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# Beam radiated from quasi-homogeneous uniformly polarized electromagnetic source scattering on quasi-homogeneous media

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**Abstract**

We study the coherence properties of the field generated by beam radiated from quasi-homogeneous (QH) electromagnetic source scattering on QH media. Formulas for the spectral density and spectral degree of coherence of the three dimensional scattered field are derived. The results show under assumption that the diagonal correlation coefficients of the source are proportional to each other, the far field of the scattered light satisfy two reciprocity relations analogous to that in the scalar case, that, the spectral density is proportional to the convolution of the spectral density of the source and the spatial Fourier transform of the correlation coefficient of the scattering potential; the spectral degree of coherence is proportional to the convolution of the diagonal correlation coefficients and the strength of the scattering potential.


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## 1. Introduction

Light scattering by random media is now of intensive interest in various field [1–3]. A general and important class of model random media is the quasi-homogeneous (QH) media [4]. This kind of random media has a property that the strength of the scattering potential $S^{(F)}(\mathbf{r},\omega)$ varies much more slowly with position $\mathbf{r}$ at a particular frequency $\omega$ than the correlation coefficient $\mu^{(F)}\ (\mathbf{r}_1,\mathbf{r}_2,\omega)\equiv\mu^{(F)}(\mathbf{r}_2-\mathbf{r}_1,\omega)$ with position $\mathbf{r}'=\mathbf{r}_2-\mathbf{r}_1$. In the framework of scalar scattering of light, an interesting result analogous to that in the radiation situation [5] was obtained by Visser et al. [6], that within the accuracy of the first-order Born approximation, for the far field of scattered light generated by light from a QH scalar source scattering by QH random media, the angular distribution of spectral density and the spectral degree of coherence form two reciprocity relations. Recently, the QH scalar source was generalized by Korotkova et al. [7] to the electromagnetic case and it was shown that for the radiated far field, analogous reciprocity relations to that of the scalar case were satisfied by the spectral density and spectral degree of coherence. Theoretically, it is desirable to know the coherence properties of the scattered electromagnetic field generated by light from a QH electromagnetic source scattering by a QH random media, however, this problem has not been studied yet. In the present paper, we discuss the coherence property of the scattered field of electromagnetic beam from a QH source scattering from a QH random media. We first study the coherence property of scattered field of a plane wave, next, we consider the case of beam scattering as a more general problem, at last, we make a conclusion.

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## 2. Electromagnetic plane wave scattering on a QH media

Consider a linearly polarized plane electromagnetic wave $\mathbf{E}^{(i)}(\mathbf{r},t)=\mathbf{E}^{(i)}(\mathbf{r},\omega)\times\exp(-\mathrm{i}\omega t)$ incident on a QH random media. $\mathbf{E}^{(i)}(\mathbf{r},\omega)=[E_a^{(i)},E_b^{(i)}]$, $E_a^{(i)},E_b^{(i)}$ are two components of the incident electric field along two mutually orthogonal direction $\mathbf{a}_i,\mathbf{b}_i$ perpendicular to the direction of wave propagation. For the sake of simplicity, we assume that both components have unit amplitude:

$$E_a^{(i)}(\mathbf{r},\omega)=E_b^{(i)}(\mathbf{r},\omega)=\exp(\mathrm{i}k\mathbf{s}_0\cdot\mathbf{r}), \tag{1}$$

where $\mathbf{r}$ is the position vector, and $\mathbf{s}_0$ is the direction of propagation, $k=\omega/c$ is the wave number, $\omega$ is the frequency, $c$ is the speed of light in vacuum. Within the accuracy of the first-order Born approximation, the amplitude of the scattered field is given by [8]:

$$\mathbf{E}^{(s)}(r\mathbf{s},\omega)=-\mathbf{s}\times\left[\mathbf{s}\times\int_D F(\mathbf{r}',\omega)\mathbf{E}^{(i)}(\mathbf{r}',\omega)G(\mathbf{r},\mathbf{r}',\omega)\,\mathrm{d}^3r'\right], \tag{2}$$

where superscript (s) denotes quantities pertaining to scattered field, $\mathbf{s}$ is the unit vector along a typical scattering path, $D$ is the domain the scatterer occupies. $F(\mathbf{r}',\omega)$ is the scattering potential of the random media. $G(\mathbf{r},\mathbf{r}',\omega)$ is the free space outgoing Green's function of the Helmholz operator, in the far zone of the scatter, a asymptotic approximation may be used for the Green's function:

$$G(r\mathbf{s},\mathbf{r}',\omega)=\frac{\exp(\mathrm{i}kr)}{r}\exp[-\mathrm{i}k\mathbf{s}\cdot\mathbf{r}']. \tag{3}$$

The correlation function of the scattering potential is defined as:

$$\begin{aligned}C^{(F)}(\mathbf{r}_1,\mathbf{r}_2,\omega)&=\langle F^*(\mathbf{r}_1,\omega)F(\mathbf{r}_2,\omega)\rangle\\&=\sqrt{S^{(F)}(\mathbf{r}_1,\omega)S^{(F)}(\mathbf{r}_2,\omega)}\eta^{(F)}(\mathbf{r}_1,\mathbf{r}_2,\omega).\end{aligned} \tag{4}$$

