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Spontaneous emission of a three-level lambda-type atom coupled to separate reservoirs

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ABSTRACT

Considering the anisotropic dispersion model, the upper state population and spontaneous emission spectrum of three-level lambda-type atom with two transitions coupled to separate reservoirs are investigated using the resolvent operator. The upper state population reaches to a steady state value after a weak oscillation when the decay rate is zero and one transition frequency is inside the bandgap. The spectrum associated with each transition was given. Compared with results that were obtained by using isotropic dispersion model, the shape of spectrum changes significantly, and no dark line appears in the spectra.

1. Introduction

It is well known that spontaneous emission depends not only on the energy level structures of an atom but also on the nature of the surrounding environment, more specifically on both the electric field per photon and on the density of states (DOS) of the radiation field. Modified coupling between the radiating atom and the field may be obtained when the atom is in a high-Q cavity, as this changes the electric field per photon. The DOS in Photonic crystals (PCs) is quite distinct to that of vacuum, therefore the emergence of PCs provides a new way to control spontaneous emission and leads to prediction of many interesting effects, such as the appearance of photon–atom bound states and spectral splitting [1], enhanced quantum interference effects [2], coherent control of spontaneous emission [3], modified reservoir induced transparency [4] and the coherent phenomena in PCs [5]. Moreover, Lambropoulos et al. have given an overview about basic quantum electrodynamics and quantum optics aspects of two and multi-level atomic system in structured reservoirs including damped cavity and photonic crystals [6]. In many early studies, isotropic dispersion model of PCs is adopted, namely, the photon dispersion relation near the band edge is one-dimensional [7,8] and the DOS is proportional to $(\omega - \omega_e)^{-1/2}$ (ω_e is the upper band edge frequency). Considerable discrepancy may be caused since the DOS in isotropic model presents singularity at the band edge. Thus, a modified model, i.e. the anisotropic dispersion model [7,8] was introduced to improve the theoretical prediction [9,10].

The dynamics of a three-level atom in a cascade and Lambda configuration with both transitions coupled to a single structured reservoir has been investigated [11–15], such as Garraway and Dalton have investigated the spontaneous emission property for a cascade atom in a high-Q cavity [15]. However, in the case of the two transitions coupled to separate reservoirs, the decay properties of a three-level lambda-type atom have been studied considering one-dimensional dispersion model in which a dark line was found in the spectrum [1]. Yang et al. [16] studied the spontaneous emission properties of a three-level Lambda-type atom embedded in anisotropic photonic crystals using Laplace transform method, and they discussed mainly the components of spontaneous emission field and the influence of bandgap on spontaneous emission spectrum. However, in our paper, for the same configuration, we will throw some light on the effect of the decay rates of the system and the photonic bandgap on decay properties of the atom under anisotropic model, and the results presented here are considerably different from that in Ref. [16]. It is found that, the trapping of upper state population occurs when the decay rate is zero and the frequency of transition coupled to modified reservoir is inside

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the bandgap. Furthermore, the shape of spectral line is dependent on the bandgap reservoir and decay rate of the atomic system. Moreover, we give the spectral line corresponding to the transition which is coupled to photonic bandgap reservoir when decay rate is zero.

2. Theoretical model

We consider a three-level Λ-type atom with two lower levels $|b\rangle$ and $|c\rangle$ coupled by the electric dipole to a common excited level $|a\rangle$, as shown in Fig. 1. The atom is assumed to be initially in the state $|a\rangle$. The transition $|a\rangle \to |c\rangle$ is coupled to a modified reservoir i.e. photonic bandgap reservoir, while the transition $|a\rangle \to |b\rangle$ is assumed to occur in free space. The Hamiltonian which describes the dynamics of this system, in the rotating wave approximation, is given by,

