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Splitting a qudit state via Greenberger–Horne–Zeilinger states of qubits

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ABSTRACT

We present a tripartite quantum information splitting scheme which splits a qutrit state via two GHZ states. The scheme is then generalized to splitting a qudit state among any number of receivers. We show that this scheme is also applicable to splitting any multi-qudit entangled states.

1. Introduction

Quantum information splitting (QIS) is one of the most striking applications of quantum entanglement. It is the counterpart of classical secret sharing [1,2] in the complete quantum scenario and was initially introduced by Hillery, Bŭzek and Berthiaume (HBB) in 1999 [3]. HBB's QIS scheme can be briefly described as follows: a splitter (named Alice) and two receivers (respectively named Bob and Charlie) safely share a 3-qubit GHZ state as quantum channel and each one possesses a qubit. Alice also has another qubit in an arbitrary state. To partition the arbitrary quantum state between Bob and Charlie, Alice carries out a Bell-state measurement on her two qubits and publishes her measurement result via a classical channel. By using this method neither Bob nor Charlie is able to solely obtain the secret quantum state unless they collaborate together. QIS has many potential applications, e.g., secure operations of distributed quantum computation [4], sharing difficult-to-construct ancilla states, joint sharing of quantum money [5], and so on. Consequently, it has attracted much attention after HBB's pioneering work and many QIS schemes have already been proposed over the past decade [6–22].

By far all the problems treated in QIS [3,6–19], we think, can be simply classified into six types, i.e., the participant number, the split quantum information, the employed quantum channel, the necessary operation, the classical communication cost and their tradeoff. Among these six types of issues, the split quantum information and the employed quantum channel as two indispensable parts in QIS, are essentially quantum states. Intuitively, any quantum state should inhabit a particle or particles. Conventionally, the particle which is initially inhabited the quantum information partially or fully is called as information particle, while the particles consisting of the quantum channel are called as channel particles. Through extensive investigations we find that, so far in all existing QIS schemes [3,6–19] the degree of freedom of single information particle is equal to that of single channel particle. As a matter of fact, recently people have already started to consider in other quantum information processes [23–26] the so-called degree-mismatch problem, where the degree of freedom of single information particle is different from that of single channel particle. Nonetheless, to our best knowledge, no QIS work is devoted to such degree-mismatch issue. However, in the future quantum network, different entangled states may be used as quantum channels and different quantum states may be needed to be split among sharers. Hence it is quite possible that one may encounter the degree-mismatch issue in QIS. Surely, it is of interest and significance to treat such problem in QIS.

In this paper, we will only preliminarily study the degree-inequality problem in QIS. In Section 2, we will detailedly introduce a tripartite scheme for splitting an unknown single-qutrit state by using two 3-qubit GHZ states as the quantum channel.

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In Section 3, we will extend the scheme from the qutrit case to a higher-dimensional (i.e., qudit) case. In Section 4, we will expand the tripartite scheme to a more-partite one. We will give a concise summary and make some brief remarks and proposals in Section 5.

2. Tripartite scheme for splitting a qutrit state via two GHZ states

Instead of proceeding direct to the most general scenario, it is instructive to first consider in details a simpler situation, namely, a tripartite QIS scheme for splitting a single-qutrit state with two GHZ states. The schematic demonstration of the tripartite scheme is shown in Fig. 1. Let the three legitimate parties in the scheme be Alice (sender) and Bob and Charlie (receivers). Alice has a secret qutrit state given by

$$|\psi\rangle_t = \xi_0|0\rangle_t + \xi_1|1\rangle_t + \xi_2|2\rangle_t, \tag{1}$$

where $|\xi_0|^2 + |\xi_1|^2 + |\xi_2|^2 = 1$. The quantum channel linking the three parties consists of two 3-qubit GHZ states

$$\begin{aligned} |\phi\rangle_{a_0b_0c_0} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{a_0b_0c_0}, \\ |\phi\rangle_{a_1b_1c_1} &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{a_1b_1c_1}. \end{aligned} \tag{2}$$

Alice, Bob, and Charlie hold the qubit pairs $(a_0, a_1), (b_0, b_1)$, and (c_0, c_1), respectively.

