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Spontaneous emission spectra of a four-level atom in photonic crystals driven by two coherent fields

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ABSTRACT

We investigate the spontaneous emission spectra of a four-level tripod-type atom embedded in a photonic crystal and driven by two coherent fields. It is found that due to the quantum interference caused by two driving fields, the spontaneous emission spectra have different features from the case of only one driving field. The spectra are sensitively dependent on the detuning of the driving fields. A dark line occurs for some particular initial states. By appropriately adjusting the external driving fields, the spectral-line can be narrowed, enhanced or suppressed.

1. Introduction

It is well known that spontaneous emission depends not only on the properties of the excited atomic system, but also on the surrounding environment [1–3] which modifies the density of states (DOS) of the radiation field. Due to the strong modification of the DOS of the photonic band gap (PBG) structures [4,5], there are many novel optical phenomena, for example, the trapping population in the atomic systems [6,7], the dark lines in spontaneous emission spectra [8–10], the enhancement of quantum interference in spontaneous emission [11–13], the transient lasing without inversion [14] and the induced transparency by the modified reservoir [15,16].

Knight and co-workers have investigated the transparency and the coherent phenomena in photonic crystals of a Λ-type atomic model, where one transition interacts with free vacuum modes and the other interacts with modes near the PBG's edges [17]. Zhang et al. studied the spontaneous emission spectra of double V-type atoms embedded in a double-band photonic crystal and found two types of quantum interference [18], in which the double V-type transitions are respectively coupled to the free vacuum modes and the photonic band modes. Jiang and co-workers [19] considered the transition from the upper level to an auxiliary level driven by a laser field and investigated the spontaneous emission of a four-level atom with two transitions coupled to the same modified reservoir. Yang et al. calculated the spontaneous emission spectrum of a four-level atom coupled by three kinds of reservoirs [20,21]. Xu et al. investigated the transparency induced by the quantum interference of a six-level atom in a photonic crystal with defect modes [22].

Recently Yang et al. have carried out a series of research about the spontaneous emission of atoms embedded in the photonic crystals, including the anisotropic of the photonic crystal [23–29], the isotropic of the photonic crystal [30–32] and the double-band photonic crystal [33–35]. It is shown that the control of spontaneous emission can be achieved by placing atoms into proper circumstances. An alternative way to control the spontaneous emission is letting the atom be driven by a coherent field. In this paper we investigate the spontaneous emission spectra of a four-level atom embedded in a single-band photonic crystal and driven by two coherent fields, and discuss the dependence of the spectra on the relative position of the photonic band gap and the detuning of external driving field.

2. Theoretical model and equations

We consider a four-level atom, as shown in Fig. 1, one upper level $|0\rangle$, and three lower levels $|1\rangle$, $|2\rangle$ and $|i\rangle$, embedded in photonic crystals. The excited level $|0\rangle$ is respectively coupled to the lower levels $|1\rangle$ and $|2\rangle$ by two coherent fields with frequencies ω_p and ω_c. The transition $|0\rangle \rightarrow |i\rangle$ is coupled to the modified reservoir. Under the rotating-wave-approximation and the electric-dipole approximation, the interaction Hamiltonian can be written as (let $\hbar = 1$)

$$H = \Omega_p e^{i\Delta_p t}|0\rangle\langle 1| + \Omega_c e^{i\Delta_c t}|0\rangle\langle 2| + \sum_\lambda g_\lambda e^{-i(\omega_\lambda - \omega_{0i})t}|0\rangle\langle i|a_\lambda + H.C, \quad (1)$$

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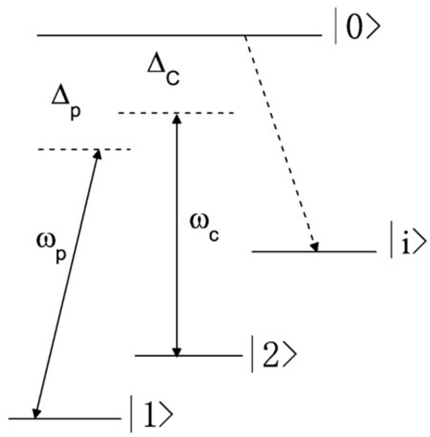

Fig. 1. Schematic diagram of a driven four-level atomic system, the thin dashed line denotes the coupling to the modified reservoir (PBG).

where g_λ is the coupling constant between the atomic transition $|0\rangle \rightarrow |i\rangle$, a_λ and a_λ^+ are the annihilation operator and creation operator for the λth reservoir mode with frequency ω_λ, Ω_p and Ω_c are the Rabi frequencies of the external driving fields, $\Delta_p = \omega_{01} - \omega_p$ and $\Delta_c = \omega_{02} - \omega_c$ are the detuning of the external driving fields from the corresponding resonances, ω_{01}, ω_{02} and ω_{0i} are the transition frequencies from level $|0\rangle$ to levels $|1\rangle$, $|2\rangle$ and $|i\rangle$, respectively.