$$\begin{aligned} H &= H_0 + V \\ H_0 &= \hbar\omega_2\sigma_{aa} + \hbar(\omega_2-\omega_1)\sigma_{bb} + \hbar\sum_\lambda \omega_\lambda a_\lambda^+ a_\lambda + \hbar\sum_k \omega_k b_k^+ b_k \\ V &= i\hbar\sum_\lambda \left[g_\lambda^{ab}\sigma_{ba}a_\lambda^+ - (g_\lambda^{ab})^*\sigma_{ab}a_\lambda\right] + i\hbar\sum_k \left[g_k^{ac}\sigma_{ca}b_k^+ - (g_k^{ac})^*\sigma_{ac}b_k\right], \end{aligned} \tag{1}$$

where ω_1 and ω_2 are the atomic transition frequencies $|a\rangle \to |c\rangle$ and $|a\rangle \to |b\rangle$, respectively, $\sigma_{i,j}$ represent atomic pseudo-spin operators, while $a_\lambda^+(a_\lambda)$ denotes the creation (annihibition) operator for the λth vacuum mode with frequency ω_λ, and $b_k^+(b_k)$ is the creation (annihibition) operator for the kth modified reservoir mode with frequency ω_k. $g_k^{ac}(g_\lambda^{ab})$ denotes the coupling constants between the kth modified reservoir mode (λth vacuum mode) of the atomic transition from $|a\rangle$to$|c\rangle$ and $|b\rangle$. Operators H_0 and V represent the non-interaction Hamiltonian and the interaction Hamiltonian, respectively. The resolvent operator [17] is a very convenient tool for studying the dynamics of system with a well-defined initial state and a total Hamiltonian independent of time. Setting $\hbar = 1$, we apply the resolvent operator defined as $G(z) = 1/(z - H)$, where z is a complex variable and H is the Hamiltonian of the system given by Eq. (1). From the definition of the resolvent operator we have $(z - H_0)G(z) = 1 + VG(z)$. As the atom is initially prepared in excited state $|a, 0\rangle$, the resolvent operator equations read

$$\begin{aligned} (z-\omega_a)G_{aa} &= 1 + \sum_\lambda V_{ab_\lambda}G_{b_\lambda a} + \sum_k V_{ac_k}G_{c_k a}, \\ (z-\omega_{b_\lambda})G_{b_\lambda a} &= V_{b_\lambda a}G_{aa}, \\ (z-\omega_{c_k})G_{c_k a} &= V_{c_k a}G_{aa}, \end{aligned} \tag{2}$$

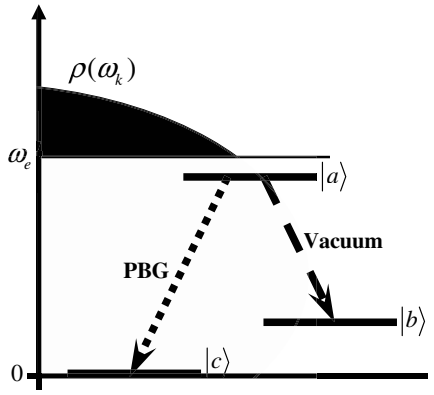

Fig. 1. Three-level atom of a Λ configuration. The thick dotted line denotes the coupling to the PCs, and the thick dashed line denotes the background decay.

where $\omega_a = \omega_2$, $\omega_{b_\lambda} = \omega_2 - \omega_1 + \omega_\lambda$, $\omega_{c_k} = \omega_k$ and $V_{ij} = \langle i|V|j\rangle$, $G_{ij} = \langle i|G|j\rangle$ ($i, j \in \{a, b, c\}$). Solving these coupled, algebraic equations, we get

$$G_{aa} = \frac{1}{z - \omega_a - R_{ac} - R_{ab}}, \tag{3}$$

$$G_{b_\lambda a} = \frac{V_{b_\lambda a}}{z-\omega_{b_\lambda}} \cdot \frac{1}{z - \omega_a - R_{ac} - R_{ab}}, \tag{4}$$

$$G_{c_k a} = \frac{V_{c_k a}}{z-\omega_{c_k}} \cdot \frac{1}{z - \omega_a - R_{ac} - R_{ab}}, \tag{5}$$

where $R_{ac} = \sum_k V_{ac_k}^2/(z - \omega_{c_k})$, $R_{ab} = \sum_\lambda V_{ab_\lambda}^2/(z - \omega_{b_\lambda})$.

