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Coupling constant of microcavity waveguides based on coupled mode theory

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ABSTRACT

We analyze the microcavity waveguides and derive the coupling constant based on the coupled mode theory. The formula contains only two parameters with clear physical meanings, the quality factor of the cavity modes and the phase shift that the lightwave acquires when tunnelling between two cavities. It provides an easy way to express and modulate the properties of the waveguides. Our analytical results are supported by the simulations using the transfer matrix method.

1. Introduction

Photonic crystals (PCs) attract much attention due to their flexible control over the flow of light in micro-optical systems [1]. The PC defects, considered as the optical microcavities, are the focus of study at all times for their potential applications in building optical devices [2–6]. It is well known that the coupled PC defects can develop a new kind of waveguides referred to as the coupled cavity waveguides (CCWs) or the coupled-resonator optical waveguides (CROWs) [7–12]. These waveguides provide unique properties that are not available for their traditional counterparts, such as low group velocity, extremely large group velocity dispersion, etc. Accordingly, they can be used in building optical delay lines, optical buffers, dispersion compensators, or pulse compressors [13–19].

No matter what device function is explored, the coupling strength between the PC defects in CCWs is most concerned because the dispersion relation of the waveguides depends on it. In the framework of tight-binding (TB) approximation, the coupling strength is embodied by the coupling constant that involves two overlap integrals reflecting the energy exchanges among the defect modes [8,10]. In practice, however, it is more convenient to extract the coupling constant from the experimental or simulation results [9–12]. For example, by measuring the resonant frequency of the single-defect mode ω_0 and the pass bandwidth of the microcavity waveguide $\Delta\Omega$, the coupling constant κ can be obtained by $\kappa = \Delta\Omega/(2\omega_0)$ [10]. We know that the coupled mode theory (CMT) is widely used in the research of the PC defect cavities [20–27]. It demonstrates high efficiency in describing the transmission property of the structures, just like the scattering formalism does [28,29]. In this paper, we show that CMT is also valid in describing the dispersion property of the CCWs. We first determine the transfer matrix of the PC defect and obtain the formula of the coupling constant using the Bloch's theorem. Similar derivations can be found in Refs. [24,29]. Then we discuss how to extract the phase shift parameter from the lineshape of the double-defect structures. Finally, numerical simulations are performed to verify the analytic results.

2. Coupling constant based on CMT

Without losing the physics and for the sake of simplicity, we choose to study an one-dimensional CCW created in a multilayer structure. As shown in Fig. 1a, it consists of GaAs (red) and air (light purple) layers with identical thickness of 0.5*a*, where *a* is the lattice constant. The defects (blue) are introduced by changing the thickness of GaAs layer from 0.5*a* to *d* for every nine layers of GaAs. From the viewpoint of transfer matrix, the CCW can be considered as the cascade structure of many unit cells. Each cell consists of one single-defect component and one "connecting waveguide" component, as shown in Fig. 1b. Here, the tunnelling of electromagnetic wave from one PC defect to another is considered equivalently as the propagation of lightwave along a connected waveguide [24,25]. Such an approach is simple and proves valid, as will be shown later.

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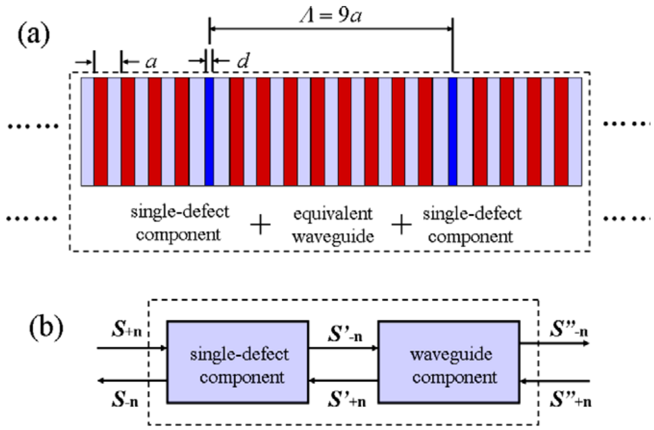

Fig. 1. Schematic of the CCW where the red, blue, and light purple parts represent the normal GaAs, the defect GaAs, and the air layers, respectively. (a) Two adjacent PC defects in the CCW are dashed to indicate the simplest coupled-defect structure. (b) Input and output wave amplitudes of an unit cell (dashed box) in the CCW.