In the interaction picture, the state vector of the atomic system at time t can be expressed as

$$|\psi(t)\rangle = a_0(t)|0,0\rangle + a_1(t)|1,0\rangle + a_2(t)|2,0\rangle + \sum_\lambda a_\lambda(t)|i,1_\lambda\rangle. \quad (2)$$

Substituting (2) into the Schrödinger equation we get the coupled equations of motion for the probability amplitudes as

$$\frac{\partial a_0(t)}{\partial t} = -i\sum_\lambda g_\lambda e^{-i(\omega_\lambda-\omega_{0i})t}a_\lambda(t) - i\Omega_p e^{i\Delta_p t}a_1(t) - i\Omega_c e^{i\Delta_c t}a_2(t), \quad (3)$$

$$\frac{\partial a_1(t)}{\partial t} = -i\Omega_p^* e^{-i\Delta_p t}a_0(t), \quad (4)$$

$$\frac{\partial a_2(t)}{\partial t} = -i\Omega_c^* e^{-i\Delta_c t}a_0(t), \quad (5)$$

$$\frac{\partial a_\lambda(t)}{\partial t} = -ig_\lambda^* e^{i(\omega_\lambda-\omega_{0i})t}a_0(t). \quad (6)$$

We proceed by performing a formal time integration of Eq. (6), and substitute the result into Eq. (3) to obtain the integral–differential equation

$$\frac{\partial a_0(t)}{\partial t} = -\int_0^t dt' a_0(t') \sum_\lambda g_\lambda^2 e^{-i(\omega_\lambda-\omega_{0i})\left(t-t'\right)} - i\Omega_p e^{i\Delta_p t}a_1(t) - i\Omega_c e^{i\Delta_c t}a_2(t). \quad (7)$$

If the reservoir is Markovian, then we have $\sum_\lambda g_\lambda^2 e^{-i(\omega_\lambda-\omega_{0i})\left(t-t'\right)} = (\gamma_1/2)\delta(t-t')$ with γ_1 being the decay rate from the state $|0\rangle$ to the state$|i\rangle$. However, for the single-isotropic model of the photonic crystal considered here, it is not the case because the density of modes of this reservoir varies more quickly than that in the free space. To tackle this problem, we assume the memory kernel [9]

$$K(t-t') = \sum_\lambda g_\lambda^2 e^{-i(\omega_\lambda-\omega_{0i})\left(t-t'\right)} \approx \beta^{3/2}\int d\omega \rho(\omega) e^{-i(\omega_\lambda-\omega_{0i})\left(t-t'\right)}, \quad (8)$$

where β is the coupling constant of the atom and the modified reservoir. The above kernel is calculated by using the appropriate DOS of the modified reservoir $\rho(\omega)$. Substituting Eq. (8) into Eq. (7), we get

$$\frac{\partial a_0(t)}{\partial t} = -\int_0^t dt' a_0(t') K(t-t') - i\Omega_p e^{i\Delta_p t}a_1(t) - i\Omega_c e^{i\Delta_c t}a_2(t). \quad (9)$$

For convenience, we suppose

$$a_0(t) = b_0(t), \quad (10)$$

$$a_1(t) = b_1(t)e^{-i\Delta_p t}, \quad (11)$$

$$a_2(t) = b_2(t)e^{-i\Delta_c t}. \quad (12)$$

Then, Eqs. (3)–(5) can be written as

$$\frac{\partial b_0(t)}{\partial t} = -i\Omega_p b_1(t) - i\Omega_c b_2(t) - \int_0^t dt' b_0(t') K(t-t'), \quad (13)$$

$$\frac{\partial b_1(t)}{\partial t} = i\Delta_p b_1(t) - i\Omega_p^* b_0(t), \quad (14)$$

$$\frac{\partial b_2(t)}{\partial t} = i\Delta_c b_2(t) - i\Omega_c^* b_0(t). \quad (15)$$

After carrying out the Laplace transformation $\tilde{b}_j(s) = \int_0^\infty e^{-st} b_j(t)dt$, we get the solution to the probability amplitude $\tilde{b}_0(s)$ as

$$\tilde{b}_0(s) = \frac{b_0(0) - i\Omega_p b_1(0)/\left(s - i\Delta_p\right) - i\Omega_c b_2(0)/(s - i\Delta_c)}{s + \Omega_p^2/\left(s - i\Delta_p\right) + \Omega_c^2/(s - i\Delta_c) + \tilde{K}(s)}, \quad (16)$$

where $\tilde{K}(s)$ is the Laplace transform of the kernel defined in Eq. (8). The memory kernel function can be obtained as

$$\tilde{K}(s) = \beta^{3/2}\int d\omega \frac{\rho(\omega)}{s + i(\omega - \omega_{0i})}. \quad (17)$$

For an ideal, perfect photonic crystal with infinite, periodic structure and without dissipation, the isotropic DOS near the band edge has the form of $\rho(\omega) = \frac{1}{\pi}\frac{1}{\sqrt{\omega-\omega_g}}\Theta(\omega-\omega_g)$, where Θ is the Heaviside step function, and ω_g is the frequency of the upper band gap edge. This kind of DOS corresponds to the case that the photonic dispersion curve has an exact band edge, and it is divergent at the band edge. However, for an actual photonic crystal, the total size is finite. The sizes of the repeated cells in the periodic structure are not exactly the same for the precision error in the process or preparation of the photonic crystals, and the absorption (or dissipation) is always inevitable. All these factors make the band edge indistinct. So we use a smooth parameter ε [36] to express the density of states as

$$\rho(\omega) = \frac{1}{\pi}\frac{\sqrt{\omega-\omega_g}}{\varepsilon + \omega - \omega_g}\Theta\left(\omega - \omega_g\right). \quad (18)$$

Substituting Eq. (18) into Eq. (17) we can get

$$\tilde{K}(s) = \frac{\beta^{3/2}}{i\sqrt{\varepsilon} + \sqrt{is - \omega_g + \omega_{0i}}}. \quad (19)$$