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Entanglements induced by cavity interacting with two-coupled atoms

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ABSTRACT

We consider a quantum optics model where the cavity interacts with two-coupled atoms. The atom–atom entanglement, atoms–cavity entanglement and the mixture for the two atoms are investigated, and discuss the effects of the initial conditions, atom–atom coupling and the mean number of photons on the entanglements and mixture. We find that atom–atom coupling plays an important role in the entanglement and mixture. Numerical results show that under some conditions the phenomena of "entanglement sudden death" and "entanglement collapse and revival" emerge.

1. Introduction

Entanglement has been identified as a key resource for many practical applications, such as quantum computation, quantum teleportation and quantum cryptography [1]. Therefore it is important to study the entanglement properties in realistic physical system, where the subsystem of our interest, two-qubit subsystem, interacts with other subsystems as the environment of the two-qubit subsystem. This type of interaction causes the indirect coupling between the two qubits. The entanglement between qubits and an environment unavoidably causes decoherence of qubits, one of the biggest obstacles in quantum information Processing. It has believed that qubits should be isolated from environment. However It has been shown that two qubits which do not interact directly with each other but interact with environment can be entangled or disentangled [2–5]. In addition to the indirect coupling with each other, there are another type of two-qubit coupling: direct coupling such as dipole coupling in NMR [6] and Coulomb coupling in superconducting charge qubits [7]. Obviously, direct coupling usually causes two-qubit entanglement and disentanglement. Since the two types of two-qubit couplings both result to two-qubit entanglement and disentanglement, what happens when the two types of coupling both exist? Do they compete with each other for entangling or disentangling two qubit? In this paper we address this problem by studying a two-coupled atoms Tavis–Cummings model (TCM), where two-coupled atoms as two qubits interact with a cavity as an environment, as shown in Fig. 1.

On the other hand, a thorough understanding of entanglement dynamical evolution in quantum physical systems, such as, quantum optics systems, has obvious implications for quantum information Processing, as well as for understanding of fundamental quantum mechanics. Vast efforts has been devoted to studying bipartite entanglement dynamics in the one-atom model [9–12], and two-atom model [13–16]. However, the atom–atom coupling was not taken into account. Here we consider the atom–atom coupling and focus on the mediating roles of the atom–cavity indirect coupling and atom–atom direct coupling by studying the evolutions of the two-qubit entanglement measured by concurrence [8] and qubits-environment entanglement measured by von Neumann entropy.

2. The physical model and some formulas

Consider system of two-coupled two-level atoms A_1 and A_2 as two qubits interacting with a cavity serving as the environment, described by the Hamiltonian [17]

$$H = H_0 + H_{AC} + H_{A_1A_2}, \tag{1}$$

where the Hamiltonian of the two atoms plus cavity H_0, the interaction between the two atoms with cavity H_{AC} and the dipole–dipole interaction between the two atoms $H_{A_1A_2}$ [18,19], are given by

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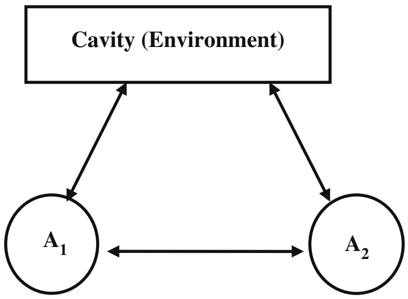

Fig. 1. Schematic illustration of a set-up. Two atoms, A_1 and A_2 coupled through dipole–dipole interaction, interact with a cavity as an environment.

$$H_0 = \frac{1}{2}\omega_0(\sigma_z^{A_1} + \sigma_z^{A_2}) + \omega a^\dagger a(\hbar = 1), \tag{2}$$

$$H_{AC} = g\sum(a^\dagger S_-^{A_i} + aS_+^{A_i}), \tag{3}$$

$$H_{A_1A_2} = \Omega(S_+^{A_1}S_-^{A_2} + S_-^{A_1}S_+^{A_2}), \tag{4}$$

where $S_+ = |+\rangle\langle-|$ and $S_- = |-\rangle\langle+|$ are the raising and lowering operators of an atom, $|-\rangle$ and $|+\rangle$ stand for the ground and exited states of an atom, g and Ω are coupling constants, $a^\dagger$ is a creation operator of the cavity, and σ_z is the z component of the Pauli spin matrices. The interaction between the two atom is untrivial because $[H_{AC}, H_{A_1A_2}] \neq 0$. For simplicity we assume that the two atoms are resonantly coupled to a single mode of the cavity field, $\omega_0 = \omega$. The Hilbert space spanned by the standard basis of the two atoms is $s = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ with $|+\rangle$ and $|-\rangle$ being the exited and ground states of an atom. According the eigenvectors of $H_{A_1A_2}$ we split the standard basis space into two parts: zero-eigenvalue subspace $s_1 = \{|++\rangle, |--\rangle\}$, and nonzero-eigenvalue invariant subspace $s_2 = \{|+-\rangle, |-+\rangle\}$. As can be seen below, the introduction of s_1 and s_2 allows a convenient analysis of the entanglement in Section 3.

