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Entanglement evolution of a two-qubit system interacting with a quantum spin environment

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ABSTRACT

We investigate the entanglement dynamics and decoherence of a two-qubit system under a quantum spin environment at finite temperature in the thermodynamics limit. For the case under study, we find different initial states will result in different entanglement evolution, what deserves mentioning here is that the state $|\Psi\rangle = \cos\alpha|01\rangle + \sin\alpha|10\rangle$ is most robust than other states when $\pi/2 < \alpha < \pi$, since the entanglement remains unchanged or increased under the spin environment. In addition, we also find the anisotropy parameter Δ can suppress the destruction of decoherence induced by the environment, and the undesirable entanglement sudden death arising from the process of entanglement evolution can be efficiently controlled by the inhomogeneous magnetic field ζ.

1. Introduction

Entanglement is one of the most striking features of quantum mechanical systems that have no classical analog. It has been the focus of much work recently due to its key role in the topical area of quantum information processing [1]. Over the past few years, there has been considerable interest in investigating entanglement in quantum spin systems with Heisenberg interactions [2–7], since the Heisenberg model, as a simple but realistic solid-state system, not only have been used to simulate a quantum computer, as well as quantum dots [8,9], nuclear spins [10], electronic spins [11], and optical lattice [12], but also display useful applications in quantum state transfer [13]. However, in the previous studies, the detailed interaction between the system and the environment is not an essential part of the matter. On the other hand, from the practical point of views, the real quantum systems will unavoidably interact with the surrounding environments and thus leads to decoherence. This is one fundamental obstacle to perform quantum computation. Therefore, it is indispensable to take into account of the decoherence caused by the interaction between the system and the environment.

Recently, the dynamic behavior of a single spin or several spins interacting with a spin bath has attracted much attention [14–18]. However, in most case it is very difficult to obtain an exact solution to the evolution of the reduced density matrix with the environment modes traced over in the case of the non-Markovian process. In Ref. [19], using a novel operator technique, the authors present an exact calculation of the dynamics of the reduced density matrix of two coupled spins in a spin environment in the thermodynamics limit at finite temperature. The results show that the dynamics of the entanglement depend strongly on the initial state of the system, the coupling between the two-spin qubits, the interaction between the qubit system and environment, the interaction between the constituents of the spin environment, the environment temperature, as well as the detuning controlled by a locally applied external magnetic field. Later, the authors in Ref. [19] study the dynamics of a central spin coupling with its environment at finite temperature under the thermodynamics limit [20]. In view of the above results, we find only the initial state $|00\rangle$, $|11\rangle$, and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ are considered in Ref. [19], and other initial states and parameters, such as inhomogeneous magnetic field and anisotropy are rarely included. Therefore, in this paper, considering other initial states, we study the entanglement evolution of two-qubit system interacting with a quantum spin environment at finite temperature in the thermodynamics limit with taking into account the influence of inhomogeneous magnetic field and anisotropy.

To quantify the amount of entanglement between the two qubits, we consider concurrence defined by Wootters [21]

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{1}$$

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where the quantities $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4$ are the square roots of the eigenvalues of the matrix $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. ρ^* denotes the complex conjugate of ρ and σ_y is the normal pauli operators. The concurrence $C = 0$ corresponds to an unentangled state and $C = 1$ for a maximally entangled state. For the special case

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{2}$$

the concurrence can be easily obtained

$$C = 2\max\{|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}\}. \tag{3}$$

This letter is organized as follows. In Section 2, we introduce the model and derive the time evolution by the novel operator technique. In Section 3, we present the result of entanglement dynamics for different initial states, and discuss the effects of inhomogeneous magnetic field and anisotropy parameter on the entanglement evolution. Finally, we conclude in Section 4.

2. Hamiltonian evolution

Here, we extend the system Hamiltonian of the model in Ref. [19] by considering the inhomogeneous magnetic field and anisotropy while keep the spin environment unchanged. The Hamiltonian of our model is $H = H_S + H_{SB} + H_B$, where H_S, H_{SB} and H_B denote the Hamiltonian of the system, system-bath interaction and bath, respectively. They can be written as

$$H_S = (\mu_0 + \zeta)S_{01}^z + (\mu_0 - \zeta)S_{02}^z + \Omega(S_{01}^+ S_{02}^- + S_{01}^- S_{02}^+) + \Delta S_{01}^z S_{02}^z, \tag{4}$$

$$H_{SB} = \frac{g_0}{\sqrt{N}}\left[(S_{01}^+ + S_{02}^+)\sum_{i=1}^{N} S_i^-\right] + \frac{g_0}{\sqrt{N}}\left[(S_{01}^- + S_{02}^-)\sum_{i=1}^{N} S_i^+\right], \tag{5}$$

$$H_B = \frac{g}{N}\sum_{i \neq j}^{N}(S_i^+ S_j^- + S_i^- S_j^+), \tag{6}$$

where the external magnetic fields are assumed to be along the z-direction, μ_0 describes the uniformity of the field while ζ measures the degree of the inhomogeneity of the field. Ω is the coupling constant between any two-qubit spins and Δ is the anisotropy parameter. S_{0i}^+ and $S_{0i}^-(i = 1, 2, 3)$ are the spin–flip operators of the spin system, respectively. S_i^+ and S_i^- are the corresponding of the ith qubit spin in the bath. N is the number of the bath atoms. g_0 is the coupling constant between the qubit system spins and bath spins, whereas g is that between the bath spins. Both constants are rescaled as $g_0/\sqrt{N}$ and $g/\sqrt{N}$ [22–25]. Using the collective angular momentum operators $J_\pm = \sum_{i=1}^N S_i^\pm$, the Hamiltonian Eqs. (5) and (6) turn out to be

