Optics Communications 281 (2008) 5078–5082

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Phase singularity annihilation of focused partially coherent beams

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ARTICLE INFO

Article history:
Received 18 January 2008
Received in revised form 22 May 2008
Accepted 25 June 2008

Keywords:
Singular optics
Focused Gaussian Schell-model beam
Annihilation of phase singularities

ABSTRACT

Expressions for focused Gaussian Schell-model (GSM) beams are derived and used to study the annihilation and subwavelength structures of phase singularities in the focal region, and to compare with the case of fully coherent Gaussian beams. It is found that the truncation parameter δ and normalized coherence length ε both affect the presence and spatial distribution of phase singularities in the focal plane. Additionally, during the creation and annihilation process the saddle point near the phase singularity does not disappear in the focal plane for GSM beams.

1. Introduction

Recently, much interest has been exhibited in singular optics, which has been extended to treating partially coherent wavefields [1–3]. When a partially coherent field is considered, its phase becomes random and is no longer well defined. However, the spectral degree of coherence of the field is found to possess phase singularities called coherence vortices, at which the spectral degree of coherence has zero value. The behavior of coherence vortices and their connection with the corresponding fully coherent intensity vortices, as well as phase singularities generated by the interference of partially coherent fields have been studied by several groups [1–8]. Although there indeed exist phase singularities of partially coherent fields in the focal region, which was indicated, e.g., see Refs. [9,10]. As yet, to the best of our knowledge, the annihilation of phase singularities of focused partially coherent fields has not been examined.

The purpose of the present paper is to study the phase singularity annihilation of apertured partially coherent fields in the focal region, where the Gaussian Schell-model (GSM) beam is taken as a typical example, and the effect of truncation parameter and normalized coherence length on the phase singularities is analyzed. Specifically, the annihilation process and subwavelength structures of phase singularities in the focal plane are studied. A comparison with the case of fully coherent Gaussian beams is also made.

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doi:10.1016/j.optcom.2008.06.053

2. Focusing of GSM beams

As shown in Fig. 1, assume that a partially coherent beam is incident upon an aperture of half width a in an opaque screen and converging toward a point 0 at a distance f from the aperture. The cross-spectral density of the focused field at two points $P(\mathbf{r}_1)$ and $P(\mathbf{r}_2)$ is, within the paraxial approximation, given by the formula

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega)=\left(\frac{k}{2\pi}\right)^2\int\int_{\Sigma}W^{(0)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)\frac{\exp\left[\mathrm{i}k(R_2R_1)\right]}{R_1-R_2}\mathrm{d}^2r'_1\mathrm{d}^2r'_2, \quad (1)$$

where k is the wave number related to the wavelength λ by $k=2\pi/\lambda$, $W^{(0)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)$ is the cross-spectral density of the incident fields in the aperture, ω is the angular frequency, and $R_1=|\mathbf{r}_1-\mathbf{r}'_1|$, $R_2=|\mathbf{r}_2-\mathbf{r}'_2|$. The integration extends over the aperture plane Σ.

From now on, we omit the explicit dependence of the various quantities on the angular frequency ω. For GSM beams considered in this paper we have [11]

$$W^0(\mathbf{r}'_1,\mathbf{r}'_2)=\exp\left[-\frac{r'^2_1+r'^2_2}{w_0^2}-\frac{(\mathbf{r}'_1-\mathbf{r}'_2)^2}{\sigma_g^2}\right], \quad (2)$$

where w_0 is the waist width, σ_g denotes the effective spectral coherence length of the field in the aperture. When $w_0 \gg \sigma_g$, the source is quasi-homogeneous (globally essentially spatially incoherent), and when $w_0 \ll \sigma_g$, the source is essentially spatially coherent.

In the paraxial domain the distances R_1 and R_2 can be approximated by the expressions

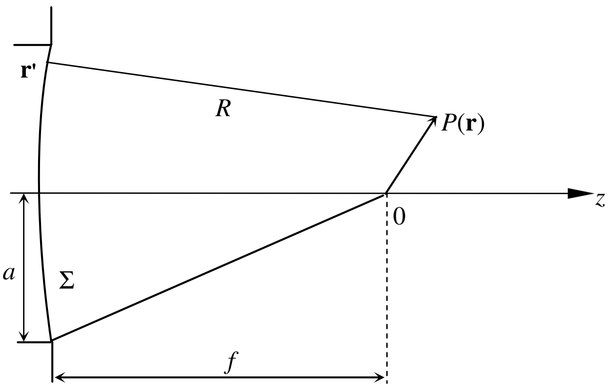

Fig. 1. Notation relating to the focusing system.

