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Three-mode entanglement via atomic coherence induced by strong classical field

Pei-Jun Song [a,*], Xiang-Ying Hao [a], Pei Huang [a], Zhi-Ming Zhan [b]

[a] School of Physics and Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, PR China
[b] School of Physics and Information Engineering, Jianghan University, Wuhan 430056, PR China

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ABSTRACT

We propose a new scheme to achieve fully three-mode entanglement based on the standard criteria [P. van Loock and A. Furusawa, Phys. Rev. A 67, 052315 (2003)] in a four-level atomic system driven by two strong classical fields. Via numerically simulating the dynamics of the system, we investigate the generation and evolution of entanglement. Based on our scheme, it is demonstrated that the three-mode continuous-variable (CV) entanglement can be achieved under different initial conditions and the entangled period will be extended by enhancing the intensity of the classical field. Moreover, our numerical results also show that the present system can be considered as a three-mode entanglement amplifier.

1. Introduction

Entanglement, as a peculiar and fundamental feature of quantum mechanics, plays a significant role in quantum information field. It provides promising and wide practical applications on quantum information processing, such as quantum cryptography, quantum computing and quantum teleportation [1–6]. Therefore, the generation and measurement of entangled state has become a research focus and been studied intensively in recent years. In the domain of discrete variables systems [7–13], such as trapped ions, quantum dot, nuclear magnetic resonance, there already exists lots of work on the generation of entangled states. At the same time, the CV entanglement has aroused increasing attentions [14–19] because of its unconditionalness for implementation of quantum information processes [20]. In recent years, based on the CV entangled state, much theoretical work has been done in various aspects of quantum information area, such as unconditional quantum teleportation [21], quantum error correction [22,23], entanglement swapping [24] and quantum computation [25]. On the other hand, physicists have also dedicated to the experimental researches of quantum information process for CV entanglement, for instance, quantum dense coding [26], unconditional quantum teleportation [27] and entanglement swapping [28].

Recently, with the development of multi-node quantum network technology, much attention has been paid on how to generate, detect and employ multipartite CV entanglement for quantum communication in various of linear optics and nonlinear optics systems [29–42]. For example, P. van Loock and Samuel L. Braunstein have investigated a truly multipartite entangled state generated by one single mode squeezed state distributed among N parties using linear optics [29]. Subsequently, a sufficient condition for genuine multipartite CV entanglement was proposed by P. van Loock and A. Furusawa [31]. Using this criterion, a theoretical scheme for generating three-mode entangled states based on cascaded nonlinear processes was presented [34]. Zhai et al. demonstrated that the optical cavity with second-order harmonic generation can produce three-mode entangled light beams when it operates below threshold [40]. Very recently, physicists have experimentally demonstrated the generation of CV multipartite entanglement among three bright beams of light, all of different wavelengths, in the system of an optical parametric oscillator [42]. The above work has shown that the three-mode CV entanglement with different frequencies for each entangled mode is important because of the requirement for storage and communication of quantum information in the nodes of quantum networks.

On the other hand, the correlated spontaneous emission laser (CEL) [43] is another usable system to generate CV entanglement and it has already attracted much attention of researchers. Based on the sufficient inseparability criteria for two-mode CV system, a large number of schemes for generating two-mode CV entanglement have been proposed in the CEL [44–50]. For instance, Xiong et al. have proposed the scheme to built a macroscopic entangled state between two modes of the radiation field in a CEL system [43]. Subsequently, Tan et al. studied the generation and evolution of entangled light in a CEL under the Wigner representation and they have found that the entanglement period is extended with the increase of the driving field's intensity [46]. Moreover, the entanglement amplification in the nondegenerate three-level cascade laser coupled to a two-mode vacuum reservoir was thoroughly analyzed in Ref. [47] and it has been pointed out that the entanglement is better when squeezing is stronger.

In this paper, via extending the existing research about generation of bipartite CV entanglement, we propose an alternative scheme for the generation of fully three-mode CV entanglement in a three-mode CEL, which consists of a four-level atomic system interacting with a

* Corresponding author.
E-mail address: peijunsong@gmail.com (P.-J. Song).