Alice wants to split the qutrit state $|\psi\rangle_t$ between Bob and Charlie so that neither of them can solely reconstruct the secret state unless they cooperate with each other. To proceed, Alice performs a joint measurement on her three particles (ta_1a_0) and publishes her outcome. The measurement basis consists of the following states,

$$\left.\begin{aligned}
|\mathscr{B}_1\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|000\rangle + |101\rangle + |210\rangle)_{ta_1a_0}, \\
|\mathscr{B}_2\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|001\rangle + |110\rangle + |211\rangle)_{ta_1a_0}, \\
|\mathscr{B}_3\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|010\rangle + |111\rangle + |200\rangle)_{ta_1a_0}, \\
|\mathscr{B}_4\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|011\rangle + |100\rangle + |201\rangle)_{ta_1a_0}, \\
|\mathscr{B}_5\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|000\rangle + e^{\frac{2\pi i}{3}}|101\rangle + e^{\frac{4\pi i}{3}}|210\rangle)_{ta_1a_0}, \\
|\mathscr{B}_6\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|001\rangle + e^{\frac{2\pi i}{3}}|110\rangle + e^{\frac{4\pi i}{3}}|211\rangle)_{ta_1a_0}, \\
|\mathscr{B}_7\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|010\rangle + e^{\frac{2\pi i}{3}}|111\rangle + e^{\frac{4\pi i}{3}}|200\rangle)_{ta_1a_0}, \\
|\mathscr{B}_8\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|011\rangle + e^{\frac{2\pi i}{3}}|100\rangle + e^{\frac{4\pi i}{3}}|201\rangle)_{ta_1a_0}, \\
|\mathscr{B}_9\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|000\rangle + e^{\frac{4\pi i}{3}}|101\rangle + e^{\frac{2\pi i}{3}}|210\rangle)_{ta_1a_0}, \\
|\mathscr{B}_{10}\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|001\rangle + e^{\frac{4\pi i}{3}}|110\rangle + e^{\frac{2\pi i}{3}}|211\rangle)_{ta_1a_0}, \\
|\mathscr{B}_{11}\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|010\rangle + e^{\frac{4\pi i}{3}}|111\rangle + e^{\frac{2\pi i}{3}}|200\rangle)_{ta_1a_0}, \\
|\mathscr{B}_{12}\rangle_{ta_1a_0} &= \tfrac{1}{\sqrt{3}}(|011\rangle + e^{\frac{4\pi i}{3}}|100\rangle + e^{\frac{2\pi i}{3}}|201\rangle)_{ta_1a_0}.
\end{aligned}\right\} \tag{3}$$

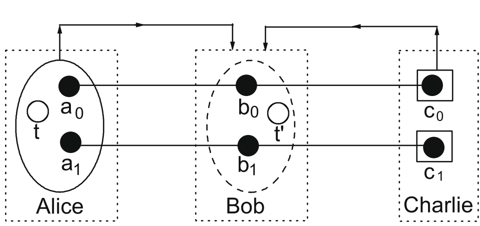

Fig. 1. Tripartite scheme for splitting a qutrit state with two 3-qubit GHZ states. Hollow (solid) circles represent qutrits (qubits). Solid lines indicate entanglement. Solid and dashed ellipses denote respectively a collective measurement and an unitary operation on the enclosed particles. Small squares denote single-qubit measurements. Arrows indicate the flow of classical information. See the text for more details.

In terms of the basis states $\mathscr{B}_j$, the combined state of the qutrit t and the quantum channel particles can be rewritten as

$$\begin{aligned}
|\Psi\rangle_{ta_0b_0c_0a_1b_1c_1} &= |\psi\rangle_t \otimes |\phi\rangle_{a_0b_0c_0} \otimes |\phi\rangle_{a_1b_1c_1} \\
&= \frac{1}{2\sqrt{3}}\Big[|\mathscr{B}_1\rangle_{ta_1a_0}(\xi_0|0000\rangle + \xi_1|0101\rangle + \xi_2|1010\rangle)_{b_1b_0c_1c_0}. \\
&\quad + |\mathscr{B}_2\rangle_{ta_1a_0}(\xi_0|0101\rangle + \xi_1|1010\rangle + \xi_2|1111\rangle)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_3\rangle_{ta_1a_0}(\xi_0|1010\rangle + \xi_1|1111\rangle + \xi_2|0000\rangle)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_4\rangle_{ta_1a_0}(\xi_0|1111\rangle + \xi_1|0000\rangle + \xi_2|0101\rangle)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_5\rangle_{ta_1a_0}\left(\xi_0|0000\rangle + \xi_1 e^{-\frac{2\pi i}{3}}|0101\rangle + \xi_2 e^{-\frac{4\pi i}{3}}|1010\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_6\rangle_{ta_1a_0}\left(\xi_0|0101\rangle + \xi_1 e^{-\frac{2\pi i}{3}}|1010\rangle + \xi_2 e^{-\frac{4\pi i}{3}}|1111\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_7\rangle_{ta_1a_0}\left(\xi_0|1010\rangle + \xi_1 e^{-\frac{2\pi i}{3}}|1111\rangle + \xi_2 e^{-\frac{4\pi i}{3}}|0000\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_8\rangle_{ta_1a_0}\left(\xi_0|1111\rangle + \xi_1 e^{-\frac{2\pi i}{3}}|0000\rangle + \xi_2 e^{-\frac{4\pi i}{3}}|0101\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_9\rangle_{ta_1a_0}\left(\xi_0|0000\rangle + \xi_1 e^{-\frac{4\pi i}{3}}|0101\rangle + \xi_2 e^{-\frac{2\pi i}{3}}|1010\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_{10}\rangle_{ta_1a_0}\left(\xi_0|0101\rangle + \xi_1 e^{-\frac{4\pi i}{3}}|1010\rangle + \xi_2 e^{-\frac{2\pi i}{3}}|1111\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_{11}\rangle_{ta_1a_0}\left(\xi_0|1010\rangle + \xi_1 e^{-\frac{4\pi i}{3}}|1111\rangle + \xi_2 e^{-\frac{2\pi i}{3}}|0000\rangle\right)_{b_1b_0c_1c_0} \\
&\quad + |\mathscr{B}_{12}\rangle_{ta_1a_0}\left(\xi_0|1111\rangle + \xi_1 e^{-\frac{4\pi i}{3}}|0000\rangle + \xi_2 e^{-\frac{2\pi i}{3}}|0101\rangle\right)_{b_1b_0c_1c_0}\Big].
\end{aligned} \tag{4}$$