In the interaction picture and in the standard basis, the time-evolution operator of the system is given by

$$U(t) = \begin{pmatrix} aK_1^N a^\dagger + \frac{N+2}{2N+3} & -aK_2^N & -aK_2^N & a\left(K_1^N - \frac{1}{2N+1}\right)a \\ -K_2^N a^\dagger & \frac{1}{2}\left(e^{i\Omega t} + K_3^N\right) & \frac{1}{2}\left(-e^{i\Omega t} + K_3^N\right) & -K_2^N a \\ -K_2^N a^\dagger & \frac{1}{2}\left(-e^{i\Omega t} + K_3^N\right) & \frac{1}{2}\left(e^{i\Omega t} + K_3^N\right) & -K_2^N a \\ a^\dagger\left(K_1^N - \frac{1}{2N+1}\right)a^\dagger & -a^\dagger K_2^N & -a^\dagger K_2^N & a^\dagger K_1^N a + \frac{N-1}{2N-1} \end{pmatrix} \tag{5}$$

where

$$N = a^\dagger a,$$

$$K_1^N = \frac{2g^2}{X_1^N - X_2^N}\left(\frac{e^{iX_1^N t}}{X_1^N} - \frac{e^{iX_2^N t}}{X_2^N}\right),$$

$$K_2^N = \frac{g}{X_1^N - X_2^N}\left(e^{iX_1^N t} - e^{iX_2^N t}\right),$$

$$K_3^N = \frac{X_1^N e^{iX_1^N t} - X_2^N e^{iX_2^N t}}{X_1^N - X_2^N},$$

$$X_1^N = -\frac{1}{2} + \sqrt{\frac{1}{4}\Omega^2 + (4N+2)g^2},$$

$$X_2^N = -\frac{1}{2} - \sqrt{\frac{1}{4}\Omega^2 + (4N+2)g^2}. \tag{6}$$

The time-evolution operator is symmetric under atom-exchange $A_1 \leftrightarrow A_2$, and provides us an important solvable model of the atom–field interaction in quantum optics. We take the initial state of the total system to be a pure product state $|\Psi(0)\rangle_{AF} = |\psi(0)\rangle_A \otimes |\psi(0)\rangle_C$, where $|\psi(0)\rangle_A$ and $|\psi(0)\rangle_C$ are the initial states of the atoms and cavity, respectively. The initial cavity state is a vacuum state discussed in Section 3 and a coherent state discussed in Section 4. The time evolution of the total state reads

$$|\Psi(t)\rangle_{AF} = U|\Psi(0)\rangle_{AF}. \tag{7}$$

In the framework of the system-plus-environment, the two two-level atoms are considered as the system of interest and serve as two qubits. The state of the atoms at any time t can be expressed in terms of a Kraus representation [20]

$$\rho_A(t) = \sum_{\mu=0} = K_\mu(t)\rho_A(0)K_\mu^\dagger(t), \tag{8}$$

where $K_\mu(t) = \langle\mu|U|\psi(0)\rangle_C$ are the Kraus operators satisfying $\sum_{\mu=0}K_\mu^\dagger K_\mu = I$.

Using the above results, we can conveniently investigate the entanglement for the two bipartite partitions: atom–atom entanglement $E(A_1 - A_2)$, atoms–cavity entanglement $E(A_1A_2 - C)$. Because the system remains in an overall pure state at all times, $E(A_1 - A_2)$ is quantified by the von Neumann entropy expressed in terms of the density matrix ρ_A of the atoms.

$$S(\rho_A) = -\mathrm{Tr}(\rho_A \log \rho_A). \tag{9}$$

Usually the atom is a mixed state by tracing over the freedom degrees of the cavity, so $E(A_1 - A_2)$ is suitably measured by concurrence [8], which may be calculated explicitly from the density matrix ρ_A

$$C(\rho_A) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{10}$$

where λ_i are the eigenvalues of the matrix $\varrho = \rho_A(\sigma_y^{A_1} \otimes \sigma_y^{A_2})\rho_A^*(\sigma_y^{A_1} \otimes \sigma_y^{A_2})$, arranged in decreasing order. Here σ_y is the well-known Pauli matrix and ρ_A^* denotes the complex conjugation of ρ_A in the standard basis $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$. Concurrence C varies from 0 for a disentangled state to 1 for a maximally entangled state. What the entanglement is associated with is the mixture quantified by linear entropy (LE) [21], here we focus on the mixture of the two atoms, as a comparison with entanglement,

$$S_L(\rho_A) = \frac{4}{3}(1 - \mathrm{Tr}\rho_A^2). \tag{11}$$

LE reflects the degree of mixture and ranges from 0 for a pure state to 1 for a maximally mixed state. $E(A_1A_2 - C)$ and $E(A_1 - A_2)$, as the functions of density operator of the two atoms, depend on the initial conditions and the coupling constants g and Ω. We first study the case where the cavity is initially in vacuum state $|\psi(0)\rangle_C = |0\rangle$.

3. Entanglements for a vacuum cavity

We will capture the key features of the evolutions of entanglement for two different partitions and mixture for the two atoms. Let's begin with the simple case of a vacuum cavity interacting