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# Schemes for preparing atomic qubit cluster states in cavity QED

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ABSTRACT

Two schemes are proposed for generating atomic qubits cluster states in cavity quantum electrodynamics (QED). In the first scheme, only two-atom–cavity interactions are involved, and cluster states can be directly generated by using constructed two-qubit controlled phase gates. The second scheme needs the assistance of additional single-qubit rotations, but takes less time than the first one for two-atom operations in the cavity. In this scheme, two projective operators are constructed to prepare two-dimension or more complicated configurations of cluster states. Both schemes are insensitive to the cavity decay due to the fact that the cavity is only virtually excited during the interaction between atoms and the cavity. The idea can also be applied to the ion trap system.



Entanglement between quantum-mechanical particles is the most intriguing characteristic of quantum mechanics. Entangled states not only provide possibilities to test quantum mechanics against local hidden-variable theory, but also are the center resource for quantum information processing [1]. Bipartite entanglement is well understood. Tripartite entangled states can be classified into two inequivalent classes, the Greenberger–Horne–Zeilinger (GHZ) class [2] and the *W* class [3], under stochastic local operations and classical communication. While entanglement for more particles is still under extensive exploration. Recently, Briegel and Raussendorf introduced a interesting type of multi-qubit entangled states, i.e., the so-called cluster states [4]. This kind of states have high persistence of entanglement, and can be regarded as an entanglement resource for the GHZ states but are more immune to decoherence than them [5]. It has been shown that a new inequality is maximally violated by the four-particle cluster states, but not the four-particle GHZ states [6]. The cluster states have extensive applications in quantum physics. They can be used to test nonlocality without inequalities [6], and more importantly, constitute a universal resource for one-way quantum computation [7] and implement quantum communication [8].

The preparation of cluster states has attracted much attention because of its unique features and extensive applications. And some schemes have been proposed in linear optics system [9], atomic ensembles [10], and other kinds of systems [11,12]. There are also some experimental reports on the observation of cluster states [13,14] and demonstration of the one-way quantum computation [14,15].

On the other hand, the microwave cavity QED, with Rydberg atoms crossing superconducting cavities, provides an almost ideal system for the realization of quantum information processing [16]. Lately, two schemes for generating cluster states with resonant interaction between atoms and cavities were proposed in Refs. [10,17]. In this paper, we propose two schemes for preparation of multi-atom cluster states. In the first scheme, only two-atom–cavity interactions are involved under the condition of large detuning between the atoms and the cavity, and cluster states can be directly generated by using constructed two-qubit controlled phase gates. The second scheme works with the assistance of additional single-qubit rotations, but it can save the two-atom operations time. In this scheme, two projective operators are constructed to prepare two-dimension or more complicated configurations of cluster states. Both schemes are insensitive to cavity decay due to the fact that they only involve atom-field interaction with large detuning and do not require the transfer of quantum information between the atoms and the cavity. All the facilities used in our schemes are well within state of the art.

We now consider two identical ladder-type three-level Rydberg atoms simultaneously interacting with a cavity field. The three

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levels of each atom are denoted by $|f\rangle, |g\rangle$ and $|e\rangle$, and they correspond to the principal quantum numbers 49, 50 and 51, respectively. The $|e\rangle \leftrightarrow |g\rangle$ and $|f\rangle \leftrightarrow |g\rangle$ transitions are at 51.1 and 54.3 GHz, respectively. Thus, we can choose the frequency of the cavity mode in a way that only the levels $|e\rangle$ and $|g\rangle$ are appropriately affected by the cavity field. The transition frequency between the states $|g\rangle$ and $|f\rangle$ is highly detuned from the cavity frequency and thus the state $|f\rangle$ is not affected during the atom–cavity interaction. Under the rotating-wave approximation, the Hamiltonian in the interaction picture is (let $\hbar = 1$)

$$H_I = g\sum_{j=1,2}\left(\mathrm{e}^{-\mathrm{i}\delta t}a^{+}\sigma_j^{-} + \mathrm{e}^{\mathrm{i}\delta t}a\sigma_j^{+}\right), \tag{1}$$

where $\sigma_j^{-} = |g_j\rangle\langle e_j|$ and $\sigma_j^{+} = |e_j\rangle\langle g_j|$, $a^{+}$ and $a$ are, respectively, the creation and annihilation operators for the cavity mode, $g$ is the atom–cavity coupling strength, and $\delta$ is the detuning between the atomic transition frequency $\omega_0$ and cavity frequency $\omega$. In the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. The energy-conversing transition is between $|e_1g_2n\rangle$ and $|g_1e_2n\rangle$. The Rabi frequency $\Omega$ for the transition between them, mediated by $|g_1g_2n+1\rangle$ and $|e_1e_2n-1\rangle$, is given by [18]