Consider the single-defect component shown in Fig. 1b, where $s_{\pm n}$ *and* $s'_{\pm n}$ are the input and output wave amplitudes of the component at the left and right sides. If we denote A, ω_0 and γ as the energy amplitude, the resonant frequency, and the total decay rate of the defect mode, respectively, then according to CMT, the stable state of the defect follows [20,22]

$$\begin{cases} j\omega A = (j\omega_0 - \gamma)A + \sqrt{\gamma}s_{+n} + \sqrt{\gamma}s'_{+n} \\ s_{-n} = -s_{+n} + \sqrt{\gamma}A, \\ s'_{-n} = -s'_{+n} + \sqrt{\gamma}A \end{cases} \tag{1}$$

where j is the imaginary unit and ω is the input frequency. By eliminating the energy amplitude A in Eq. (1), we get the matrix expression of the component as

$$\begin{bmatrix} s'_{-n} \\ s'_{+n} \end{bmatrix} = M_\mathrm{d} \begin{bmatrix} s_{+n} \\ s_{-n} \end{bmatrix} = \begin{bmatrix} 1+j\delta & j\delta \\ -j\delta & 1-j\delta \end{bmatrix} \begin{bmatrix} s_{+n} \\ s_{-n} \end{bmatrix}, \tag{2}$$

where $\delta = (\omega_0 - \omega)/\gamma$ is the detuning of input frequency to the defect resonant frequency in the unit of γ, and M_d is the transfer matrix of the single-defect component. The lineshape of the defect mode is represented by the reciprocal of the square module of $(M_\mathrm{d})_{22}$. It is a Lorentizian shape. Similarly, under the lossless condition, the transfer matrix of the equivalent waveguide component in Fig. 1b, denoted as M_w, can be obtained from

$$\begin{bmatrix} s''_{-n} \\ s''_{+n} \end{bmatrix} = M_\mathrm{w} \begin{bmatrix} s'_{-n} \\ s'_{+n} \end{bmatrix} = \begin{bmatrix} e^{j\varphi} & 0 \\ 0 & e^{-j\varphi} \end{bmatrix} \begin{bmatrix} s'_{-n} \\ s'_{+n} \end{bmatrix}, \tag{3}$$

where $s''_{\pm n}$ are the input and output wave amplitudes of the component at its right side, φ is the equivalent phase shift acquired by the lightwave when it propagates along the waveguide component. Based on Eqs. (2) and (3), the matrix expression of the nth unit cell is

$$\begin{bmatrix} s''_{-n} \\ s''_{+n} \end{bmatrix} = M_\mathrm{w} M_\mathrm{d} \begin{bmatrix} s_{+n} \\ s_{-n} \end{bmatrix}. \tag{4}$$

As can be seen from Fig. 1b, s''_{-n} and s''_{+n} are also the input and output wave amplitudes of the $(n+1)$th unit cell at its left side, i.e., $s''_{-n} = s_{+(n+1)}$ and $s''_{-n} = s_{-(\ n+1)}$. So, for all the input and output wave amplitudes of two adjacent units at their left sides, we have

$$\begin{bmatrix} s_{+(n+1)} \\ s_{-(n+1)} \end{bmatrix} = M_\mathrm{w} M_\mathrm{d} \begin{bmatrix} s_{+n} \\ s_{-n} \end{bmatrix} = \lambda \begin{bmatrix} s_{+n} \\ s_{-n} \end{bmatrix} \tag{5}$$

