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Generalized transfer function: A simple model applied to active single-mode microring resonators

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ABSTRACT

The spectral properties of an active single-mode microring resonator are investigated in the frame of the generalized transfer function (GTF) approach, as derived from extended scattering and/or transfer matrix formalism. Spontaneous emission, looked upon as the driving source of the radiation, is described in a semi-classical way in the spectral domain. The internal and emitted fields are filtered into the resonance modes of the whole structure. The generalized transfer function expresses the spectral density of internal saturating intensity and includes all essential mechanisms at work in a laser oscillator: gain, losses and sources. The active zone is saturated through amplified spontaneous emission (ASE), integrated over its whole spectral range. Continuously valid across threshold, the method enables one to derive in a simple way the main steady-state properties of the laser oscillation, with the pumping rate as the only external parameter.

1. Introduction

In the past few years, optical open resonators based on total internal reflection (TIR) such as microspheres, microdisks or microrings have been subject to numerous studies [1,2]. The unique combination of strong temporal and spatial confinement of light makes these systems particularly attractive, not only for fundamental research [3,4] but also as new building blocks for fiber optics and photonic applications [5]. For instance, a review of photonic structures based on highly integrated passive coupled microrings can be found in Refs. [6,7]. Naturally enough, active structures have also been heavily investigated. Since the observation of the first continuous-wave (CW) laser oscillation in a large solid-state Nd:YAG sphere [8], laser action has been demonstrated in many different rare-earth-doped glass spheres [9,10], as well as semiconductor microdisks [11,12] or microrings [13].

As soon as optical gain is involved, spontaneous emission is known to play quite important a role: far from being a mere noise to be avoided, it acts as the very driving source of the electromagnetic field. In that respect, the laser behavior of single-mode Fabry–Perot (FP) resonators is well depicted in the spectral domain by the semi-classical generalized transfer function (GTF) [14,15]. The GTF expresses the spectral density of the internal field, where the gain, the source and the refractive index of the active zone are uniformly saturated through amplified spontaneous emission (ASE), longitudinally averaged and integrated over its whole spectral range. Assuming the total saturating intensity as the only internal parameter, a self-consistent calculation enables one to derive the total power as well as the lineshape of the emitted radiation, as functions of the pumping level.

In its turn, the GTF of a one-dimensional active cavity can be conveniently established in the frame of the extended (3×3) transfer matrix formalism. This elegant way of dealing with internal sources had been originally proposed by Weber and Wang [16,17] for investigating active distributed feedback (DFB) devices, and further developed by one of us well into the laser regime. An analytical expression for the GTF is easily derived in terms of structural parameters, leading to a self-consistent calculation of all parameters as soon as the saturation of the active medium is correctly taken into account [18,19].

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Considering the enormous potential of such a formalism for the active FP cavity, or more generally for any kind of one-dimensional resonator, including multisection structures, it seemed natural to investigate its adaptation to active resonators with other geometries. The transposition is not straightforward, though: from a physical point of view, a ring resonator differs from a FP cavity by its traveling wave behaviour (quite different from the standing wave pattern). Above all, from a technical point of view, because of its loop topology, a ring cannot be decomposed into cascadable matrices. Therefore, a specific approach is needed: in that respect, "extended scattering parameters" (as introduced hereafter) appear as a useful tool for expressing the relationships between the relevant waves that travel inside the ring resonator, with the spontaneous contributions included.

The present work is devoted to the specific case of an active single-mode microring resonator, coupled to a passive straight waveguide that acts as input/output ports. We would like to emphasise that the main appeal of our model lies with its astounding simplicity. Besides, it can be used to describe, indifferently, either a selective amplifier (below threshold) or a laser oscillator (above threshold). We show how the spontaneous field generated by an active zone is coupled into the resonance modes of the structure, that eventually determine the spectral lineshape. We give simple and generic expressions for the spontaneous contributions in terms of equivalent fields that couple into the mode. Since we are mainly concerned with structural properties, no restriction is made a priori regarding the materials themselves. For the sake of simplicity, thermal effects are not considered until the very end: an external control of temperature is therefore implicitly assumed.

Our paper is organized as follows. In Section 2, we recall the main features of the "classical" transfer function of the device. The spectral density of emitted radiation is derived in the frame of extended transfer and/or scattering matrix formalisms including sources. The spectral density of internal saturating field, or generalized transfer function, is exposed in Section 3; the effect of material dispersion is illustrated on a concrete example, since we present a complete self-consistent determination, in normalized parameters, of the threshold-crossing in a ring filled with an homogeneously broadened atomic gain medium. We derive analytical expressions that remain continuously valid across threshold. Possible further developments and perspectives, such as thermal effects, extended cavity schemes, a structure made of a ring connected to two parallel straight waveguides or the case of optical seeding, are briefly outlined, along with conclusions, in Section 4.

