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Diffraction of a plane wave by a soft–hard strip

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ABSTRACT

In this paper we have studied the problem of diffraction of a plane wave by a finite soft–hard strip. By using the Fourier transform the boundary value problem is reduced to a matrix Wiener–Hopf equation. Using the matrix factorization of the kernel matrix, the problem is solved for two coupled equations using the Wiener–Hopf technique and the method of steepest descent. It is observed that the diffracted field is the sum of the fields produced by the two edges of the strip and an interaction field. Some graphs showing the effects of various parameters on the field produced by two edges of the strip are also plotted.

1. Introduction

Scattering of waves by strips/slits is an important topic both in acoustics and electromagnetics. It has attracted the attention of many researchers [1–15]. A variety of analytical and numerical techniques have been used to study scattering from strips/slits. To name a few only e.g., Morse and Rubenstein [1] studied the diffraction of acoustic waves by a strip using method of separation of variables. Bowman et al. [2] summarized and reviewed much of the work done on strip. Another important detail of works, using the Wiener–Hopf (WH) technique, consists of Jones [3], Kobayashi [4], Noble [5], Faulkner [6], Cinar and Büyükaksoy [7], Serbest and Büyükaksoy [8], Büyükaksoy and Alkumru [9], Asghar [10], Asghar et al. [11] and Ayub et al. [12,13]. Recently Ahmad and Naqvi [14] and Imran et al. [15] studied the electromagnetic scattering from a two dimensional perfect electromagnetic conductor (PEMC) strip and PEMC strip grating and by an infinitely long conducting strip on dielectric slab by using numerical simulation and Kobayashi potential method. When the strip length is large as compared to the incident wavelength a high frequency approximate solution can be obtained by using the concept of the geometrical theory of diffraction GTD [16]. (Also we have asymptotically evaluated the integrals I_1 to I_6 in the Appendix under the assumption that strip length is large [5, pp. 201] with respect to the wavelength.).

In this paper we have studied the diffraction of a plane wave by a soft–hard strip. The continued interest in the problem is due to the fact that it constitutes the simplest half plane problem which can be formulated as a system of coupled WH equations that cannot be decoupled trivially. Rawlins [17] took the lead in the discussion of diffraction of a plane acoustic waves by a semi-infinite barrier satisfying the soft (pressure release) boundary condition on its upper surface while the hard (rigid) boundary condition on its lower surface. The author [18] reconsidered the problem solved by [17] and factorized the kernel matrix appearing in the problem by Daniele–Kharapkov methods [19,20] to give the solution of the matrix WH problem.

The WH technique [5] proves to be a powerful tool to tackle, not only, the problems of diffraction by a single half plane but it may further be extended to the case of parallel half planes. In the present work, we examine a more general problem of plane-wave diffraction by a finite soft–hard strip. By using the Fourier transform technique, three-part boundary value problem is reduced to a matrix WH equation. The solution of this matrix WH problem requires the factorization of the kernel matrix appearing in the problem. This matrix factorization has been done by [18]. With the matrix factorization known, we then follow Noble's approach [5] closely to calculate the diffracted field produced by the finite soft–hard strip. It is observed that the two edges of the strip give rise to two diffracted fields (one from each edge) and the interaction of one edge upon the other edge. Finally the diffracted field is calculated using the method of steepest descent. Some graphs, showing the effect of different parameters on the diffracted field produced by the two edges of the soft–hard strip, are also plotted and discussed.

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2. Mathematical formulation of the problem

Consider the diffraction of a plane wave incident on the finite soft–hard plane $S = \{x \in (p,q),\ y = 0,\ z \in (-\infty,\infty)\}$. The plane is assumed to be of infinitesimal thickness and soft (pressure release) at the top and hard (rigid) at the bottom. A time factor $e^{-i\omega t}$ is assumed and suppressed throughout. The geometry of the problem is depicted in Fig. 1.

For harmonic vibrations of time dependence $e^{-i\omega t}$, we require the solution of the wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\psi_t(x,y)=0, \tag{1}$$

where ψ_t is the total velocity potential and the boundary and continuity conditions are given

$$\psi_t(x,0^+)=0,\quad \text{on}\quad p<x<q, \tag{2a}$$

$$\frac{\partial\psi_t(x,0-)}{\partial y}=0,\quad \text{on}\quad p<x<q, \tag{2b}$$

and

$$\psi_t(x,0^+)-\psi_t(x,0^-)=0,\quad \text{on}\quad \left\{\begin{matrix}-\infty<x<p\\ q<x<\infty\end{matrix}\right\}, \tag{3a}$$

$$\frac{\partial\psi_t(x,0+)}{\partial y}-\frac{\partial\psi_t(x,0-)}{\partial y}=0,\quad \text{on}\quad \left\{\begin{matrix}-\infty<x<p\\ q<x<\infty\end{matrix}\right\}. \tag{3b}$$

Let a plane wave

$$\psi_i(x,y)=e^{-ik(x\cos\theta_0+y\sin\theta_0)}, \tag{4}$$

be incident upon the soft–hard half finite plate occupying the position $p<x<q$, $y=0$. In Eq. (4), θ_0 is the angle of incidence and for the analytic convenience it is assumed that the wave number k has positive imaginary part. For the analysis purpose it is convenient to express the total field ψ_t as [5]

$$\psi_t(x,y)=\psi_i(x,y)+\psi(x,y), \tag{5}$$

where $\psi(x,y)$ is the diffracted field and $\psi_i(x,y)$ is the incident field given by Eq. (4). For the unique solution of the problem, the edge conditions require that ψ_t and its normal derivative must be bounded and satisfy [17,18]

$$\psi_t(x,0)=\begin{cases}-1+O(x-p)^{\frac{1}{4}} & \text{as } x\to p,\\ -1+O(x-q)^{\frac{1}{4}} & \text{as } x\to q,\end{cases} \tag{6}$$

$$\frac{\partial\psi_t(x,0)}{\partial y}=\begin{cases}O(x-p)^{-\frac{3}{4}} & \text{as } x\to p,\\ O(x-q)^{-\frac{3}{4}} & \text{as } x\to q.\end{cases} \tag{7}$$