In Eq. (4), the angle brackets denote average taken over a statistical ensemble of realizations of the scatterer, the asterisk denotes complex conjugate. $S^{(F)}(\mathbf{r},\omega)=C^{(F)}(\mathbf{r},\mathbf{r},\omega)$ is the strength of the scattering potential, $\eta^{(F)}(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is the correlation coefficient of the scattering potential. For a QH random media, the correlation coefficient depends on $\mathbf{r}_1,\mathbf{r}_2$ only through the difference $\mathbf{r}_2-\mathbf{r}_1$, and the strength of the scattering potential varies much more slowly with its spatial argument than the correlation coefficient with its spatial argument. In other words, over the effective with of $|\eta^{(F)}|$ the function $S^{(F)}$ is essentially constant. Consequently, over regions of the scatterer for which $|\eta^{(F)}(\mathbf{r}_2'-\mathbf{r}_1',\omega)|$ is appreciable, it is reasonable to make the approximation:

$$S^{(F)}(\mathbf{r}_1,\omega)\approx S^{(F)}(\mathbf{r}_2,\omega)\approx S^{(F)}[(\mathbf{r}_1+\mathbf{r}_2)/2,\omega]. \tag{5}$$

On substituting Eq. (5) into Eq. (4) we obtain for the correlation function:

$$C^{(F)}(\mathbf{r}_1,\mathbf{r}_2,\omega)\approx S^{(F)}[(\mathbf{r}_1+\mathbf{r}_2)/2,\omega]\eta^{(F)}(\mathbf{r}_2-\mathbf{r}_1,\omega). \tag{6}$$

Let $[E_a^{(s)},E_b^{(s)}]$ be the two components of scattered electric field vector along two mutually orthogonal direction $\mathbf{a}_s$, $\mathbf{b}_s$ perpendicular to the scattering path $\mathbf{s}$. $\mathbf{a}_s$, $\mathbf{b}_s$ are chosen in the following way:

$$\begin{aligned}\mathbf{a}_i&=\mathbf{a}_s=\frac{\mathbf{s}\times\mathbf{s}_0}{|\mathbf{s}\times\mathbf{s}_0|},\\\mathbf{b}_i&=\mathbf{s}_0\times\mathbf{a}_i,\\\mathbf{b}_s&=\mathbf{s}\times\mathbf{a}_s,\end{aligned} \tag{7}$$

we also assume that the cartesian coordinate is chosen as:

$$\mathbf{x}=\mathbf{a}_i,\quad \mathbf{y}=\mathbf{b}_i,\quad \mathbf{z}=\mathbf{s}_0. \tag{8}$$

The scattered electromagnetic field is three dimensional, and its three components may be calculated as:

$$\begin{aligned}E_x^{(s)}&=A_x(\phi)\int_D F(\mathbf{r}',\omega)E_x^{(i)}(\mathbf{r}',\omega)G(\mathbf{r},\mathbf{r}',\omega)\,\mathrm{d}^3r',\\E_y^{(s)}&=A_y(\phi)\int_D F(\mathbf{r}',\omega)E_y^{(i)}(\mathbf{r}',\omega)G(\mathbf{r},\mathbf{r}',\omega)\,\mathrm{d}^3r',\\E_z^{(s)}&=A_z(\phi)\int_D F(\mathbf{r}',\omega)E_y^{(i)}(\mathbf{r}',\omega)G(\mathbf{r},\mathbf{r}',\omega)\,\mathrm{d}^3r',\end{aligned} \tag{9}$$

where $\phi$ is the angle $\mathbf{s}$ makes with $\mathbf{s}_0$. The geometrical factors $A_i$ $(i=x,y,z)$ in front of the integrals in Eq. (9) have the form:

$$\begin{aligned}A_x(\phi)&=1,\\A_y(\phi)&=\cos^2\phi,\\A_z(\phi)&=-\sin\phi\cos\phi.\end{aligned} \tag{10}$$

Based on the unified theory of coherence and polarization [9], the coherence properties of the scattered field can be represented by a third-order matrix in terms of the three components of the scattered electric field:

$$\begin{aligned}\mathbf{W}^{(s)}&\equiv[W_{ij}^{(s)}(\mathbf{r}_1,\mathbf{r}_2,\omega)]\\&=[\langle E_i^{(s)^*}(\mathbf{r}_1,\omega)E_j^{(s)}(\mathbf{r}_2,\omega)\rangle]\quad(i,j=x,y,z).\end{aligned} \tag{11}$$

In Eq. (11) the superscript (s) represents quantities pertaining to the scattered field, $E_i^{(s)}$ $(i=x,y,z)$ denote the cartesian components of the typical member of the statistical ensemble of the scattered electric field, and the angle brackets denote the average taken on a statistical ensemble of realizations of the electric filed, the asterisk denotes complex conjugation. The $\omega$ was included in equations above to indicate that we treat the problem in space-frequency domain and the equations hold for a single frequency component of the field, but to avoid unnecessary cluttering of the formulas, we henceforth omit $\omega$ in various notations. On substituting Eq. (9) into Eq. (11) and using Eqs. (1), (3) we obtain:

$$\begin{aligned}W_{ij}^{(s)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2)&=A_i(\phi_1)A_j(\phi_2)\frac{\exp[\mathrm{i}k(r_2-r_1)]}{r_1r_2}\\&\quad\times\int_D\int_D C^{(F)}(\mathbf{r}_1',\mathbf{r}_2')\exp[\mathrm{i}k\mathbf{s}_0\cdot(\mathbf{r}_2'-\mathbf{r}_1')]\\&\quad\times\exp[\mathrm{i}k(\mathbf{s}_1\cdot\mathbf{r}_1'-\mathbf{s}_2\cdot\mathbf{r}_2')]\,\mathrm{d}^3r_1'\,\mathrm{d}^3r_2'\end{aligned} \tag{12}$$