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Discussion

# Estimation of the degree of asphericity of a glass sphere using a vectorial shearing interferometer

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ABSTRACT

The degree of asphericity is estimated by determining the average radius of curvature in different sections, at various points on the surface of a sphere, and the deviation from it. We employ the *vectorial shearing interferometer* (VSI) as the instrument to determine the radius of curvature from two subapertures of the transparent glass sphere. We incorporate the sphere as a thick lens into the interferometric setup, illuminating it with an expanded beam. The spherical aberration, introduced by the sphere in the wave front, depends on the local sphere radius, on the refraction index of the glass, and on the cone angle of the source. The wave front aberrated by the sphere impinges on the VSI. Here, the wave front is divided in two in amplitude, it is sheared vectorially, and it is superimposed with itself. The fringe pattern is formed in the intersection of the wave fronts. The shape of the resulting fringe pattern is directly related to spherical aberration. We estimate qualitatively the degree of asphericity, comparing the phase gradients in different sections of the sphere. Here, we report on the experimental setup to test the asphericity, the results with different vectorial shearing (magnitude and direction). Finally, we perform a comparison with the theoretical predictions.



## 1. Introduction

Aspherical surfaces are of great interest in optics due to the ability of some such surfaces to form a perfect point image of a point object, excluding the diffraction effects. Additionally, in the last 40 years, the development of increasingly more sophisticated software resulted in diffraction limited designs incorporating small number of aspherical optical elements.

### *1.1. Glass sphere*

We are investigating the degree of asphericity of a glass sphere. It is used as a primary density standard by metrology laboratories [1]. Density measurement is useful both for the industrial production and in the scientific work. In the petroleum industry, for example, the measurement of the density of the produced oils is crucial for the assessment of the quality control. In scientific applications, glass spheres are used to determine a precise value of the Avogadro constant [2].

The density ($D$) may be calculated by applying its definition, $D = m/V$, where $m$ and $V$ are the mass and the volume of the sphere, respectively. The value of the mass is obtained by comparisons with the mass standard and the volume is determined in terms of dimensional measurements [3]. The primary density standard is kept under controlled conditions of temperature and humidity, until it is used for creating secondary standards. It is then compared to the primary reference in weight and volume.

### *1.2. Optical characteristics of a glass sphere*

The spherical form of the primary density standard is selected because it is much less susceptible to damage than a cube or a cylinder, with their sharp and clearly defined edges. Furthermore, the volume of a sphere with an excellent sphericity may be determined with a high degree of certainty using the mean of diameters over many directions and at many points. We studied the degree of asphericity of a sphere fabricated of BK7 glass with high homogeneity and high mechanical and thermal resistance.

The glass sphere is believed to have been finished optically with a high precision; however no specific values are provided due to the absence of established procedures and adequate tools to measure sphericity of a sphere. This experimental determination is in general considered challenging because its relatively small radius of curvature, over a large sphere segment produces a high fringe density relative to those of traditional optical surfaces. Thus, a detector with a very high resolution is necessary to resolve individual fringes. Finally, an instrument is needed that may be adjusted for variable surface quality, such as a vectorial shearing interferometer that allows precise control over measurement sensitivity.

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A small number of optical techniques to measure the radius of curvature of a surface exist, such as a cross-section contour or profilometer [4,5]. Traditional interferometric methods are also used to certify the sphericity of the glass spheres. Currently, the spheres are tested in specialized standard laboratories, the *Physikalisch-Technische Bundesanstalt* (PTB) in Germany or the *National Institute of Standards and Technology* (NIST) in USA [6,7].

PTB employs a spherical interferometer to determine the diameter of the sphere. This interferometer consists of a spherical etalon formed by the spherical reference faces of two Fizeau lenses; the phase stepping is performed by wavelength tuning [8]. The NIST developed an interferometric instrument called XCALIBIR, also based on a spherical Fizeau interferometer. A variant of the radius bench method is used to measure the radius of curvature [9]. These methods require acquisition and fabrication of high quality reference surfaces [10,11]. These, in turn, have to be tested to demonstrate their quality.

We propose the *vectorial shearing interferometer* (VSI) as an alternative instrument to determine the degree of asphericity of a glass sphere. The advantage of using this instrument is that it requires no high quality reference surfaces, has adjustable sensitivity, and it is economical. Other configurations of shearing interferometers have also been studied with varying degree of success [12,13].

We use the VSI to analyze the aberrations introduced by the sphere in an expanded laser beam. Local sphere radius is calculated from the value of spherical aberration coefficient. Thus, we use the VSI to determine how the radius of curvature changes from reference sub-aperture at a number of locations on the sphere surface.

