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Transmission properties of Fibonacci quasi-periodic one-dimensional photonic crystals containing indefinite metamaterials

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ABSTRACT

The transmission properties of Fibonacci quasi-periodic one-dimensional photonic crystals (1DPCs) containing indefinite metamaterials are theoretically studied. It is found that 1DPCs can possess an omnidirectional zero average index (zero-$\bar{n}$) gap which exists in all Fibonacci sequences. In contrast to Bragg gaps, such zero-$\bar{n}$ gap is less sensitive to the incidence angle, the scale length and the polarizations of electromagnetic waves. When an impurity is introduced, a defect mode appears inside the zero-$\bar{n}$ gap with a very weak dependence on the incidence angle and scaling.

1. Introduction

Photonic crystal (PC) is a type of artificial dielectric or metallic periodical structure and it has been intensively studied due to the novel electromagnetic characteristics and important scientific and engineering applications [1,2]. The essential property of PCs is the photonic band gap. The photonic band gap of conventional PCs originates from Bragg scattering, which is strongly dependent on the incidence angle, lattice constant and polarization. Recently, a new type of photonic band gap coming from the mechanisms beyond Bragg scattering is realized in the PCs containing negative-index materials (NIMs). Such photonic band gap is called zero-averaged refractive-index gap (zero-$\bar{n}$ gap) [3–6] which is invariant upon the change of scaling, insensitive to the disorder and also independent of the incidence angle and polarization. NIM has simultaneously negative permittivity (ε) and negative permeability (μ), in which the electric field vector **E**, the magnetic field vector **H** and the wave vector **k** form a left-handed triplet [7]. Therefore, NIM is also named as left-handed material. Although NIMs do not exist in nature, they can be processed artificially by using sub-wavelength microstructures with the unit cell of split-ring resonators and conducting wires [8], or LC-loaded left-handed transmission lines [9] etc.

Recently, a kind of metamaterial called indefinite metamaterial (IMM) [10] has been fabricated successfully in experiments [8]. IMMs are anisotropic and have different signs for the components of the permittivity and permeability tensors. As only a part of the constitutive parameters is required to be negative, IMMs are much easier to fabricate compared with the isotropic NIMs. Many properties and applications of the IMMs have been studied extensively, e.g. the characteristics of surface modes, spatial filtering property and Goos-Hänchen effects [11–13]. Besides the periodic structures containing isotropic NIM, the omnidirectional zero-$\bar{n}$ gap and the defect mode are also found in the photonic crystals containing IMMs [14]. Fibonacci quasi-periodic one-dimensional photonic crystals composed of positive-index materials and isotropic NIMs have also been a concern in theory and experiments [15–18], but those quasi-periodic structures containing IMMs are not studied. In this paper, Fibonacci quasi-periodic photonic crystals composed of isotropic positive-index material and IMMs are investigated theoretically.

2. Model and theory

A Fibonacci quasi-periodic structure is based on the Fibonacci generation scheme, the sequence is expressed as $S_{j+1}=\{S_jS_{j-1}\}$ for level $j\geq 1$, with $S_0=\{B\}$ and $S_1=\{A\}$, the first few sequences are $S_2=\{AB\}$, $S_3=\{ABA\}$, $S_4=\{ABAAB\}$ and so on. In a Fibonacci quasi-periodic one-dimensional photonic crystal, two types of layer, A and B are arranged in a Fibonacci sequence (FS), where A and B are considered as the isotropic positive-index material and the IMM respectively. For the jth generation of the considered Fibonacci one-dimensional photonic crystal, the sequence can be expressed as $P_j=(S_j)_N$, in which N is the number of periods. As an example, the third sequence of P_3 is $P_3=(ABA)_N$ as shown in Fig. 1.

