



# Prospect for detecting squeezed states of light created by a single atom in free space

Magdalena Stobińska<sup>a,b,c,\*</sup>, Markus Sondermann<sup>a</sup>, Gerd Leuchs<sup>a</sup>

<sup>a</sup>Institute for Optics, Information and Photonics, Erlangen-Nürnberg University, Staudtstr. 7/B2, 91058 Erlangen, Germany

<sup>b</sup>Max Planck Institute for the Science of Light, Günter-Scharowsky-Str. 1, Bau 24, 91058 Erlangen, Germany

<sup>c</sup>Institute of Theoretical Physics and Astrophysics, University of Gdańsk, Poland

## ARTICLE INFO

### Article history:

Received 5 August 2009

Received in revised form 12 October 2009

Accepted 19 October 2009

### Keywords:

Squeezing

Resonance fluorescence

Single photon

Single atom

## ABSTRACT

We discuss the possibilities of studying in detail the dynamics of spontaneous emission of a single photon by a single atom and measuring the transient degree of squeezing by means of full solid angle fluorescence detection.

© 2009 Elsevier B.V. All rights reserved.

## 1. Introduction

*We are mourning our teacher and colleague Krzysztof Wódkiewicz whom we will fondly remember as a skillful scientist, sharp thinker and friend.*

Quadrature squeezed states of light are a basic resource for continuous variable (CV) based quantum communication. Two independent CV squeezed states interfering on a beam splitter form at its output CV Einstein–Podolsky–Rosen entangled states [1,2]. Experimentally, this entanglement was demonstrated for vacuum states [3] and for intense laser beams [4]. Based on squeezed-state entanglement quantum teleportation of coherent states was experimentally achieved [5]. Recently it was shown that a single quadrature squeezed photon also enables obtaining teleportation of coherent state qubits [6]. Moreover, quadrature squeezing can be transformed into polarization squeezing [7]. CV squeezed states have also been useful for developing various other tools for quantum information processing. For example, they have been used for realizing quantum nondemolition coupling and thus CV quantum erasing [8]. Furthermore, a method for dense quantum coding for the quadrature amplitudes of the electromagnetic field has been proposed [9].

Whereas in the majority of the references mentioned so far squeezed states have been used as a tool, we want to focus on a

more fundamental aspect here. We propose an experiment that is expected to enable the successful observation of a squeezed state that is generated in one of the most fundamental settings of quantum optics: the spontaneous emission of a single photon by a single two-level atom that is prepared in a suitable superposition state.

The name ‘squeezed’ states emerges from their property that the quantum uncertainty in one of two noncommuting observables  $[A, B] = iC$  in these states is decreased  $(\Delta A)^2 < \frac{1}{2}|\langle C \rangle|$  at the cost of increasing uncertainty in the other one  $(\Delta B)^2 > \frac{1}{2}|\langle C \rangle|$ , in order to obey the Heisenberg uncertainty relation  $\Delta A \Delta B \geq \frac{1}{2}|\langle C \rangle|$ . Amplitude and phase quadrature operators are defined as

$$X_1 = 1/2(a + a^\dagger), \quad X_2 = i/2(a^\dagger - a), \quad (1)$$

where  $a$  and  $a^\dagger$  denote annihilation and creation operators, respectively,  $X_{1,2}$  are noncommuting observables  $[X_1, X_2] = i/2$  for which  $\Delta X_1 \Delta X_2 \geq 1/4$ . Therefore, a state for which  $(\Delta X_k)^2 < 1/4$  for  $k = 1, 2$  is called quadrature squeezed.

Usually, quadrature squeezed states are produced via a squeezing transformation generated by a Hamiltonian quadratic in annihilation  $a$  and creation  $a^\dagger$  operators applied to a coherent state  $|\alpha\rangle$ . For a single-mode of light the transformation takes the following form

$$\hat{S} = \exp \left\{ \frac{\xi^*}{2} a^2 - \frac{\xi}{2} a^{\dagger 2} \right\}, \quad (2)$$

where  $\xi$  is called the squeezing parameter.

Early on it was realized that applying the operator  $\hat{S}$  to the vacuum state leads to a superposition of even number Fock states [10],

\* Corresponding author. Address: Institute for Optics, Information and Photonics, Erlangen-Nürnberg University, Staudtstr. 7/B2, 91058 Erlangen, Germany.

