j. In the above equation, $\langle\varphi^2\rangle=\sum_{j=1}^{\infty}\langle|a_j|^2\rangle$ is the total phase variance expressed by infinite Zernike polynomials. While employing the phase spectrum in Eq. (2), the expression of Zernike coefficient variance reveals [16]

$$\langle|a_j^2|\rangle = 4.1196(n+1)\left(\frac{D}{\tilde{r}_0}\right)^{\alpha-2}\times\frac{\int_0^\infty[\xi^2+(\pi\frac{D}{L_0})^2]^{-\alpha/2}\xi^{-1}J_{n+1}^2(\xi)d\xi}{\int_0^\infty[\xi^2+(\pi\frac{D}{L_0})^2]^{-\alpha/2}\xi\left(1-\frac{4J_1^2(\xi)}{\xi^2}\right)d\xi}, \tag{7}$$

where n is the radial degree in Zernike polynomials and there is a definitive correspondence relationship between the mode j and n. With infinite outer scale, the Zernike coefficient variance takes the form as

$$\langle|a_j^2|\rangle = 8A_\alpha\left(\frac{D}{\tilde{r}_0}\right)^{\alpha-2}(n+1)\pi^{\alpha-1}\times\frac{\Gamma(\alpha+1)\Gamma\left(\frac{2n+2-\alpha}{2}\right)}{2^{\alpha+1}\left[\Gamma\left(\frac{\alpha+2}{2}\right)\right]^2\Gamma\left(\frac{2n+4+\alpha}{2}\right)}. \tag{8}$$

When setting the parameter $\alpha=11/3$, the Zernike coefficient variances are equal to those in [15]. Consequently, the residual phase error can be obtained by Eq. (6) through Zernike coefficient variances.

3. BER performance of heterodyne receivers

Coherent detection involves the incoming signal mixing with the local oscillator beam and a heterodyne receiver downconverts the carrier signal to an intermediate frequency carrier. Here, a binary-phase-shift-keying (BPSK) modulation with a synchronous heterodyne system is only considered. The heterodyne system uses a pair of photodetectors to comprise a balanced receiver. It is assumed that the dominant noise is local oscillator shot noise and the receiver is able to track any phase fluctuation induced by turbulence as well as the laser phase noise, i.e., ideal coherent demodulation is implemented. To evaluate the impact of turbulence, the combined effects of log-amplitude and phase fluctuation in non-Kolmogorov turbulence should be considered. Consequently, the field in the receiver plane can be expressed as [19]

$$A = A_S\exp[\chi(r)-j\phi(r)], \tag{9}$$

where A_S is the amplitude in absence of turbulence, χ_r and ϕ_r represent the log-amplitude and phase fluctuations, respectively. In a heterodyne detection system, the photocurrent coming out of the balanced receiver is

$$i_S = \eta A_0A_S\int d\mathbf{r}W(\mathbf{r})\exp[\chi(\mathbf{r})]\cos[2\pi\Delta ft+\Delta\phi-\phi(\mathbf{r})], \tag{10}$$

where η is the quantum efficiency of the photodetecter, $W(\mathbf{r})$ is the aperture function, A_0 is the amplitude of local light, and Δf and $\Delta\phi$ are, respectively, the differences between the frequencies and phases of the signal and local light. When the cosine item is expanded, the signal power can be written as $\overline{i_S^2}=\frac{1}{2}\left(\eta\frac{\pi}{4}D^2A_0A_S\right)^2\left(\xi_r^2+\xi_i^2\right)$ with D as the diameter of the receiver, and we have

$$\begin{aligned}\xi_r &= \left(\frac{\pi}{4}D^2\right)^{-1}\int drW(r)\exp[\chi(r)]\cos[\phi(r)]\\ \xi_i &= \left(\frac{\pi}{4}D^2\right)^{-1}\int drW(r)\exp[\chi(r)]\sin[\phi(r)].\end{aligned} \tag{11}$$

If we only consider the shot noise, the SNR can be written as

$$\gamma = i_S^2/i_N^2 = \gamma_0\left(\xi_r^2+\xi_i^2\right) = \gamma_0\xi^2. \tag{12}$$

Here, γ_0 is the original turbulence-free SNR, ξ^2 stands for the fading factor induced by turbulence. Following the approximation in speckle statistics of imaging optics [22], the receiver aperture is assumed to