$$\begin{cases} R_1 \approx f - \mathbf{q}_1 \cdot \mathbf{r}_1 \\ R_2 \approx f - \mathbf{q}_2 \cdot \mathbf{r}_2 \end{cases} \tag{3}$$

with $\mathbf{q}_1$ and $\mathbf{q}_2$ being unit vectors along the directions $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. The substitution from Eqs. (2) and (3) into Eq. (1) yields

$$W(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\lambda^2 f^2}\int\int_\Sigma \exp\left[-\frac{r_1'^2 + r_2'^2}{w_0^2} - \frac{(\mathbf{r}_1' + \mathbf{r}_2')^2}{\sigma_g^2}\right] \times \frac{\exp[ik(\mathbf{q_1}\cdot\mathbf{r_1} - \mathbf{q_2}\cdot\mathbf{r_2})]}{R_1 R_2}\mathrm{d}^2 r_1' \mathrm{d}^2 r_2'. \tag{4}$$

Without loss of generality, we consider the cross-spectral density of the focused field at two points $P(\mathbf{r}_1)$ and $P(\mathbf{r}_2)$ in the $x0z$ plane, i.e., $\mathbf{r}_1 = (x_1,0,z_1)$ and $\mathbf{r}_2 = (x_2,0,z_2)$. Then

$$\mathbf{q}_1 \cdot \mathbf{r}_1 = \frac{r_1' x_1 \cos\theta_1'}{f} - z_1\left(1 - \frac{r_1'^2}{2f^2}\right), \tag{5}$$

$$\mathbf{q}_2 \cdot \mathbf{r}_2 = \frac{r_2' x_2 \cos\theta_2'}{f} - z_2\left(1 - \frac{r_2'^2}{2f^2}\right), \tag{6}$$

Transforming Cartesian (x,y,z) to polar coordinates (ρ,θ,z) and substituting Eqs. (5) and (6) into Eq. (4), after integral calculations, the final result can be expressed as

$$W(\mathbf{r}_1,\mathbf{r}_2) = \left(\frac{ka}{f}\right)^2 \sum_{l=-\infty}^{\infty}\int_0^1\int_0^1 \exp\left\{ik\left[z_2\left(1 - \frac{a^2 t_2^2}{2f^2}\right) - z_1\left(1 - \frac{a^2 t_1^2}{2f^2}\right)\right] - (t_1^2 + t_2^2)\left(\delta^2 + \frac{1}{\varepsilon^2}\right)\right\} \times J_l\left(\frac{kax_1 t_1}{f}\right) J_l\left(\frac{kax_2 t_2}{f}\right) I_l\left(\frac{2t_1 t_2}{\varepsilon^2}\right) t_1 t_2 \mathrm{d}t_1 \mathrm{d}t_2, \tag{7}$$

with $J_l(\cdot)$ the Bessel function of the first kind and order l, $I_l(\cdot)$ being the modified Bessel function of order l, and

$$\delta = \frac{a}{w_0} \quad \text{(truncation parameter)}, \tag{8}$$

$$\varepsilon = \frac{\sigma_g}{a}. \quad \text{(normalized coherence length)}. \tag{9}$$

If pairs of axial points $\mathbf{r}_1 = (0,0,z_1)$ and $\mathbf{r}_2 = (0,0,z_2)$ are considered, the cross-spectral density simplifies to

$$W(0,0,z_1;0,0,z_2) = \left(\frac{ka}{f}\right)^2 \int_0^1\int_0^1 \exp\left\{ik\left[z_2\left(1 - \frac{a^2 t_2^2}{2f^2}\right) - z_1\left(1 - \frac{a^2 t_1^2}{2f^2}\right)\right] - (t_1^2 + t_2^2)\left(\delta^2 + \frac{1}{\varepsilon^2}\right)\right\} \times I_0\left(\frac{2t_1 t_2}{\varepsilon^2}\right) t_1 t_2 \mathrm{d}t_1 \mathrm{d}t_2. \tag{10}$$