E-mail address: [magda.stobinska@mpl.mpg.de](mailto:magda.stobinska@mpl.mpg.de) (M. Stobińska).

hence the name two-photon coherent states for this special class of squeezed states. In experiments squeezing is realized either in the parametric amplification or in four wave mixing process in  $\chi^{(3)}$  nonlinear medium (e.g. in optical fiber).

Squeezing is present not only in a superposition of two macroscopic coherent states [11] but also in the superposition of vacuum and a single photon arising in the process of spontaneous emission of an atom into a single cavity mode [12]. At the time this came a bit as a surprise<sup>1</sup>.

Several years later, different schemes for the generation of arbitrary quantum states of light – including the superposition states mentioned above – have been proposed [13–16]. All these schemes have in common that they are based on the interaction of atoms with the single-mode light field of a cavity. However, here we concentrate on the interaction dynamics of light and single atoms in free space, in particular in the dynamics of spontaneous emission. In other words, the squeezing in the fluorescence of a single atom [17,18] is a sensitive tool for studying the dynamics of spontaneous emission in an unprecedented way. Therefore, we sacrifice the possibility of generating arbitrary quantum states and restrict the discussion to the most simple superposition state, namely the one described in Ref. [12]. As we will outline below, this state can be generated by preparing a single two-level atom in the corresponding superposition of its ground state and its excited state and full solid angle collection of the spontaneously emitted photon.

The paper is organized as follows. In Section 2 we summarize the theoretical model for obtaining the single photon squeezed state put forward in Ref. [12]. Section 3 is devoted to the discussion about the possibilities for realization and detection of squeezing in superposition states in a suitable free space experimental setup.

## 2. Model

According to the squeezing transformation in Eq. (2), the squeezed vacuum state is a superposition of even Fock number states [10]

$$\begin{aligned}\hat{S}|0\rangle &= e^{-\frac{1}{2}\ln\chi\xi} \exp\left\{-\frac{\text{th}\xi}{2}a^\dagger a\right\}|0\rangle \\ &= e^{-\frac{1}{2}\ln\chi\xi} \left\{|0\rangle - \sqrt{2!}\frac{\text{th}\xi}{2}|2\rangle + \sqrt{4!}\frac{\text{th}^2\xi}{4\cdot 2!}|4\rangle + \dots\right\},\end{aligned}\quad (3)$$

where the squeezing parameter  $\xi$  was assumed to be real for simplicity. Wódkiewicz et al. [12] discovered that in one-photon superposition states

$$|\varphi\rangle = \gamma|0\rangle + \beta|1\rangle \quad (4)$$

where  $|\gamma|^2 + |\beta|^2 = 1$ , quadrature squeezing is present as well for  $|\beta| < 1/\sqrt{2}$  and some special values of relative phase  $\phi$  between the probabilities amplitudes

$$\begin{aligned}(\Delta X_1)^2(\phi = 0, \pi) &= (\Delta X_2)^2(\pi/2, 3\pi/2) \\ &= 1/4 + |\beta|^2(|\beta|^2 - 1/2).\end{aligned}\quad (5)$$

The values of  $\phi = 0, \pi(\pi/2, 3\pi/2)$  correspond to the amplitude (phase) quadrature squeezing, respectively. The state Eq. (4) is presented in terms of a Wigner function in Fig. 1 for  $\gamma = \sqrt{2/3}$  and  $\beta = \sqrt{1/3}$ .

The superposition given in Eq. (4) arises naturally during interaction of a single quantized electromagnetic field mode initially in the vacuum state with a two-level atom initially in a superposition

of excited and ground states. The system is described by the Jaynes–Cummings Hamiltonian

$$H = 1/2\hbar\omega_0\sigma_z + \hbar\omega a^\dagger a + \hbar\lambda(a^\dagger\sigma_- + \sigma_+a) \quad (6)$$

where  $\omega_0$  is the atomic transition frequency,  $\sigma_z$  is the inversion operator,  $\omega$  is the field mode frequency,  $a$  and  $a^\dagger$  are field annihilation and creation operators,  $\lambda$  is interaction strength,  $\sigma_-$  and  $\sigma_+$  are lowering and rising atomic operators. For the relation between the Jaynes–Cummings single-mode model and the Wigner–Weisskopf model with a continuum of modes see Section 3 below.