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nondegenerate three-mode resonant cavity. The root for the generation of entanglement in our scheme is the atom coherence which is induced by two strong classical fields. By numerically simulating the dynamics of the system, we demonstrate that a three-mode CV entanglement can be achieved based on our present scheme. We also point out that the entanglement periods can be extended via enhancing the intensity of the classical field. It is known that until now, there has been other schemes for generating tripartite CV entanglement based upon CEL. Our scheme has the following characteristics. In our paper, we have showed that the CV entanglement can be realized under different initial conditions based on our scheme. Also we numerically demonstrated the entanglement time manifests linear increase property as the intensity of classical field increases and the initial conditions affect little on the entanglement time, we think the study of this point can show the controllability of our program. In addition, we have pointed out that our entangled state may have three modes with three different frequencies, which can be seen as a "three-colour entanglement" [42], and it has important applied foreground in quantum network science.

The remainder of this paper is organized into four parts as follows. In Section 2, we firstly describe the system model then present the equation of motion for reduced density operator ρ_f of the cavity fields. In Section 3, the generation of three-mode CV entanglement is demonstrated. Finally, we conclude with a brief summary in Section 4.

2. Model and equation

We consider that a three-mode CEL involves injection of four-level atoms in a triply resonant cavity at a rate γ_a. It is noticed that we choose an open system, and we would like to illustrate that actually the closed system and the open system are both suitable for realizing continuous-variable (CV) entanglement based on a spontaneous emission laser (CEL). For instance, Refs. [47] and [50] are examples of generation CV entanglement in a closed system, for which the atoms will be placed in a cavity at the initial time. The experimental implementation for these schemes relies on the trapping technology of cold atoms. On the other hand, researches on CV entanglement in an open system also attract physicists attention, e.g. Refs. [46] and [49]. In this case, the atoms are prepared in a certain state initially, and injected into a cavity for the following generation of CV entanglement. Thus whether the model system is open or closed, there may exist the possibility of putting forward a scheme to achieve CV entanglement. We choose an open system in our present work because we think it reasonable and we are more interested in it. The atom level configuration is shown in Fig. 1(b). The dipole allowed transitions $|3\rangle \leftrightarrow |2\rangle$, $|2\rangle \leftrightarrow |0\rangle$ and $|2\rangle \leftrightarrow |1\rangle$ are resonantly coupled with the three nondegenerate cavity modes ν_1, ν_2 and ν_3, respectively, and two classical fields with angular frequency ω_{p1} and ω_{p2} induce the dipole forbidden (magnetic dipole allowed) transitions $|3\rangle \leftrightarrow |0\rangle$ and $|3\rangle \leftrightarrow |1\rangle$. In the interaction picture, under the dipole and rotating wave approximation, the total interaction Hamiltonian of our system can be written as [51–54]

$$H_I = g_1 a_1 |3\rangle\langle 2| + g_2 a_2 |2\rangle\langle 0| + g_3 a_3 |2\rangle\langle 1| - \Omega_{p1} |3\rangle\langle 0| - \Omega_{p2} |3\rangle\langle 1| + \text{H.c.}. \quad (1)$$

Here we have set $\hbar = 1$, $a_j^\dagger$ and a_j $(j = 1, 2, 3)$ are the creation and annihilation operators for the three cavity modes. $\Omega_{p1} = |\Omega_{p1}| \exp(\phi_{p1})$, $\Omega_{p2} = |\Omega_{p2}| exp(\phi_{p2})$ represent the Rabi frequencies of the two classical fields, and g_1, g_2, g_3 are the atom-field coupling constants.

Using the standard methods of laser theory [16], the equation of motion for the reduced density operator of field ρ_f can be obtained by taking a trace over the atom

$$\begin{aligned}\dot{\rho_f} &= -iTr_{atom}\left[H_I, \rho_{a-f}\right] \\ &= \left(-ig_1\left[a_1^\dagger, \rho_{32}\right] - ig_2\left[a_2^\dagger, \rho_{20}\right] - ig_3\left[a_3^\dagger, \rho_{21}\right] + \text{H.c.}\right),\end{aligned} \quad (2)$$

where ρ_{a-f} denotes the full atom-field density operator and $\rho_{32} = \langle 3|\rho_{a-f}|2\rangle$, $\rho_{20} = \langle 2|\rho_{a-f}|0\rangle$, $\rho_{21} = \langle 2|\rho_{a-f}|1\rangle$.