The total wavefunction $|\Psi(t)\rangle$ of the system in the time domain reads $|\Psi(t)\rangle = U_{aa}(t)|a, 0\rangle + \sum_\lambda U_{b_\lambda a}(t)|b, 1_\lambda\rangle + \sum_k U_{c_k a}(t)|c, 1_k\rangle$, where the amplitudes $U_{ia}(t)$ are obtained by performing the inversion integral for $G_{ij}(z)$

$$U_{ij}(t) = \frac{1}{2\pi i}\int_{+\infty+i\varepsilon}^{-\infty+i\varepsilon} dz e^{-izt} G_{ij}(z), \tag{6}$$

where ε is an infinitesimal small positive quantity.

3. Results and discussion

3.1. Time evolution of upper state population

Using the identity $1/(x \pm i\eta) = p(1/x) \mp i\pi\delta(x)$ (p denoting the principal values part of the integral) we can get $R_{ij} = \Delta_{ij} - i\Gamma_{ij}/2$, where Δ_{ij} is level shift operator and Γ_{ij} is spontaneous emission rate. Here we neglect the Δ_{ij} and using the identity $\Gamma_{ij}(z) = 2\pi|g_{ij}|^2\rho(z)$ derive from R_{ab} and R_{ac}, where the $\rho(z)$ is the DOS of emitted photon. Because the transition $|a\rangle \to |b\rangle$ is coupled to the free space, we can use the Weiskopf–Wigner approximation to get $R_{ab} = -i\gamma/2$, where γ represents spontaneous emission rate of a photon from the excited state to a lower level. However, since the transition $|a\rangle \to |c\rangle$ is coupled to the photonic bandgap reservoir, the Weiskopf–Wigner approximation is no longer valid, however, we can obtain $R_{ac} = -i\pi c\rho(z)$ from $\Gamma_{ij}(z) = 2\pi|g_{ij}|^2\rho(z)$ where c represents the effective coupling of the atomic transition to the reservoir which is equal to the square of the coupling constants. Substituting R_{ab} and R_{ac} into Eq. (3), we have $G_{aa}(z) = 1/(z - \omega_a + ic\pi\rho(z) + i\gamma/2)$. The isotropic dispersion relation is a result of two serious approximations. First, the vectorial nature of electromagnetic waves has been neglected. The second assumption concerns the 'effective-mass' approximation for atomic transitions close to the band edge. However, in anisotropic model, the vectorial nature of electromagnetic waves is preserved but the second approximation remains. The corresponding dispersion relation reads $\omega_{\mathbf{k}} \approx \omega_e + A(\mathbf{k} - \mathbf{k_0})^2$, where ω_e is the upper band edge frequency and $A \approx \omega_e^2/k_0^2$ [18]. In the dispersion relation, it is obvious that, as $\mathbf{k}$ moves away from $\mathbf{k_0}$, both the direction and magnitude of the band edge wave vector $\boldsymbol{\omega}_{\mathbf{k}}$ are modified. Using the dispersion relation and $\rho(\omega) = \sum_k \delta(\omega(\mathbf{k}) - \omega)$, we can get the DOS as $\rho(z) = f(A)\sqrt{z - \omega_e}\theta(z - \omega_e)$, where $f(A)$ is a constant and θ is the Heaviside step function. For the convenience of calculation of time evolution operator we set $f(A) = 1/\pi$, then, the DOS can be written as $\rho(z) = \sqrt{z - \omega_e}\theta(z - \omega_e)/\pi$. Through change of variable $z \to z + \omega_a$, we obtain

$$G_{aa}(z) = \frac{1}{z + i(\gamma/2 + c\sqrt{z + \delta_2})}. \tag{7}$$

where $\delta_2 = \omega_2 - \omega_e$. In order to obtain the $|U_{aa}(t)|^2$, we should perform the inversion integral of Eq. (6). To evaluate the integral, method applied in Ref. [19] can be used. We close the contour as a semicircle in the lower half of the complex plane. Since the expres-