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Self-focusing and defocusing of TEM_{0p} Hermite–Gaussian laser beams in collisionless plasma

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A B S T R A C T

The authors have investigated the self-focusing and defocusing of first six TEM_{0p} Hermite–Gaussian laser beams in collisionless plasma. In case of collisionless plasma the nonlinearity in the dielectric constant is mainly due to the ponderomotive force. It is found that modes with odd *p*-values defocuses and that with even *p*-values exhibit oscillatory as well as defocusing character of beam-width parameters variation during their propagation in collisionless plasma. The entire theoretical formulation is established under parabolic wave equation approach. The numerical computation is completed by using fourth order Runge–Kutta method. Finally the behavior of beam-width parameters with the dimensionless distance of propagation is presented graphically.

1. Introduction

Study of the phenomena related to self-focusing of intense laser light propagating in a plasma has become a subject of considerable interest, since these phenomena play an important role in a large amount of high power laser applications, such as X-ray lasers [1], harmonic generation [2,3], laser-driven plasma accelerators [4–9], and fast igniter concept of inertial confinement fusion [10]. For these applications, preformed plasma channels are required to further guide the laser beam beyond the Rayleigh length, after which the beam expands infinitely in vacuum due to natural diffraction. It has been shown that the beam spot size performs periodic oscillations along the propagation distance in the presence of preformed channel and has equilibrium solution (i.e. constant spot size) when the laser power is equal to the matched powers with different nonlinear effects [5,11–14]. Gupta et al. [15] have observed a plasma density ramp of a suitable length can reduce these oscillations. Furthermore, they have also predicted a magnetic field acts as a catalyst for self-focusing of a laser beam during propagation in a plasma density ramp [16].

When an intense laser beam acts on collisionless plasma, the quiver velocity of electrons is relativistic so that their mass is intensity dependant but for long pulse experiments, the relativistic effects can be ignored [17] and ponderomotive force of the beam nonlinearity perturbs electron density resulting in the excitation of electron plasma wave (wakefield) [18]. Recently, Hermite–Gaussian beams have attracted the researchers as a useful optical trap wherein the central trap-depth depends on waist-size of the beam [19]. It is also noticed that frequency of optical trap increases for smaller waist-size and larger power of the beam. Since waist-size is finally related to beam-width parameters, studies on beam-width parameter variation have added flavor in the subject of high power laser beams. In this paper, we present the propagation of first six TEM_{0p} Hermite–Gaussian laser beams in collisionless plasma by a ponderomotive mechanism. In addition to adopting a different intensity profile, we have studied the higher order Hermite–Gaussian modes and manipulate numerically by using fourth order Runge–Kutta method in this paper instead of analytical treatment exposed for cylindrically symmetric beams by Patil et al. [20,21]. We have also employed two different transverse beam-width parameters in Cartesian coordinate system and assumed the medium to be non-absorptive and aberrationless.

In Section 2, the field distribution of TEM_{0p} Hermite–Gaussian beams propagating along *z*-axis and nonlinear dielectric constant for collisionless plasma are presented. In Section 3, we have set up and derived the differential equations for beam-width parameters by parabolic wave equation approach under Wentzel–Kramers–Brillouin (WKB) and paraxial approximations. Section 4 is devoted to the discussion of important results, supported by numerical analysis. Finally, a brief conclusion is added in Section 5.

2. Theoretical considerations

2.1. Field distribution of Hermite–Gaussian beams

We employ the Hermite–Gaussian laser beam propagating along *z*-axis with the field distribution in the following form

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$$E(x,y,z) = \frac{E_0}{[f_1(z)f_2(z)]^{1/2}} H_0\left(\frac{\sqrt{2}x}{r_0 f_1(z)}\right) H_p\left(\frac{\sqrt{2}y}{r_0 f_2(z)}\right) \times \exp\left[-\left(\frac{x^2}{r_0^2 f_1^2(z)} + \frac{y^2}{r_0^2 f_2^2(z)}\right)\right] \quad (1)$$

where $H_j(\cdot)$ denotes the jth order Hermite polynomial; (j = 0, p) with p = 0, 1, 2, 3, 4, 5. r_0 is the spot size of the beam, E_0 is the amplitude of Gaussian beam for the central position at $r = z = 0$, is a constant, $f_1(z)$ and $f_2(z)$ are the dimensionless beam-width parameters in x and y directions.

2.2. Nonlinear dielectric constant

Further we consider such propagation in a nonlinear medium characterized by dielectric constant of the form [22]

$$\varepsilon = \varepsilon_0 + \Phi(EE^*), \quad (2a)$$

with

$$\varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2}, \quad (2b)$$

$$\omega_p = \left(\frac{4\pi n_0 e^2}{m}\right)^{\frac{1}{2}} \quad (2c)$$

where ε_0 and Φ represents the linear and nonlinear parts of the dielectric constant respectively, ω_p is the plasma frequency.