Let us consider an atom prepared in a coherent superposition of excited  $|e\rangle$  and ground  $|g\rangle$  states which interacts with a vacuum field  $|0\rangle$

$$|\Psi(t=0)\rangle = \cos(\theta/2)|e, 0\rangle + e^{i\phi}\sin(\theta/2)|g, 0\rangle. \quad (7)$$

The atom can spontaneously decay to its ground state and emit a photon. In the Schrödinger picture and on resonance the total wave function of the system at time  $t$  reads

$$\begin{aligned}|\Psi(t)\rangle &= \cos(\theta/2)\cos(\lambda t)e^{-i\omega t}|e, 0\rangle + e^{i\phi}\sin(\theta/2)|g, 0\rangle \\ &\quad - i\cos(\theta/2)\sin(\lambda t)e^{-i\omega t}|g, 1\rangle.\end{aligned}\quad (8)$$

The uncertainties in quadrature operators for this state measured by a homodyne detector are equal to

$$(\Delta X_1)^2 = 1/4 + \cos^2(\theta/2)\sin^2(\lambda t)\left(1/2 - \sin^2\phi\sin^2(\theta/2)\right), \quad (9)$$

$$(\Delta X_2)^2 = 1/4 + \cos^2(\theta/2)\sin^2(\lambda t)\left(1/2 - \cos^2\phi\sin^2(\theta/2)\right). \quad (10)$$

The phase  $\phi$  denotes the phase difference between the field and the local oscillator in homodyne detection. For  $\theta = 2\pi/3, 4\pi/3$  and  $\phi = \pi/2$  squeezing appears in the amplitude quadrature

$$(\Delta X_1)^2 = 1/4 - 1/16\sin^2(\lambda t). \quad (11)$$

Note that for time  $t = \pi/2\lambda$  we obtain the desired superposition Eq. (4). Furthermore, no squeezing arises for a purely excited ( $\theta = 0$ ) or de-excited ( $\theta = \pi$ ) atom.

As indicated by Eqs. (9), (10) the amount of squeezing in the emitted photon superposition state is dependent on the state in which the atom was prepared. Thus, a precise measurement of the photonic state would give some insight into the state – possibly unknown a priori – in which the atom was before the emission process.

Moreover, in [12] a link between atomic dipole  $D = |g\rangle\langle e| = D_1 + iD_2$  squeezing and radiated field squeezing has been established. If the initial atomic state satisfies that  $\langle[D^\dagger, D]\rangle < 0$  and the dipole is squeezed:  $(\Delta D_1)^2 < 0$  the field will be squeezed as well. Since in resonance fluorescence the normally ordered variance of the electric field in the far field zone is related to the normally ordered variance of the atomic dipole, it is possible to observe the quadrature squeezing present in the one-photon superposition state by means of resonance fluorescence preparing the atomic state properly.

## 3. Experimental prospects

The maximum amount of squeezing predicted by Eqs. (5) and (11) is 1.25 dB for  $|\beta| = 1/2$ . This small reduction of quadrature fluctuations calls for a sophisticated detection scheme.

First, the atom has to be prepared in the desired superposition state by application of a suitable optical pulse. Identifying  $|\beta|$  with  $|\cos(\theta/2)|$  determines the area of the excitation pulse that prepares the atom in the superposition state that enables maximum squeezing. The state of the radiation field then has to be detected transiently during the emission process via time resolved homodyne measurements (e.g., [19,20]).

<sup>1</sup> One of us (GL) vividly remembers Krzysztof Wódkiewicz coming to his office at the Max Planck Institute for Quantum Optics in Garching emphasizing that he had just found an unexpected result: The superposition of the  $|0\rangle$  and the  $|1\rangle$  Fock states may show squeezing. The surprise was that squeezed states generated from vacuum were not exclusively superpositions of even Fock states.

Download English Version:

<https://daneshyari.com/en/article/1538699>

Download Persian Version:

<https://daneshyari.com/article/1538699>

[Daneshyari.com](https://daneshyari.com)