$$\begin{aligned}\Omega &= \frac{\langle e_1g_2n|H_I|g_1g_2n+1\rangle\langle g_1g_2n+1|H_I|g_1e_2n\rangle}{\delta} \\ &\quad + \frac{\langle e_1g_2n|H_I|e_1e_2n-1\rangle\langle e_1e_2n-1|H_I|g_1e_2n\rangle}{-\delta} \\ &= \frac{g^2}{\delta}.\end{aligned} \tag{2}$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of photon-number of the cavity mode. Then the effective Hamiltonian can be written as [19]

$$H_{\mathrm{E}}' = \Omega\sum_{j=1}^{2}(|e_j\rangle\langle e_j|aa^{+} - |g_j\rangle\langle g_j|a^{+}a) + \Omega(\sigma_1^{+}\sigma_2^{-} + \sigma_1^{-}\sigma_2^{+}). \tag{3}$$

The first and second terms describe the photon-number dependent Stark shifts, and the third and fourth terms describe the dipole coupling between the two atoms induced by the cavity mode. Assuming the cavity field is initially in the vacuum state, the effective Hamiltonian reduces to

$$H_{\mathrm{E}} = \Omega\left(\sum_{j=1}^{2}|e_j\rangle\langle e_j| + \sum_{k,j=1,k\neq j}^{2}\sigma_k^{+}\sigma_j^{-}\right). \tag{4}$$

The time evolution operator of the system is

$$U_{1,2}(t) = \exp(-\mathrm{i}H_e t). \tag{5}$$

Then the relevant states of the atomic system at any time $t$ can be given by

$$\begin{aligned}&|g_ke_j\rangle \to \mathrm{e}^{-\mathrm{i}\Omega t}[\cos(\Omega t)|g_ke_j\rangle - \mathrm{i}\sin(\Omega t)|e_kg_j\rangle], \\ &|g_1g_2\rangle \to |g_1g_2\rangle, |f_ke_j\rangle \to \mathrm{e}^{-\mathrm{i}\Omega t}|f_ke_j\rangle, \\ &|f_kg_j\rangle \to |f_kg_j\rangle \quad (k,j=1,2,\ k\neq j).\end{aligned} \tag{6}$$

By choosing $\Omega t = \pi$, we can obtain a two-qubit controlled phase gate $CPG$ from (6) as follows [20]:

$$\begin{aligned}&|g_1e_2\rangle \to |g_1e_2\rangle, \quad |g_1g_2\rangle \to |g_1g_2\rangle, \\ &|f_1g_2\rangle \to |f_1g_2\rangle, \quad |f_1e_2\rangle \to -|f_1e_2\rangle.\end{aligned} \tag{7}$$

The two-qubit controlled phase gate $CPG$ (7) can be used to generating atomic qubits cluster states. For example, assuming there are N atoms are initially in the state

$$|\Psi\rangle_{1,N}^{0} = \bigotimes_{j=1}^{N}|+_j\rangle, \tag{8}$$

where as $j$ are odd numbers $|+_j\rangle = (|g_j\rangle + |e_j\rangle)/\sqrt{2}$, otherwise $|+_j\rangle = (|g_j\rangle + |f_j\rangle)/\sqrt{2}$, then an N-atom one-dimensional ($1D$) cluster state can be obtained by using the controlled phase gate of (7), i.e.,

$$|\Psi\rangle_{1,N}^{C} = \bigotimes_{j=1}^{N}CPG_{j,j+1}|+_j\rangle = \bigotimes_{j=1}^{N}\left(|0_j\rangle + |1_j\rangle\sigma_{j+1}^{z}\right) \tag{9}$$

with the convention $CPG_{N,N+1} = 1, \sigma_{N+1}^{z} = 1$ and $\sigma_j^{z} = |0_j\rangle\langle 0_j| - |1_j\rangle\langle 1_j|$. Here we have encoded $|g\rangle \to |0\rangle$ and $|e\rangle(|f\rangle) \to |1\rangle$. Eq. (9) implies that a multipartite atomic qubits cluster state can be created by sequential application of the controlled phase gate of (7) in neighboring atoms. Note that the operations $CPG_{j,j+1}$ and $CPG_{j+2,j+3}$ are separate and can also be simultaneously performed, which can be understood by the quantum mechanical communication $[CPG_{j,j+1}, CPG_{j+2,j+3}] = 0$. In this sense, the presented scheme may take less time than previous ones [10,17] for the same purpose preparing cluster states. As for the experimental design, we can, respectively, put the cavities and atoms, related to $CPG_{j,j+1}$ and $CPG_{j+2,j+3}$, in parallel, and operate them simultaneously. Similarly, arbitrary configurations as well as two-dimension (2D) of multi-atom cluster states can be generated in accordance with this idea.