Since the modified Bessel function $I_0(\cdot)$ is not equal to zero, we can infer from Eq. (10) that there are no zeros of $W(0,0,z_1;\ 0,0,z_2)$ for GSM beams.For pairs of points in the focal plane $\mathbf{r}_1 = (x_1,0,0)$ and $\mathbf{r}_2 = (x_2,0,0)$ the cross-spectral density reduces to

$$W(x_1,0,0;x_2,0,0) = \left(\frac{ka}{f}\right)^2 \sum_{l=-\infty}^{\infty}\int_0^1\int_0^1 \exp\left[-(t_1^2 + t_2^2)\left(\delta^2 + \frac{1}{\varepsilon^2}\right)\right] \times J_l\left(\frac{kax_1 t_1}{f}\right) J_l\left(\frac{kax_2 t_2}{f}\right) I_l\left(\frac{2t_1 t_2}{\varepsilon^2}\right) t_1 t_2 \mathrm{d}t_1 \mathrm{d}t_2. \tag{11}$$

One can notice from Eq. (11) that $W(x_1,0,0;\ x_2,0,0)$, and hence also the spectral degree of coherence $\mu(x_1,0,0;\ x_2,0,0)$ in the focal plane is real-valued.

The spectral degree of coherence is defined as [11]

$$\mu(\mathbf{r}_1,\mathbf{r}_2) = \frac{W(\mathbf{r}_1,\mathbf{r}_2)}{[S(\mathbf{r}_1)S(\mathbf{r}_2)]^{1/2}}, \tag{12}$$

where $S(\mathbf{r}_i)$ specifies the spectral density (intensity) at the position $\mathbf{r}_i$ and is given by

$$S(\mathbf{r}_i) = W(\mathbf{r}_i,\mathbf{r}_i) \quad (i = 1,2). \tag{13}$$

It is noted that the intensity $S(\mathbf{r}_i)$ is always a real-valued function, whereas $\mu(\mathbf{r}_1,\mathbf{r}_2)$ is complex-valued in general [11,12].

With the help of Eqs. (7) and (12), we can study the phase singularities of apertured GSM beams in the focal region. In the

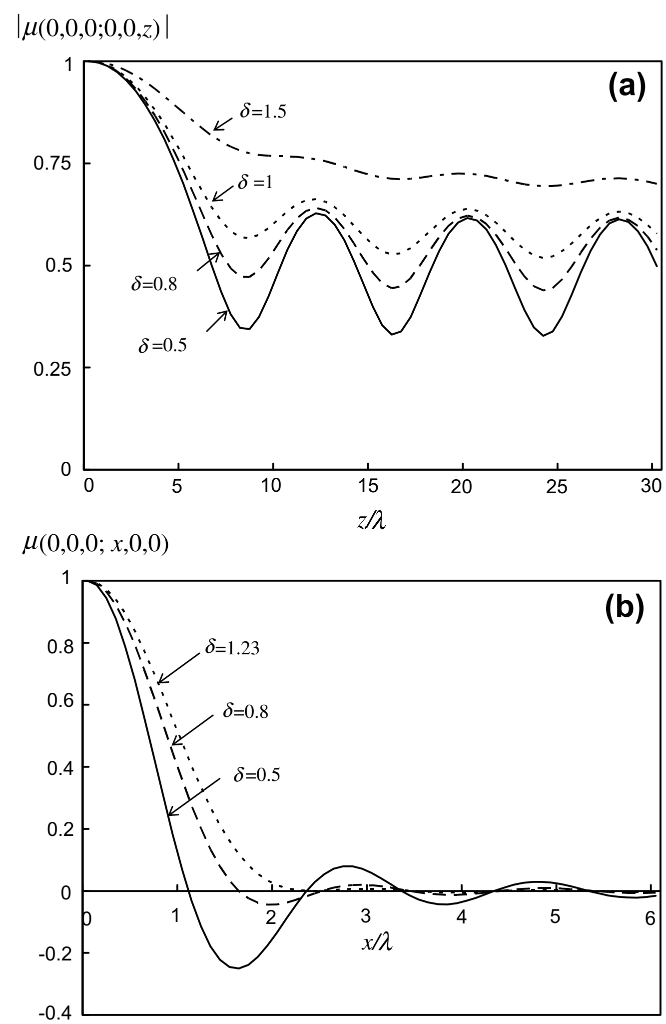

Fig. 2. (a) Modulus of the axial spectral degree of coherence $|\mu(0,0,0;\ 0,0,z)|$ and (b) spectral degree of coherence $\mu(0,0,0;\ x,0,0)$ in the focal plane for different values of the truncation parameter δ, $\lambda = 0.6328\ \mu\mathrm{m}$, $f = 20$ mm, $a = 10$ mm, $\varepsilon = 0.4$.