Thus, scattered field satisfies the Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\psi(x,y)=0, \tag{8}$$

subject to the boundary conditions

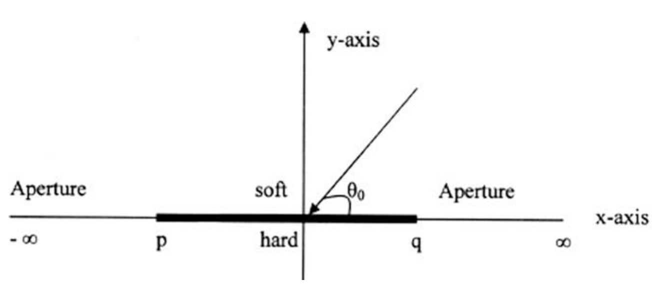

Fig. 1. Geometry of the problem.

$$\begin{cases}\psi(x,0^+)=-e^{-ikx\cos\theta_0}\\ \frac{\partial\psi(x,0^-)}{\partial y}=ik\sin\theta_0 e^{-ikx\cos\theta_0}\end{cases}\quad \text{on } p<x<q, \tag{9}$$

and the continuity conditions

$$\begin{cases}\psi(x,0^+)=\psi(x,0^-)\\ \frac{\partial\psi(x,0^+)}{\partial y}=\frac{\partial\psi(x,0-)}{\partial y}\end{cases}\quad \text{on}\quad \begin{matrix}-\infty<x<p,\\ q<x<\infty.\end{matrix} \tag{10}$$

The Fourier transform pair is defined as follows

$$\begin{aligned}\overline{\psi}(\alpha,y)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(x,y)e^{i\alpha x}dx,\\ &=e^{i\alpha p}\overline{\psi}_-(\alpha,y)+\overline{\psi}_1(\alpha,y)+e^{i\alpha q}\overline{\psi}_+(\alpha,y),\end{aligned} \tag{11}$$

and

$$\psi(x,y)=\int_{-\infty}^{\infty}\overline{\psi}(\alpha,y)e^{-i\alpha x}d\alpha, \tag{12}$$

where

$$\begin{aligned}\overline{\psi}_-(\alpha,y)&=\frac{1}{2\pi}\int_{-\infty}^{p}\psi(x,y)e^{i\alpha(x-p)}dx,\\ \overline{\psi}_1(\alpha,y)&=\frac{1}{2\pi}\int_{p}^{q}\psi(x,y)e^{i\alpha x}dx,\\ \overline{\psi}_+(\alpha,y)&=\frac{1}{2\pi}\int_{q}^{\infty}\psi(x,y)e^{i\alpha(x-q)}dx.\end{aligned} \tag{13}$$

The function $\overline{\psi}_-(\alpha,y)$is regular in the lower half plane $\mathrm{Im}\ \alpha<\mathrm{Im}\ k$, $\overline{\psi}_+(\alpha,y)$ is regular in the upper half plane $\mathrm{Im}\ \alpha>\mathrm{Im}\ k\cos\theta_0$ and $\overline{\psi}_1(\alpha,y)$ is an analytic function which is regular in the common region $\mathrm{Im}\ k\cos\theta_0<\ \mathrm{Im}\ \alpha<\mathrm{Im}\ k$.

On taking the Fourier transform of the Eq. (8) we arrive at

$$\frac{d^2\overline{\psi}(\alpha,y)}{dy^2}+K^2\overline{\psi}(\alpha,y)=0, \tag{14}$$

where $K(\alpha)=\sqrt{k^2-\alpha^2}$.

Defining $K(\alpha)$, the square root function, to be that branch which reduces to $+k$ when $\alpha=0$ and when the complex $\alpha-$ plane is cut either from $\alpha=k$ to $\alpha=k\infty$ or from $\alpha=-k$ to $\alpha=-k\infty$. The solution of Eq. (14), representing the outgoing waves at infinity, can formally be written as

$$\overline{\psi}(\alpha,y)=\begin{cases}A(\alpha)e^{iK(\alpha)y} & y>0,\\ B(\alpha)e^{-iK(\alpha)y} & y<0,\end{cases} \tag{15}$$

where $A(\alpha)$ and $B(\alpha)$ are the unknown coefficients which are to be determined. The Fourier transform of the boundary conditions (9) and (10) yields

$$\overline{\psi}_1(\alpha,0^+)=-\frac{1}{i\sqrt{2\pi}}G(\alpha), \tag{16a}$$

$$\frac{\partial\overline{\psi}_1(\alpha,0^-)}{\partial y}=\frac{k\sin\theta_0}{\sqrt{2\pi}}G(\alpha), \tag{16b}$$

and

$$\overline{\psi}_\pm(\alpha,0^+)=\overline{\psi}_\pm(\alpha,0^+), \tag{17a}$$

$$\frac{\partial\overline{\psi}_\pm(\alpha,0^+)}{\partial y}=\frac{\partial\overline{\psi}_\pm(\alpha,0^-)}{\partial y}, \tag{17b}$$

where

$$G(\alpha)=\frac{e^{i(\alpha-k\cos\theta_0)q}-e^{i(\alpha-k\cos\theta_0)p}}{\sqrt{2\pi}(\alpha-k\cos\theta_0)}. \tag{18}$$