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Focusing and phase compensation of paraxial beams by a left-handed material slab

Hailu Luo [a,*], Wei Hu [b], Zhongzhou Ren [a], Weixing Shu [a], Fei Li [a]

[a] Department of Physics, Nanjing University, Nanjing 210008, China

[b] Laboratory of Light Transmission Optics, South China Normal University, Guangzhou 510630, China

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Abstract

On the basis of angular spectrum representation, a formalism describing paraxial beams propagating through an isotropic left-handed material (LHM) slab is presented. The treatment allows us to introduce the ideas of beam focusing and phase compensation by LHM slab. Because of the negative refractive index of LHM slab, the inverse Gouy phase shift and the negative Rayleigh length of paraxial Gaussian beam are proposed. It is shown that the phase difference caused by the Gouy phase shift in right-handed material (RHM) can be compensated by that caused by the inverse Gouy phase shift in LHM. If certain matching conditions are satisfied, the intensity and phase distributions at object plane can be completely reconstructed at the image plane.

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1. Introduction

In the late 1960s, Veselago firstly introduced the concept of left-handed material (LHM) in which both the permittivity ε and the permeability μ are negative [1]. Veselago predicted that electromagnetic waves incident on a planar interface between a right-handed material (RHM) and a LHM will undergo negative refraction. Theoretically, a LHM planar slab can act as a lens and focus waves from a point source. Experimentally, the negative refraction has been observed by using periodic wires and rings structure [2–6]. In the past few years, negative refractions in photonic crystals [7–10] and anisotropic metamaterials [11–15] have also been reported.

Recently, Pendry extended Veslago's analysis and further predicted that a LHM slab can amplify evanescent waves and thus behaves like a perfect lens [16]. It is well known that in a conventional imaging system the evanescent waves are drastically decayed before they reach the image plane. While in a LHM slab system, both the phases of propagating waves and the amplitudes of evanescent waves from a near-field object could be restored at its image. Therefore, the spatial resolution of the superlens can overcome the diffraction limit of conventional imaging systems and reach the subwavelength scale. While great research interests were initiated by the revolutionary concept [17–20], hot debates were also raised [21–27].

The main purpose of the present work is to investigate the paraxial beams propagating through an isotropic LHM slab. Starting from the representation of plane-wave angular spectrum, we derive the propagation of paraxial beams in RHM and LHM. Our formalism permits us to introduce ideas for beam focusing and phase compensation of paraxial beams by using LHM slab. Because of the negative refractive index, the inverse Gouy phase shift and negative Rayleigh length in LHM slab are proposed. As an example, we obtain the analytical description for a Gaussian beam propagating through a LHM slab. We find

* Corresponding author. Tel.: +86 2583596499; fax: +86 2583326028.
E-mail address: hailuluo@163.com (H. Luo).

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that the phase difference caused by the Gouy phase shift in RHM can be compensated by that caused by the inverse Gouy phase shift in LHM.

2. The paraxial model of beam propagation

In this section, we present a brief derivation on paraxial model in RHM and LHM. Following the standard procedure, we consider a monochromatic electromagnetic field $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})\exp(-\mathrm{i}\omega t)]$ and $\mathbf{B}(\mathbf{r},t) = \mathrm{Re}[\mathbf{B}(\mathbf{r})\exp(-\mathrm{i}\omega t)]$ of angular frequency ω propagating through an isotropic material. The field can be described by Maxwell's equations [28]

$$\begin{aligned}
&\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\\
&\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t},\\
&\mathbf{D} = \varepsilon \mathbf{E},\\
&\mathbf{B} = \mu \mathbf{H}.
\end{aligned} \tag{1}$$

One can easily find that the wave propagation is only permitted in the medium with $\varepsilon,\ \mu > 0$ or $\varepsilon,\ \mu < 0$. In the former case, $\mathbf{E}$, $\mathbf{H}$ and $\mathbf{k}$ form a right-handed triplet, while in the latter case, $\mathbf{E}$, $\mathbf{H}$ and $\mathbf{k}$ form a left-handed triplet. The previous Maxwell equations can be combined straightforwardly to obtain the well-known equation for the complex amplitude of the electric field in RHM or LHM

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) + k^2 \mathbf{E} = 0, \tag{2}$$

where $k = n_{\mathrm{R,L}}\omega/c$, c is the speed of light in vacuum, $n_{\mathrm{R}} = \sqrt{\varepsilon_{\mathrm{R}}\mu_{\mathrm{R}}}$ and $n_{\mathrm{L}} = -\sqrt{\varepsilon_{\mathrm{L}}\mu_{\mathrm{L}}}$ are the refractive index of RHM and LHM, respectively [1].