This expression shows that, measuring the three particles (ta_1a_0) in the basis given in Eq. (3), Alice gets any one of the 12 possible results with equal probabilities. Suppose, for example, Alice obtains $|\mathscr{B}_{12}\rangle_{ta_1a_0}$, then Bob's and Charlie's qubits collapse to the following state,

$$|\varphi_{12}\rangle_{b_1b_0c_1c_0} = \left(\xi_0|1111\rangle + \xi_1 e^{-\frac{4\pi i}{3}}|0000\rangle + \xi_2 e^{-\frac{2\pi i}{3}}|0101\rangle\right)_{b_1b_0c_1c_0}. \tag{5}$$

It is clear that, after Alice's measurement the coefficients of the secret qutrit state are transferred to those of the four-qubit state in the receivers' hands, so that, if they collaborate with each other, either Bob or Charlie can retrieve the original qutrit information. One can easily see from Eq. (5) that (b_0b_1) and (c_0c_1) are completely symmetrical, so that Alice can choose either Bob or Charlie to be the final receiver of the secret information. Without loss of generality we may assume Bob to be the one. In this case, Charlie must measure each of his qubits c_0 and c_1 in the X basis. Let us rewrite the Eq. (5) as

$$\begin{aligned} |\varphi_{12}\rangle_{b_1b_0c_1c_0} = \frac{1}{2}(&|+\rangle_{c_1}|+\rangle_{c_0} + |+\rangle_{c_1}|-\rangle_{c_0}\sigma^z_{b_0} + |-\rangle_{c_1}| \\ &+\rangle_{c_0}\sigma^z_{b_1} + |-\rangle_{c_1}|-\rangle_{c_0}\sigma^z_{b_1}\sigma^z_{b_0})|\psi_{12}\rangle_{b_1b_0}, \end{aligned} \tag{6}$$

where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle), \sigma^z = (|0\rangle\langle 0| - |1\rangle\langle 1|)$, and $|\psi_{12}\rangle_{b_1b_0} = (\xi_0|\widetilde{0}_{12}\rangle + \xi_1|\widetilde{1}_{12}\rangle + \xi_2|\widetilde{2}_{12}\rangle)_{b_1b_0}$ with $|\widetilde{0}_{12}\rangle \equiv |11\rangle, |\widetilde{1}_{12}\rangle \equiv e^{-\frac{4\pi i}{3}}|00\rangle$ and $|\widetilde{2}_{12}\rangle \equiv e^{-\frac{2\pi i}{3}}|01\rangle$. Therefore, knowing Charlie's measurement results, Bob can effectively recover the qutrit state in the form of a two-qubit state $|\psi_{12}\rangle_{b_1b_0}$ by preforming appropriate single-qubit Pauli operations as indicated in Eq. (6). It should be emphasizes that the original qutrit information is now encoded in a three-dimensional subspace of the four-dimensional Hilbert space of the two qubits (b_1b_0) in Bob's hand. Note that, with Alice's measurement result, Bob already has amplitude information of the qutrit state but not the phase information, which is supplied by Charlie's measurement results.

Similarly, if Alice gets any other results, Bob can also effectively recover the qutrit state with Charlie's help. The corresponding procedures can be easily read off from the following expression,