Next, we introduce another scheme for preparing multi-atom cluster states by using the above system (also considering the three-level Rydberg atoms). To this end, we first assume the $K$th atom is initially in the entangled state with other subsystems $(1/\sqrt{2})(|f_K\rangle|\Phi'\rangle + |e_K\rangle|\Phi''\rangle)$, where $|\Phi'\rangle$ and $|\Phi''\rangle$ are arbitrary normalized wave functions of the subsystems, and the $(K+1)$th atom is initially in the state $|g_{K+1}\rangle$. We then let the two atoms interact simultaneously with a vacuum cavity, the effective interaction Hamiltonian is described by Eq. (4). After an interaction time $t = \pi/(2\Omega)$, the state of the atomic system evolves into

$$|\Phi\rangle_{K,K+1} = \frac{1}{\sqrt{2}}(|f_K\rangle|g_{K+1}\rangle|\Phi'\rangle - |g_K\rangle|e_{K+1}\rangle|\Phi''\rangle). \tag{10}$$

After leaving the cavity, the $(K+1)$th atom is sent through two classical fields tuned to the transitions $|e\rangle \leftrightarrow |g\rangle$ and $|g\rangle \leftrightarrow |f\rangle$, respectively. Choosing the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition

$$\begin{aligned}&|g_{K+1}\rangle \to \frac{1}{\sqrt{2}}(|g_{K+1}\rangle + |e_{K+1}\rangle) \to \frac{1}{\sqrt{2}}(|f_{K+1}\rangle + |e_{K+1}\rangle), \\ &|e_{K+1}\rangle \to \frac{1}{\sqrt{2}}(|g_{K+1}\rangle - |e_{K+1}\rangle) \to \frac{1}{\sqrt{2}}(|f_{K+1}\rangle - |e_{K+1}\rangle).\end{aligned} \tag{11}$$

These operations can be described by a unitary operator

$$\begin{aligned}R_{K+1} &= (|g_{K+1}\rangle\langle f_{K+1}| + |f_{K+1}\rangle\langle g_{K+1}|) \otimes \frac{1}{\sqrt{2}}(|g_{K+1}\rangle\langle g_{K+1}| + |e_{K+1}\rangle\langle g_{K+1}| \\ &\quad + |g_{K+1}\rangle\langle e_{K+1}| - |e_{K+1}\rangle\langle e_{K+1}|).\end{aligned} \tag{12}$$

Thus, under the action of $R_{K+1}$ the state (10) becomes

$$|\Phi\rangle_{K,K+1}^{C'} = \frac{1}{2}(|f_K\rangle|\Phi'\rangle + |g_K\rangle|\Phi''\rangle)(|f_{K+1}\rangle\sigma_K^{z} + |e_{K+1}\rangle), \tag{13}$$

where $\sigma_K^{z} = |f_K\rangle\langle f_K| - |g_K\rangle\langle g_K|$, and the subscript indicates that the Pauli operator acts on the $K$th atom.Now we show how a multi-atom 1D cluster state can be prepared. Assuming N atoms initially in the state

$$|\Phi\rangle_{1,N}^{0} = \frac{1}{\sqrt{2}}(|f_1\rangle + |e_1\rangle)|g_2\cdots g_N\rangle, \tag{14}$$

and performing the operations mentioned above on every pair of neighbor-labeled atoms, we obtain

$$\begin{aligned}|\Phi\rangle_{1,N}^{C'} &= V_{[N-1]}V_{[N-2]}\cdots V_{[1]}\frac{1}{\sqrt{2}}(|f_1\rangle + |e_1\rangle)|g_2\cdots g_N\rangle \\ &= \frac{1}{2^{N/2}}\bigotimes_{j=1}^{N-1}\left(|g_j\rangle + |f_j\rangle\sigma_{j-1}^{z}\right)(|e_N\rangle + |f_N\rangle\sigma_{N-1}^{z})\end{aligned} \tag{15}$$

with the convention $\sigma_0^{z} = 1$ and $V_{[K]} = R_{K+1}U_{K,K+1}(t = \frac{\pi}{2\Omega})$ $(K = 1,\cdots, N-1)$. The state (15) is just an N-atom 1D cluster state. Of course, we can let the $N$th atom subject to one classical pulse, which in-