Eq. (2) can be conveniently solved by employing the Fourier transformations, so the complex amplitude in RHM and LHM can be conveniently expressed as

$$\mathbf{E}(\mathbf{r}_\perp, z) = \int \mathrm{d}^2\mathbf{k}_\perp \tilde{E}(\mathbf{k}_\perp)\exp[\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{r}_\perp + \mathrm{i}k_z z]. \tag{3}$$

Here $\mathbf{r}_\perp = x\mathbf{e}_x + y\mathbf{e}_y$, $\mathbf{k}_\perp = k_x\mathbf{e}_x + k_y\mathbf{e}_y$, and e_j is the unit vector in the j-direction. Note that $k_z = \sigma\sqrt{n_{\mathrm{R,L}}^2 k_0^2 - k_\perp^2}$, $\sigma = 1$ for RHM and $\sigma = -1$ for LHM. The choice of sign ensures that power propagates away from the surface to the $+z$ direction. The field $\tilde{E}(\mathbf{k}_\perp)$ in Eq. (3) is related to the boundary distribution of the electric field by means of the relation

$$\tilde{\mathbf{E}}(\mathbf{k}_\perp) = \int \mathrm{d}^2\mathbf{r}_\perp \mathbf{E}(\mathbf{r}_\perp, 0)\exp[\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{r}_\perp], \tag{4}$$

which is a standard two-dimensional Fourier transform [29]. In fact, after the electric field on the plane $z = 0$ is known, Eq. (3) together with Eq. (4) provides the expression of the field in the space $z > 0$.

From a mathematical point of view, the approximate paraxial expression for the field can be obtained by the expansion of the square root of k_z to the first order in $|\mathbf{k}_\perp|/k$ [30,31], which yields

$$\mathbf{E}(\mathbf{r}_\perp, z) = \exp(\mathrm{i}n_{\mathrm{R,L}}k_0 z)\int \mathrm{d}^2\mathbf{k}_\perp \times \exp\left[\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{r}_\perp - \frac{\mathrm{i}\mathbf{k}_\perp z}{2n_{\mathrm{R,L}}k_0}\right]\tilde{\mathbf{E}}(\mathbf{k}_\perp). \tag{5}$$

Since our attention will be focused on beam propagating along the $+z$ direction, we can write

$$\mathbf{E}(\mathbf{r}_\perp, z) = \mathbf{A}(\mathbf{r}_\perp, z)\exp(\mathrm{i}n_{\mathrm{R,L}}k_0 z), \tag{6}$$

where the field $A(\mathbf{r}_\perp,z)$ is the slowly varying envelope amplitude which satisfies the parabolic equation

$$\left[\mathrm{i}\frac{\partial}{\partial z} + \frac{1}{2n_{\mathrm{R,L}}k_0}\nabla_\perp^2\right]\mathbf{A}(\mathbf{r}_\perp, z) = 0, \tag{7}$$

where $\nabla_\perp = \partial_x\mathbf{e}_x + \partial_y\mathbf{e}_y$. From Eq. (7) we can find that the field of paraxial beams in LHM can be written in the similar way to that in RHM, while the sign of the refractive index is reverse.

3. The propagation of paraxial Gaussian beam

The previous section outlined the paraxial model for general laser beams propagating in RHM and LHM. In this section we shall investigate the analytical description for a beam with a boundary Gaussian distribution. This example allows us to describe the new features of beam propagation in LHM slab. As shown in Fig. 1, the LHM slab in region 2 is surrounded by the usual RHM in region 1 and region 3. The beam will pass the interfaces $z = a$ and $z = a + d$ before it reaches the image plane $z = a + b + d$. To be uniform throughout the following analysis, we introduce different coordinate transformations $z_i^*(i = 1, 2, 3)$ in the three regions, respectively.

First we want to explore the field in region 1. Without any loss of generality, we assume that the input waist locates at the object plane $z = 0$. The fundamental

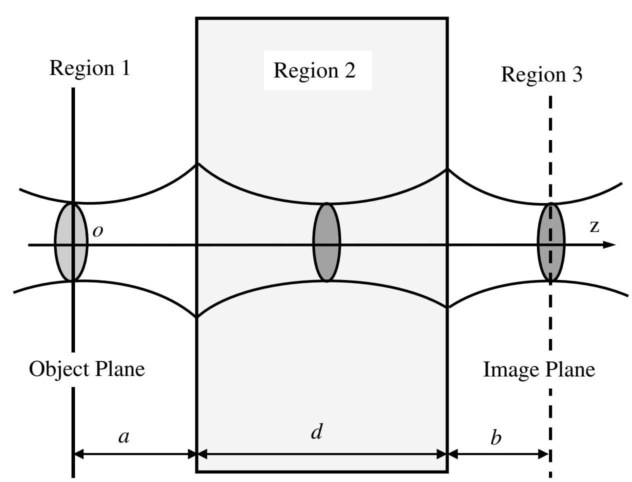

Fig. 1. The mechanisms for paraxial beams propagating through an isotropic LHM slab. The LHM slab in region 2 is surrounded by the usual RHM in region 1 and region 3. The solid line and the dashed line are the theoretical object and image planes, respectively.