Assuming the atoms which are pumped to the energy level $|3\rangle$ at the initial time, are injected into the cavity at a rate of γ_a. The density operators ρ_{32}, ρ_{20} and ρ_{21} can be evaluated to the first order in the coupling constants g_1, g_2 and g_3 as

$$\dot{\rho_{32}} = i\Omega_{p1}\rho_{02} + i\Omega_{p2}\rho_{12} - ig_1 a_1 \rho_{22}^{(0)} + ig_2 \rho_{30}^{(0)} a_2^\dagger + ig_3 \rho_{31}^{(0)} a_3^\dagger + ig_1 \rho_{33}^{(0)} a_1 - \gamma\rho_{32}, \quad (3a)$$

$$\dot{\rho_{20}} = ig_2 a_2 \rho_{00}^{(0)} - ig_3 a_3 \rho_{10}^{(0)} - ig_1 a_1^\dagger \rho_{30}^{(0)} + ig_2 \rho_{22}^{(0)} a_2 - i\Omega_{p1}\rho_{23} - \gamma\rho_{20}, \quad (3b)$$

$$\dot{\rho_{21}} = -ig_2 a_2 \rho_{01}^{(0)} - ig_3 a_3 \rho_{11}^{(0)} - ig_1 a_1^\dagger \rho_{32}^{(0)} + ig_3 \rho_{22}^{(0)} a_3^\dagger - i\Omega_{p2}\rho_{23} - \gamma\rho_{21}, \quad (3c)$$

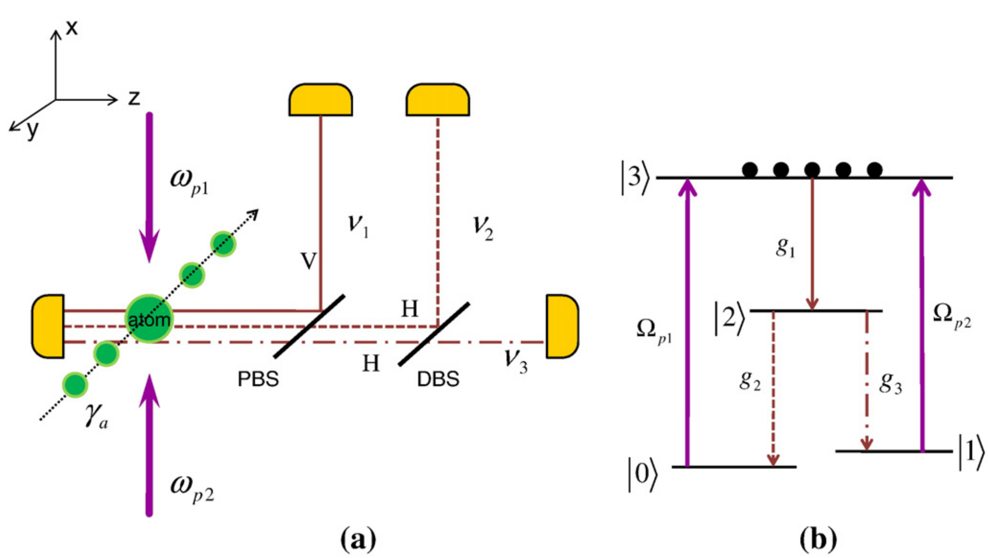

Fig. 1. (a) Schematics for the three-mode CEL. Atomic medium is injected into a triply resonant cavity with a rate of γ_a. ν_1, ν_2 and ν_3 $(\nu_1 \neq \nu_2 \neq \nu_3)$ are the frequencies of three (nondegenerate) cavity modes, respectively. PBS and DBS represent polarization beam splitter and dichroic beam splitter, respectively. H and V denote horizontal and vertical line polarizations light, respectively. (b) Level configuration of the inverse-tripod-type atom.