In case of collisionless plasma, the nonlinearity in the dielectric constant is mainly due to the ponderomotive force and the nonlinear part of dielectric constant is given by [23]

$$\Phi(EE^*) = \frac{\omega_p^2}{\omega^2}\left[1 - \exp\left(-\frac{3}{4}\frac{m}{M}\alpha EE^*\right)\right], \quad (3a)$$

with

$$\alpha = \frac{e^2 M}{6m^2\omega^2 k_B T_0}, \quad (3b)$$

where e, m and n_0 being the electronic charge, mass and the electron density, M is the mass of scatterer in the plasma, ω is the frequency of laser used, k_B is the Boltzmann's constant and T_0 is the equilibrium plasma temperature.

3. Self-focusing and de-focusing

The wave equation governing the propagation of the laser beam may be written as,

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E + \nabla\left(\frac{E\cdot\nabla\varepsilon}{\varepsilon}\right) = 0 \quad (4)$$

The last term on left hand side of Eq. (4) can be neglected provided that $k^{-2}\nabla^2(\ln\varepsilon) \ll 1$ where, k represents the wave vector. This inequality is satisfied in almost all cases of practical interest. Thus

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0 \quad (5)$$

This equation is solved by employing Wentzel–Kramers–Brillouin (WKB) approximation. For convenience, we express the solution in the Cartesian coordinate system as

$$E = A(x,y,z)\exp[i(\omega t - kz)], \quad (6a)$$

where

$$k = \frac{\omega}{c}\varepsilon_0^{1/2} \quad (6b)$$

and $A(x,y,z)$ is the complex amplitude of the electric field.

Substituting for $E(x,y,z)$ from Eq. (6) in Eq. (5) and neglecting $\partial^2 A/\partial z^2$ which implies that the characteristic distance of intensity variation is much greater than wavelength, one obtains

$$2ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{k^2}{\varepsilon_0}\Phi(AA^*)A \quad (7)$$

To solve Eq. (7), we express $A(x,y,z)$ as

$$A(x,y,z) = A_{0p}(x,y,z)\exp[-ikS(x,y,z)], \quad (8)$$

where A_{0p} and S are real functions of x, y and z. Substituting for $A(x,y,z)$ from Eq. (8) in Eq. (7) and equating the real and imaginary parts on both sides of the resulting equation, one obtains

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 = \frac{1}{k^2 A_{0p}}\left(\frac{\partial^2 A_{0p}}{\partial x^2} + \frac{\partial^2 A_{0p}}{\partial y^2}\right) + \frac{1}{\varepsilon_0}\Phi(A_{0p}^2) \quad (9)$$

and

$$\frac{\partial A_{0p}^2}{\partial z} + \frac{\partial S}{\partial x}\frac{\partial A_{0p}^2}{\partial x} + \frac{\partial S}{\partial y}\frac{\partial A_{0p}^2}{\partial y} + A_{0p}^2\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right) = 0 \quad (10)$$

Following Akhmanov et al. [24] and its extension by Sodha et al. [22,25], the solutions of Eqs. (9) and (10) for TEM_{0p} Hermite–Gaussian beams can be written as

$$S = \frac{x^2}{2}\beta_1(z) + \frac{y^2}{2}\beta_2(z) + \phi(z) \quad (11a)$$

$$A_{0p}^2 = \frac{E_0^2}{f_1(z)f_2(z)}\left[H_0\left(\frac{\sqrt{2}x}{r_0 f_1(z)}\right)\exp\left[-\left(\frac{x^2}{r_0^2 f_1^2(z)}\right)\right]\right] \times \left[H_p\left(\frac{\sqrt{2}y}{r_0 f_2(z)}\right)\exp\left[-\left(\frac{y^2}{r_0^2 f_2^2(z)}\right)\right]\right] \quad (11b)$$

and

$$\beta_1(z) = \frac{1}{f_1}\frac{df_1}{dz}, \quad \beta_2(z) = \frac{1}{f_2}\frac{df_2}{dz} \quad (11c)$$

It is obvious that the parameter $\beta_1(z)$ and $\beta_2(z)$ represents the curvature of the wavefront in x and y directions.

Employing paraxial approximation to obtain expressions for the beam-width parameters f_1 and f_2 as

$$\frac{d^2 f_1}{d\eta^2} = \frac{4}{f_1^3} - 2\left(\frac{E_0^2 R_d^2}{\varepsilon_0 r_0^2}\right)\frac{\Phi'(A_{0p}^2)\big|_{x=y=0}}{f_2 f_1^2} \quad (12)$$

and

$$\frac{d^2 f_2}{d\eta^2} = \frac{4}{f_2^3} - 2\left(\frac{E_0^2 R_d^2}{\varepsilon_0 r_0^2}\right)\frac{\Phi'(A_{0p}^2)\big|_{x=y=0}}{f_1 f_2^2}, \quad (13)$$

with

$$R_d = kr_0^2 \quad (14)$$

$$\eta = \frac{z}{R_d} \quad (15)$$

where R_d is the diffraction length and η is the dimensionless distance of propagation.

Establishing the general relation between the numerical coefficient in the second term of Eqs. (12) and (13) and the mode index p = 0, 2, 4, the general beam-width parameter differential equations for even modes are as below: