Optics Communications 281 (2008) 5701–5705

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Wavefront reconstruction by two-step generalized phase-shifting interferometry

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A R T I C L E I N F O

Article history:
Received 17 January 2008
Received in revised form 31 July 2008
Accepted 6 August 2008

PACS:
42.40.Kw
42.87.Bg
07.05.Pj
42.30.Rx

Keywords:
Holographic interferometry
Phase-shifting interferometry
Image processing
Phase retrieval

A B S T R A C T

A wavefront reconstruction method by two-step generalized phase-shifting interferometry (GPSI) with blind phase shift extraction algorithm is verified by both the computer simulations and optical experiments. This method can retrieve complex object wave field by using two interferograms, the recorded object and reference wave intensities, and an unknown phase shift without additional processing. The simulations with irregular wavefronts have shown the effectiveness and high accuracy of this method for blind phase-shift extraction and wavefront reconstruction over a wide range of phase-shifts, while the optical experiments for both the direct and indirect objects have yielded satisfactory results with a higher resolution of reconstructed image than those reported recently.

1. Introduction

Digital holography (DH) technique has been demonstrated to be a useful method in many fields of optics, particularly for wavefront reconstruction [1–5]. To suppress the zero-order diffraction and conjugate image, besides digital filtering, many other efforts [6–8] have been made, such as the recording of multi-holograms in different positions [6,7] and designing of new reconstruction algorithms [8].

Phase-shifting interferometry (PSI) has proved to be an effective way to record complex wave field digitally with the use of CCD and in-line geometry without the need of filtering or other operations to suppress the conjugate image [9–12]. Some special devices such as the piezoelectric transducer (PZT) [11], Bragg cell [13], diffraction grating [14,15], tilted glass plate [16], rotating half-wave plate [17], etc. have been used to generate phase shifts in PSI. Traditional PSI needs at least three interferograms and special values of phase shifts. To simplify the recording process, some researchers have investigated the possibility of wavefront reconstruction with only two interferograms. For example, Ramírez and Garcia-Sucerquia utilized the subtraction technique between two holograms recorded in different positions of a ground glass [18], and Chen *et al* employed two holograms and an estimation procedure to retrieve wavefront [19]. However, the former is only suitable for amplitude objects, while the latter needs Fourier spectrum processing. The algorithm of two-step generalized phase-shifting interferometry (GPSI) we recently proposed [20,21] can overcome the drawbacks mentioned above, and provide a more effective and convenient approach to find the unknown phase shift with a blind searching algorithm and then retrieve the complex object wave field.

In this paper, we give some new simulations with irregular wavefronts to test the effectiveness of our method for blind phase-shift extraction and wavefront reconstruction over a wide range of phase-shifts. In addition, to demonstrate and prove the feasibility of our method in practice, a variety of optical experiments have been made with both indirect and direct input real amplitude objects (Chinese character and resolution target); satisfactory experimental results with higher resolution than those reported recently [13,19,21] have been obtained.

We will first give a brief review of the two-step GPSI and introduce some new simulations, then, as a major part of this article, optical experimental demonstrations in two cases are discussed, which is followed by final conclusions.

2. Review of two-step GPSI

Wavefront reconstruction is a process by which several recorded interference patterns, such as holograms or interferograms,

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doi:10.1016/j.optcom.2008.08.010

are reconverted into an image of the original diffracting object [22], and its fundamental principles have been well established by a number of pioneer researchers [23,24]. Among different methods of wavefront reconstruction, PSI has become a powerful tool and found wide use in a variety of applications [11]. Standard PSI requires a special constant phase shift like $\pi/2$ or $2\pi/3$. On the contrary, in our proposed GPSI, arbitrary unknown phase shifts can be utilized in the recording process of interferograms, and then blindly extracted with some specially developed algorithms for wave reconstruction [11,25–27]. Specifically, in two-step GPSI we can retrieve the original object wave with only two interferograms and one unknown phase shift with the help of the measurement of the object and reference wave intensities [20,21].

The optical realization of the two-step GPSI is similar with standard PSI, and its experiment setup is shown in Fig. 1. A light beam is redirected by mirror M_1, and is then split by a beam splitter BS_1 into two beams, an object beam and a reference beam. The former is redirected by mirror M_2, focused by lens L_{01}, spatially filtered by a pinhole PH_1, and then collimated by lens L_1. An object (Obj) is placed in object plane behind L_1. Similarly, the reference beam is reflected by a mirror attached to a PZT controlled by a computer PC_1, then focused by a lens L_{02}, filtered by a pinhole PH_2, collimated by a lens L_2, and finally redirected by beam splitter BS_2 to propagate to a recording CCD camera where the two waves interfere forming interferograms. A_1 and A_2 are two attenuators used to modulate the light intensity, and D_1 and D_2 are two apertures. The phase of the reference plane wave can be changed by moving the PZT, and the first and second interferograms are recorded before and after a certain translation of it.