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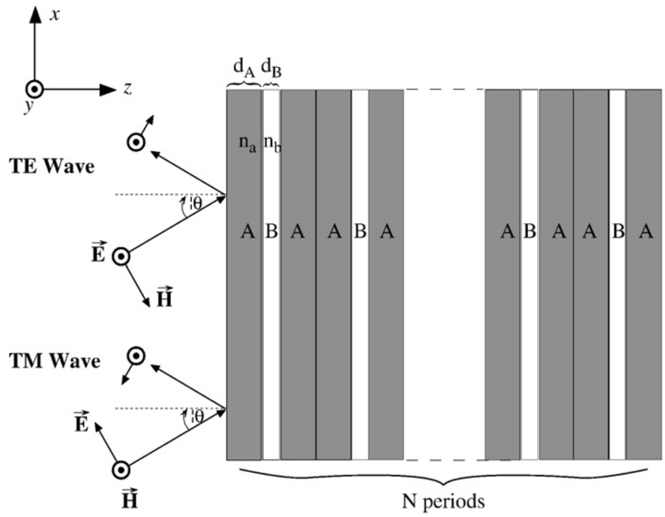

Fig. 1. Schematic diagram of Fibonacci quasi-periodic one-dimensional photonic crystal consisting of alternate positive-index material (A) and IMM (B) under any incidence angle (θ) for TE and TM waves.

The conventional technique is employed to obtain the photonic spectra for a Fibonacci structure due to a lack of periodicity in Fibonacci sequences. In a Fibonacci one-dimensional periodic structure, a certain level of Fibonacci multilayer works as a super-cell. Let a plane wave be incident from the vacuum at an angle θ onto the Fibonacci quasi-periodic one-dimensional photonic crystal as shown in Fig. 1, where the layers are parallel to the x–y plane and the wave at normal incidence is along the z-axis. The permittivity, permeability and thicknesses of the two layers are assumed to be ε_A, ε_B, μ_A, μ_B, d_A and d_B, respectively. For simplicity, both ε_B and μ_B tensors in the IMM layer are considered to be diagonalizable, which have the forms as

$$\varepsilon_B = \begin{pmatrix} \varepsilon_{Bx} & 0 & 0 \\ 0 & \varepsilon_{By} & 0 \\ 0 & 0 & \varepsilon_{Bz} \end{pmatrix}, \quad \mu_B = \begin{pmatrix} \mu_{Bx} & 0 & 0 \\ 0 & \mu_{By} & 0 \\ 0 & 0 & \mu_{Bz} \end{pmatrix}. \tag{1}$$

Without loss of generality, we consider the oblique monochromatic incident wave with the electric field and the magnetic field polarized along the y-axis for the transverse electric (TE) and transverse magnetic (TM) waves respectively. For the TE wave, the electric and magnetic fields can be expressed as

$$E_{By} = e^{ik_x x}\left(Ae^{ik_{Bz}z} + Be^{-ik_{Bz}z}\right)$$

$$H_{Bx} = \frac{-k_{Bz}}{\omega\mu_0\mu_{Bx}} e^{ik_x x}\left(Ae^{ik_{Bz}z} - Be^{-ik_{Bz}z}\right)$$

$$H_{Bz} = \frac{k_x}{\omega\mu_0\mu_{Bz}} e^{ik_x x}\left(Ae^{ik_{Bz}z} + Be^{-ik_{Bz}z}\right) \tag{2}$$

in the IMM layer, and

$$E_{Ay} = e^{ik_x x}\left(Ce^{ik_{Az}z} + De^{-ik_{Az}z}\right)$$

$$H_{Ax} = \frac{-k_{Az}}{\omega\mu_0\mu_A} e^{ik_x x}\left(Ce^{ik_{Az}z} - De^{-ik_{Az}z}\right)$$

$$H_{Az} = \frac{k_x}{\omega\mu_0\mu_A} e^{ik_x x}\left(Ce^{ik_{Az}z} + De^{-ik_{Az}z}\right) \tag{3}$$

in the positive-index material layer. $k_{Bz}^2 = \omega^2\varepsilon_{By}\mu_{Bx}/c^2 - \mu_{Bx}/\mu_{Bz}k_x^2$ and $k_{Az}^2 = \omega^2\varepsilon_A\mu_A/c^2 - k_x^2$, and k_{Bz} and k_{Az} are the z-components of the wave vector in the IMM and the positive-index material layers respectively; k_x is the x-component. A, B, C and D are the four constants which are determined by the boundary conditions at the interfaces. For a finite Fibonacci quasi-periodic structure, the electric and magnetic fields from the position z to $z+\Delta z$ in the same layer can be related via a transfer matrix for materials A and B [19]

$$T_{A,B}\left(d_{A,B},\omega\right) = \begin{pmatrix} \cos\left(k_{A,Bz}d_{A,B}\right) & i\dfrac{\mu_{A,Bx}\omega}{k_{A,Bz}c}\sin\left(k_{A,Bz}d_{A,B}\right) \\ i\dfrac{k_{A,Bz}c}{\mu_{A,Bx}\omega}\sin\left(k_{A,Bz}d_{A,B}\right) & \cos\left(k_{A,Bz}d_{A,B}\right) \end{pmatrix}. \tag{4}$$

By means of the transfer matrix method, we can obtain the transmission coefficient of a monochromatic plane wave

$$t(\omega) = \frac{2p_0}{[p_0x_{22}(\omega) + p_sx_{11}(\omega)] - [p_0p_sx_{12}(\omega) + x_{21}(\omega)]} \tag{5}$$

where $p_0 = p_s = \cos\theta$ for the air background, $x_{ij}(i,j=1,2)$ is the matrix elements of $X_N(\omega) = \prod_{j=1}^{j=2N} T_{A,B}(d_{A,B},\omega)$ which represents the total transfer matrix connecting the fields at the entrance and the exit end. The treatment for the TM wave is similar to that for the TE wave, where $k_{Bz}^2 = \omega^2\mu_{By}\varepsilon_{Bx}/c^2 - \varepsilon_{Bx}/\varepsilon_{Bz}k_x^2$.

3. Results and discussions

In the following calculation, we adopt the Drude model [5] to describe the tensor components for the permittivity and permeability of IMM, such dispersion can be realized in a composite made of periodically LC-loaded transmission lines [9]. $\varepsilon_{By} = 1–100/(2\pi f)^2$, $\mu_{Bx} = 1.21–100/(2\pi f)^2$ and $\mu_{Bz} = 2$ for the TE wave. For the TM wave, we choose $\mu_{By} = 1.21–100/(2\pi f)^2$, $\varepsilon_{Bx} = 1–100/(2\pi f)^2$ and $\varepsilon_{Bz} = 2$, where f is the operation frequency with a unit of GHz. The permittivity and permeability of the isotropic positive-index material layer are set to be $\varepsilon_A = 4$ and $\mu_A = 1$ respectively. Firstly, the energy bands for the successive Fibonacci sequences of the quasi-periodic structure (i.e., from P_2 to P_7) are investigated with $d_A = 8$ mm, $d_B = 6$ mm and $N = 16$, which are schematically shown in Fig. 2. It is noticed that two or three broad forbidden gaps are open constantly for each Fibonacci level in the considered frequency range, besides some minor gaps in other frequency ranges. In the broad gap at low frequency, IMMs show a negative refractive index. While at high frequency, they act as an anisotropic positive-index material. In fact, the Fibonacci structure P_2 is actually the photonic crystal discussed by Y. J. Xiang et al. [14]. The

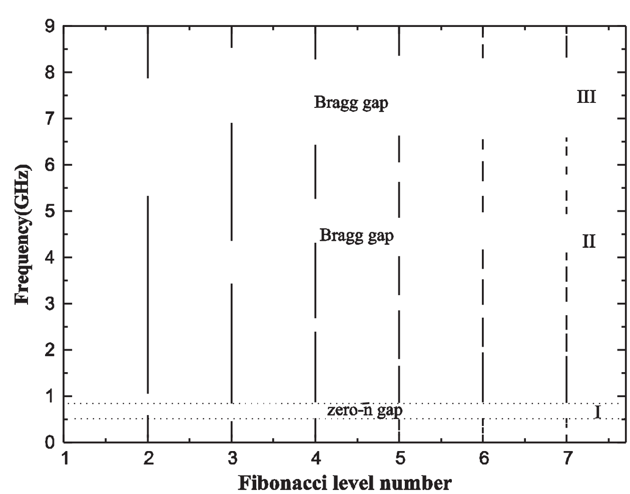

Fig. 2. The energy bands for the first six successive levels of the Fibonacci quasi-periodic one-dimensional photonic crystals.