2. Transfer function and extended matrix formalism

2.1. Classical transfer function

Consider a ring resonator (of propagation constant β_R and perimeter L_R) coupled to a straight waveguide (of propagation constant β_S and length L_S), as depicted in Fig. 1a. Both waveguides are supposed transversally single-mode, and we neglect polarization effects, so that the waves are purely scalar. Time dependence is taken as $\exp(+i\omega t)$.

Following Yariv [20], we use simple analytical formulas based on structural parameters. One key element is the linear coupler between the rectilinear waveguide and the ring, seen as a four-port network (Fig. 1b). It is completely determined by three parameters t_c, t'_c, and k_c. Let us call $\eta^2 = (t_c t'_c - k_c^2)$; $\gamma = \exp(g_R L_R/2)$ and $\exp(-i\mathrm{Re}[\beta_R]L_R)$ are the amplitude and phase change over the ring, respectively, with g_R its modal gain. This expression remains valid whatever the gain or loss level: $\gamma = 1$ in a transparent medium, $\gamma > 1$ in an amplifier, $\gamma < 1$ in case of absorption. Internal ports 3 and 4 are connected through:

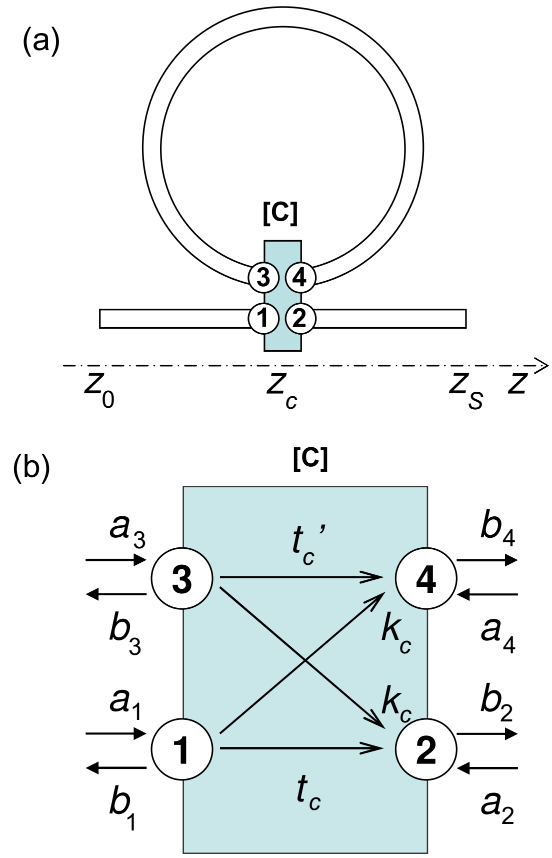

Fig. 1. Single-mode ring waveguide connected to a single-mode straight waveguide. (a) Geometrical configuration. (b) Notations for the coupler seen as a four-port network.

$$a_3 = b_4 \gamma \exp(-i\mathrm{Re}[\beta_R]L_R), \tag{1a}$$
$$a_4 = b_3 \gamma \exp(-i\mathrm{Re}[\beta_R]L_R). \tag{1b}$$

Between remaining ports 1 and 2, the complex transmittance t_R reads:

$$t_R = \frac{t_c - \eta^2 \gamma \exp(-i\mathrm{Re}[\beta_R]L_R)}{1 - t'_c \gamma \exp(-i\mathrm{Re}[\beta_R]L_R)}. \tag{2}$$

Without loss of generality, we can shift the reference planes in order for the coupler to be completely localized at abscissa z_c. In that case, it is legitimate to assume that $t'_c = t^*_c = \eta\tau_0 \exp(-i\varphi_c)$, where τ_0 denotes the value of $|t_c|$ in the lossless coupler, whereas η^2 becomes a real number at most equal to unity, representative of the coupler losses. With $\Phi = \mathrm{Re}[\beta_R]L_R + \varphi_c$ the overall phase change over a round-trip, the denominator $D_R = [1 - \tau_0\eta\gamma\exp(-i\Phi)]$ presents the typical signature of a spectrally selective resonance. The "classical" transfer function in intensity $T_R = |t_R|^2$ can be expressed as

$$T_R(\Phi) = \eta^2 \left\{ 1 + \frac{T_0 - 1}{1 + m_R \sin^2(\Phi/2)} \right\}, \tag{3a}$$
$$T_0 = \left(\frac{\tau_0 - \eta\gamma}{1 - \tau_0\eta\gamma} \right)^2, \tag{3b}$$
$$m_R = \frac{4\tau_0\eta\gamma}{(1 - \tau_0\eta\gamma)^2}. \tag{3c}$$

The whole system is thus completely determined by four independent parameters only: $(\eta, \tau_0, \gamma, \Phi)$, with $(\eta^2 T_0)$ the transmission at resonance and m_R a factor of spectral selectivity.