In the next section, we describe the vectorial shearing interferometer that we developed in our laboratory and optimized for this application. In Section 3, we use the glass sphere as an optical component. Furthermore, we analyze the aberrations introduced by the sphere on the wave front of a laser. We describe sub-aperture testing of the sphere cross-section. In Section 4, we compare the fringe patterns generated by different sections of the sphere in order to qualitatively determine the degree of sphericity. Section 5 is dedicated to conclusions and future work. In the next section, we briefly review the features of VSI and elaborate on specific changes needed to characterize the shape of the glass sphere.

## 2. Vectorial shearing interferometer (VSI)

In a VSI the wave-front under test is compared with itself. Two identical wave fronts follow paths nearly, but one of them undergoes a small displacement relative to the other, as shown in Fig. 1. The shearing vector $\Delta\vec{\rho}$ describes the displacement of the wave front $W_d$ $(x_d,y_d)$ with respect to the original one $W(x,y)$. The superposition of the wave fronts gives rise to the fringe pattern. At the interference plane, the incidence $I_T(x,y)$ of the fringe pattern is [14]:

$$I_T(x,y) = I_b(x,y) + I_m(x,y)\cos\Delta W(x,y). \tag{1}$$

Here, $I_b$ is the background offset, and $I_m$ is the modulated incidence.

This pattern carries the information of the optical path difference (OPD) in the sheared direction, and may be expressed as:

$$\begin{aligned}\Delta W &= W_d(x_d,y_d) - W(x,y),\\ \Delta W &= W(x+\Delta x, y+\Delta y) - W(x,y).\end{aligned} \tag{2}$$

From the fringe pattern, the path difference may be obtained as:

$$\Delta W = m\lambda. \tag{3}$$

Here $m$ is the order of the interference fringe and $\lambda$ is the wavelength.

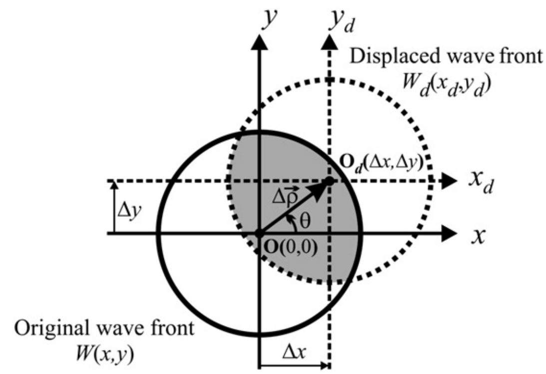


**Fig. 1.** Original and displaced wave fronts. The $\Delta x$ and $\Delta y$ values are obtained from vector $\Delta\vec{\rho}$ and the angle $\theta$.

For small displacements, the distances $\Delta x$ and $\Delta y$ may be considered infinitesimal quantities ($dx$, $dy$). Then, the pattern represents the total differential of the wave-front function in the shearing direction [15]:

$$\frac{\partial W(x,y)}{\partial x}dx + \frac{\partial W(x,y)}{\partial y}dy = m\lambda. \tag{4}$$

The shape, and density of the fringe pattern may be controlled by the direction and magnitude of the shearing vector $\Delta\vec{\rho}$. The ability of the operator to control the number of fringes is particularly useful when asymmetrical components are tested. $\Delta W$ is the OPD (optical path difference) between wave fronts, given by Eq. (2). The reconstructed wave front may be found by direct integration of its gradient, avoiding the evaluation of the arctangent function and the complex phase reconstruction methods [16–20].

In previous configurations, in one arm of a Mach Zehnder interferometer the sheared wave front was displaced in a specific direction according to the relative angle between the wedge prisms and their separation, but this configuration introduces tilt in the sheared wave front. A compensator system was necessary in the other arm of the interferometer in order to minimize the tilt deviation [21,22].

Currently, our implementation of the VSI uses a configuration of wedge prisms that do not introduce tilt in the sheared wave front and a compensator system is not needed, as illustrated in Fig. 2. The wave front under test impinges on the first beam splitter, BS1. Here, the amplitude is divided into two. In order to shear one wave front, we use a pair of wedge prisms P1 and P2 as a displacement system in one arm of the interferometer. In the other arm the wave front continues its path without being modified. Both wave fronts, partially displaced with respect to each other, are superimposed after the second beam splitter, BS2. The fringe pattern is produced in the space behind the second beam splitter. A high resolution CCD captures the digital image. The camera is interfaced to a computer, where the images are stored and further analyzed.

In the current application, it is particularly important to be able to control separately the magnitude and direction of the shear, in order to examine an area on the sphere from different directions. The displacement system introduces no tilt in the wave front and no changes in the image orientation. The degrees of freedom of the displacement system, the angle $\omega$ of rotation of both prisms and the distance $d$ between them, allow the operator the requisite control to displace the wave front vectorially (on any direction and any magnitude). Additionally, the magnitude of the displacement determines the sensitivity of the VSI [23].

In the sheared wave front (see Fig. 1), the angle $\theta$ depends on the angle of rotation $\omega$ of both prisms. The shearing magnitude $|\Delta\vec{\rho}|$ is