Assuming that an object in the object plane (x_o, y_o) is a certain distance z away from the recording plane (x, y), the complex amplitude distribution of the object wave in plane (x, y) is $U(x, y) = A_o(x, y)\exp[i\varphi(x, y)]$, the amplitude of an on-axis reference plane wave in this plane is $A_r(x, y)$. In digital simulations, $A_r(x, y)$ can be chosen as a constant greater than the maximum of $A_o(x, y)$ [28]; and in optical experiments, $A_r(x, y)$ is the square root of the recorded reference intensity $I_r(x, y)$. In two-step GPSI we may assume the constant phase of the reference plane wave in recording the first interferogram as $\delta_1 = 0$ and that in recording the second interferogram as $\delta_2 = \delta$ $(0 < \delta < \pi)$, where the phase shift δ is determined by the translation of PZT, then the intensities of the two interferograms can be expressed as

$$I_1 = A_o^2 + A_r^2 + 2A_oA_r\cos\varphi, \tag{1}$$

$$I_2 = A_o^2 + A_r^2 + 2A_oA_r\cos(\varphi - \delta). \tag{2}$$

Introducing $a = A_o^2 + A_r^2$, from Eqs. (1) and (2) we can obtain

$$\cos\varphi = \frac{I_1 - a}{2A_oA_r}, \tag{3}$$

$$\sin\varphi = \frac{I_2 - I_1\cos\delta - (1-\cos\delta)a}{2A_oA_r\sin\delta}. \tag{4}$$

The relation $\cos^2\varphi + \sin^2\varphi = 1$ yields

$$\left(\frac{I_1 - a}{2A_oA_r}\right)^2 + \left(\frac{I_2 - I_1\cos\delta - (1-\cos\delta)a}{2A_oA_r\sin\delta}\right)^2 = 1. \tag{5}$$

From this equation we can get

$$b^2\cos^2\delta - 2(I_1 - a)(I_2 - a)\cos\delta + (I_1 - a)^2 + (I_2 - a)^2 - b^2 = 0, \tag{6}$$

where $b = 2A_0A_r$. By denoting

$$p = \langle b^2\rangle, \quad q = \langle -2(I_1 - a)(I_2 - a)\rangle,$$
$$r = \langle (I_1 - a)^2 + (I_2 - a)^2 - b^2\rangle, \tag{7}$$

where $\langle\rangle$ means averaging over the whole frame, we can obtain

$$\delta = \cos^{-1}\left(\frac{-q \pm \sqrt{q^2 - 4pr}}{2p}\right) \tag{8}$$

from the average form of Eq. (6). If the correct δ is found, from Eqs. (3) and (4), the complex object wave in the recording plane (x, y) can be retrieved with the use of the measured object wave intensity $I_o(x, y) = A_o^2(x, y)$ and reference wave intensity $I_r(x, y) = A_r^2(x, y)$ as [20,21]

$$\begin{aligned} U(x,y) &= A_o(x,y)\exp[i\varphi(x,y)] \\ &= A_o(x,y)\cos\varphi(x,y) + iA_o(x,y)\sin\varphi(x,y) \\ &= \frac{I_1 - a}{2A_r} + i\frac{I_2 - I_1\cos\delta - (1-\cos\delta)a}{2A_r\sin\delta}. \end{aligned} \tag{9}$$

If necessary, the complex object field in original object plane (x_o, y_o) may be further obtained by the inverse Fresnel transform (IFrT) of $U(x, y)$ with distance z.

Eq. (8) gives two solutions for the phase shift δ. In the computer simulations and the optical experiment with a digital image as an input, the input image is computer generated with an exactly known form, so we can use correlation coefficient (CC) [29] between it and the retrieved image to verify the ability of our method for wavefront reconstruction. However, in optical experiments with direct inputs or in general case, the exact object wavefront is unknown, in order to decide which one of the two solutions of Eq. (8) is right, a total-error function ΔE is introduced [21],

$$\begin{aligned} \Delta E = \langle &|I_1 - I_o - I_r - 2\sqrt{I_oI_r}\cos\varphi| + |I_2 - I_o - I_r - 2\sqrt{I_oI_r} \\ &\times \cos(\varphi - \delta)|\rangle, \end{aligned} \tag{10}$$

where || denotes taking absolute value. For each δ of the two solutions, we can retrieve the object wave $\sqrt{I_o}\exp(i\varphi)$ and then obtain the phase function $\varphi(x, y)$ with the help of the recorded intensities I_1, I_2, I_o and I_r by Eq. (9). Substituting I_1, I_2, I_o, I_r, δ and φ into Eq. (10), the corresponding ΔE can be calculated. Obviously, the two δ values will lead to two different ΔEs and the one yielding the smaller ΔE should be the correct phase shift.

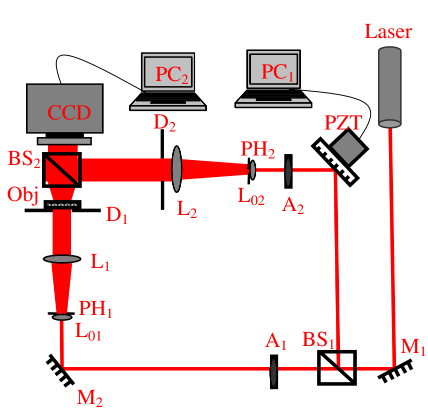

Fig. 1. The schematic diagram of the optical experiment system.

3. Computer simulations

A series of computer simulations have been made to verify the feasibility of our proposed method and investigate its performance. An irregular wavefront with uniform amplitude and the optical path function

$$\begin{aligned} W(x_o, y_o) = {} & 3(x_o + 10\Delta x_o)[(x_o - 10\Delta x_o)^2 + y_o^2] \\ & - 4.375\times10^{-4}(x_o - 10\Delta x_o) - 1.438\times10^{-4}(y_o + 50\Delta y_o) \end{aligned} \tag{11}$$