$$H_{SB} = \frac{g_0}{\sqrt{N}}(S_{01}^+ + S_{02}^+)J_- + \frac{g_0}{\sqrt{N}}(S_{01}^- + S_{02}^-)J_+, \tag{7}$$

$$H_B = \frac{g_0}{N}(J_+J_- + J_-J_+) - g. \tag{8}$$

After the Holstein–Primakoff transformation:

$$J_+ = b^\dagger(\sqrt{N - b^\dagger b}), \quad J_- = (\sqrt{N - b^\dagger b})b, \tag{9}$$

with $[b, b^\dagger] = 1$, the Hamiltonian Eqs. (7) and (8) can be written as

$$H_{SB} = g_0(S_{01}^+ + S_{02}^+)\sqrt{1 - \frac{b^\dagger b}{N}}b + g_0(S_{01}^- + S_{02}^-)b^\dagger\sqrt{1 - \frac{b^\dagger b}{N}}, \tag{10}$$

$$H_B = g\left[b^\dagger\left(1 - \frac{b^\dagger b}{N}\right)b + \sqrt{1 - \frac{b^\dagger b}{N}}bb^\dagger\sqrt{1 - \frac{b^\dagger b}{N}}\right] - g. \tag{11}$$

In the thermodynamic limit (i.e., $N \to \infty$) at finite temperature, we have

$$\begin{aligned} H_{SB} &= g_0\left[(S_{01}^+ + S_{02}^+)b + (S_{01}^- + S_{02}^-)b^\dagger\right], \\ H_B &= 2gb^\dagger b. \end{aligned} \tag{12}$$

Here we have applied the approximation that $b^\dagger b/N$ tends to be vanishing, since the energy of the excitations original from the interaction between the system and the bath is very low. In the following, we are interested in the density matrix evolution of the spin system, by which we can know the entanglement dynamics under decoherence induced by the spin environment. Since the Hamiltonian is time independent, the density matrix evolves for the total system is

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}, \tag{13}$$

where we assume that the initial density matrix $\rho(0)$ is separable between the system and the bath, i.e. $\rho(0) = \rho_S(0) \otimes \rho_B$. Here the initial state of the spin system is described by $\rho_S(0)$. The density matrix of the environment satisfies a thermal distribution, i.e. $\rho_B = e^{-H_B/kT}/Z$, where Z is the partition function and the Boltzmann constant k is set to one in this paper. Then we can obtain the reduced system density matrix by tracing out the environment degree of freedom, i.e. $\rho_S(t) = Tr_B\rho(t)$.

For two-qubit system, there are two types of Bell-like states,

$$|\Phi\rangle = \cos\alpha|00\rangle + \sin\alpha|11\rangle, \quad |\Psi\rangle = \cos\alpha|01\rangle + \sin\alpha|10\rangle, \tag{14}$$

with $0 \leqslant \alpha \leqslant \pi$. In Ref. [19], the authors have studied the entanglement evolution for the initial system state $|00\rangle$, $|11\rangle$, and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ in the absence of inhomogeneous magnetic field and anisotropy. Here we use $|\Phi\rangle$ and $|\Psi\rangle$ to serve as the initial states of the system taking into account the inhomogeneous magnetic field and anisotropy. Namely, $\rho_S(0) = |\Phi\rangle\langle\Phi|$ and $\rho_S(0) = |\Psi\rangle\langle\Psi|$. First, by taking state $|\Psi\rangle$ as the initial system state, the reduced density matrix for the system can be written as

$$\begin{aligned} \rho_S(t) = \frac{1}{Z}&\cos^2\alpha Tr_B\left[e^{-iHt}|01\rangle e^{-H_B/T}\langle 01|e^{iHt}\right] \\ &+ \frac{1}{Z}\sin^2\alpha Tr_B\left[e^{-iHt}|10\rangle e^{-H_B/T}\langle 10|e^{iHt}\right] \\ &+ \frac{1}{Z}\sin\alpha\cos\alpha Tr_B\left[e^{-iHt}|01\rangle e^{-H_B/T}\langle 10|e^{iHt}\right] \\ &+ \frac{1}{Z}\sin\alpha\cos\alpha Tr_B\left[e^{-iHt}|10\rangle e^{-H_B/T}\langle 01|e^{iHt}\right], \end{aligned} \tag{15}$$

and we can easily obtain

$$Z = \frac{1}{1 - e^{-2g/T}}. \tag{16}$$

Following the idea of operator technique introduced in Ref. [19], we first need to convert the time evolution equation of the qubit system under the action of the total Hamiltonian into a set of coupled non-commuting operator variable equations. Considering the constituents of Hamiltonian H, we can write

$$e^{-iHt}|01\rangle = A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle, \tag{17}$$

where A, B, C and D are functions of operators b, $b^\dagger$, and time t. Using the Schrödinger equation identity

$$i\frac{d}{dt}(e^{-iHt}|01\rangle) = H(e^{-iHt}|01\rangle), \tag{18}$$