The second equality of Eq. (5) is based on the Bloch's theorem, which is widely used in literatures [24,29–31]. Because CCWs are periodic structures with an infinite number of units and here the interval of $s_{\pm(n+1)}$ and $s_{\pm n}$ equals one period, the difference of $s_{\pm(n+1)}$ and $s_{\pm n}$ is only a Bloch's phase factor. It means that λ equals $e^{j\beta\Lambda}$ or $e^{-j\beta\Lambda}$ with β the Bloch wave vector and Λ the periodic length ($\Lambda = 9a$ for the CCW studied). If a nontrivial solution of Eq. (5) exists, the corresponding determinant $|M_\mathrm{w}M_\mathrm{d} - \lambda I|$ must be zero, where I is the unit matrix. It leads to the equation of

$$\cos(\beta\Lambda) = \cos\varphi + \delta\sin\varphi, \tag{6a}$$

which is the dispersion relation of the CCW. Consider that $\delta = (\omega_0 - \omega)/\gamma$, we can rewrite Eq. (6a) as

$$\omega = \omega_0\left(1 + \frac{\cot\varphi}{2Q}\right) - \frac{\omega_0}{2Q\sin\varphi}\cos\beta\Lambda, \tag{6b}$$

where $Q = \omega_0/(2\gamma) = \omega_0/\Delta\omega$ is the quality factor of the defect mode, $\Delta\omega$ is the linewidth of the defect mode. Obviously, the dispersion relation has got the same form as that of the TB approach. Based on Eq. (6b), the pass bandwidth of CCW $\Delta\Omega$, the group velocity v_g, and the coupling factor κ, can be derived as

$$\Delta\Omega = \frac{\omega_0}{Q\sin\varphi}, \tag{7}$$

$$v_g = \frac{\omega_0\Lambda}{2Q\sin\varphi}\sin\beta\Lambda, \tag{8}$$

$$\kappa = \frac{1}{2Q\sin\varphi}, \tag{9}$$

respectively. Apparently, φ plays an important role in describing the properties of CCWs. In particular, as shown in Eq. (9), Q is not the unique parameter to determine the coupling factor, which is often misunderstood by some people. Even when Q is constant, i.e., the given defect modes, we can still modulated the properties of CCWs if the way of changing φ is discovered. This is very useful in the design of microcavity waveguides.

Now the problem is how to acquire the value of φ. We are sure that φ hides its message in the lineshape of the structure that contains at least two PC defects, because different φ usually results in different lineshape, flat top or not [24,25]. Consider the dashed box shown in Fig. 1a that consists of two single-defect components being connected by one equivalent waveguide component, its transfer matrix $M_{2\mathrm{d}} = M_\mathrm{d}M_\mathrm{w}M_\mathrm{d}$. The lineshape $T_{2\mathrm{d}}$ can be obtained by using the matrix element $(M_{2\mathrm{d}})_{22}$

$$T_{2\mathrm{d}} = \frac{1}{|(M_{2\mathrm{d}})_{22}|^2} = \frac{1}{1 + 4\delta^2(\delta\sin\varphi + \cos\varphi)^2}. \tag{10}$$

Obviously, $T_{2\mathrm{d}}$ possesses two peaks of unity located at $\delta = 0$ and $\delta = -\cot\varphi$, and one valley of $4\sin^2\varphi/(4\sin^2\varphi + \cos^4\varphi)$ located at $\delta = -0.5\cot\varphi$. So we can extract φ readily by measuring one of the following values: the valley location, the valley transmittance, or the second peak location. From Eq. (10) we also find that if φ happens to be $\pi/2$ the lineshape possesses only one peak at $\delta = 0$.

3. Comparison and discussion

To verify the above analysis, we perform simulations using a TMM software developed by Reynolds. Its computational framework was established by Pendry and MacKinnon [32,33]. In TMM, the investigated structure is divided into small cells. In each cell, Maxwell's equations are solved in the way of transfer matrix, in one frequency. The cascade product of all these transfer matrices results in the transfer matrix of the whole structure, from which the stable transmission can be obtained. This is an accurate solution of Maxwell's equations without any assumptions. So we can use it to test the CMT results. Of course, if one investigates the dynamical properties of the structure, it is better to use the FDTD scheme that solves Maxwell's equations in the temporal and spatial domains. For the one-dimensional PC constructed by GaAs