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# Deterministic generation of squeezing for a cavity field with a collection of driven atoms

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**Abstract**

We propose an efficient scheme for realizing squeezing for a cavity mode. In the scheme, a collection of ladder-type three-level atoms are trapped in a cavity and driven by two classical fields. Under certain conditions, the cavity field deterministically evolves to a squeezed state. The scheme can also be used for conditional generation of superpositions of different squeezed vacuum states.


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## 1. Introduction

Recently, the problem of generating various quantum states has been a central topic in quantum optics. Of special interest are the squeezed states of an electromagnetic field, whose quantum fluctuation in one quadrature is below the vacuum level [1]. The squeezed states are defined as

$$|\xi,\alpha\rangle = S(\xi)D(\alpha)|0\rangle, \tag{1}$$

where $|0\rangle$ is the vacuum state, $D(\alpha) = \mathrm{e}^{\alpha a^+ - \alpha^* a}$ is the displacement operator, and $S(\xi) = \mathrm{e}^{(\xi^* a^2 - \xi a^{+2})/2}$ is the squeeze operator, with $a$ and $a^+$ being the annihilation and creation operators for the harmonic oscillator and $\xi = r\mathrm{e}^{\mathrm{i}\theta}$ being the squeeze factor. The variances of the quadrature phase operators $X_1 = (a^+\mathrm{e}^{\mathrm{i}\theta/2} + a\mathrm{e}^{-\mathrm{i}\theta/2})/2$ and $X_2 = \mathrm{i}(a^+\mathrm{e}^{\mathrm{i}\theta/2} - a\mathrm{e}^{-\mathrm{i}\theta/2})/2$ are $(\Delta X_1)^2 = \mathrm{e}^{-2r}/4$ and $(\Delta X_2)^2 = \mathrm{e}^{2r}/4$, respectively. Therefore, the noise in one quadrature is reduced below the quantum limit. The squeezed fields are useful in optical communications [2] and gravitational wave detection [3].

On the other hand, the two-mode squeezed states are defined as

$$|\xi,\alpha,\beta\rangle = S'(\xi)D(\alpha)D(\beta)|0,0\rangle, \tag{2}$$

where $S'(\xi) = \mathrm{e}^{(\xi^* ab - \xi a^+ b^+)/2}$ is the two-mode squeeze operator, with $a$ and $b$ being the annihilation operators for the two harmonic oscillators. The two-mode squeezed states are essentially highly entangled states. In the limit $|\xi| \to \infty$, the two-mode squeezed vacuum state is exactly the original Einstein–Podolsky–Rosen entanglement [4]. Such states can be employed to test Bell's inequalities [5] and realize quantum teleportation [6].

Cavity QED is a qualified candidate for quantum state engineering and quantum information processing [7–9]. Schemes have been proposed for the implementation of squeezing for a cavity field via interaction with a single driven three-level atom [10–12]. More recently, Prado et al. [13] have shown that the squeeze operator can be obtained by using a single driven two-level atom. Guzman et al. [14] have presented a scheme for realizing squeezing via interaction with an atomic sample. In this paper, we present an alternative scheme for deterministic generation of squeezed states for a microwave cavity field. Like the schemes of Ref. [10–13], our scheme also uses driven atoms. During the interaction, the atoms are not entangled with the cavity field and no conditional measurements are required. However, our scheme uses collective interaction of $N$ atoms with the cavity field and thus, the squeeze parameter increases

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with the number of atoms as considered in Ref. [14]. In comparison with the scheme of Ref. [14] based on four-photon transition induced by the cavity field and two weak classical fields, our scheme uses strongly driven two-photon transition and thus, the effective coupling strength is greatly increased, which is important in view of decoherence.

The paper is organized as follows. In Section 2, we study the interaction of a collection of three-level atoms with the cavity field and a strong classical field. In Section 3, we show how we can generate the squeezed state for a cavity field via this model. In Section 4, we give a brief discussion on the experimental issues. A summary appears in Section 5.

## 2. The model

We consider $N$ identical three-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. The interaction between a collection of atoms and a cavity field has been extensively studied [15]. The cavity mode is coupled to the transitions $|g\rangle \rightarrow |i\rangle$ and $|i\rangle \rightarrow |e\rangle$ with the detunings being $\varDelta$ and $-(\varDelta+\delta)$, respectively, as shown in Fig. 1. Meanwhile, the transitions $|g\rangle \rightarrow |i\rangle$ and $|i\rangle \rightarrow |e\rangle$ are driven by two strong classical fields with the detunings being $\varDelta'$ and $-\varDelta'$, respectively. In the interaction picture the Hamiltonian is

$$H = H_1 + H_2, \tag{3}$$

where

$$H_1 = \sum_{j=1}^{N} \left[ g_1 a^+ \mathrm{e}^{\mathrm{i}\varDelta t} |g_j\rangle\langle i_j| + g_2 a^+ \mathrm{e}^{-\mathrm{i}(\varDelta+\delta)t} |i_j\rangle\langle e_j| + H.c. \right], \tag{4}$$

$$H_2 = \sum_{j=1}^{N} \Omega\left( \mathrm{e}^{\mathrm{i}\varDelta' t} |g_j\rangle\langle i_j| + \mathrm{e}^{-\mathrm{i}\varDelta' t} |i_j\rangle\langle e_j| + H.c. \right), \tag{5}$$

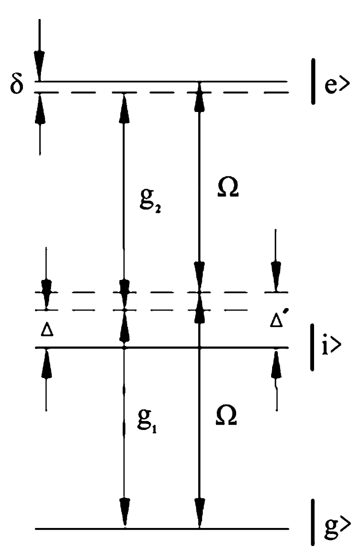


Fig. 1. The configuration of the strongly driven atom–cavity system. The cavity mode is coupled to the transitions $|g\rangle \rightarrow |i\rangle$ and $|i\rangle \rightarrow |e\rangle$ with the detunings being $\Delta$ and $-(\Delta+\delta)$, respectively. Meanwhile, the transitions $|g\rangle \rightarrow |e\rangle$ and $|i\rangle \rightarrow |e\rangle$ are driven by two strong classical fields with the detunings being $\Delta'$ and $-\Delta'$, respectively.

$a^+$ and $a$ are the creation and annihilation operators for the cavity mode, $g_1$ and $g_2$ are the coupling strengths between the cavity mode and the transitions $|g\rangle \rightarrow |i\rangle$ and $|i\rangle \rightarrow |e\rangle$, respectively. $\Omega$ is the Rabi frequency of the classical field. Here we assume that the classical fields have the same Rabi frequency $\Omega$ and phase 0 by adjusting the amplitudes and phases of the classical fields appropriately.

Under the conditions $\varDelta, \varDelta', |\varDelta' - \varDelta| \gg \sqrt{N} g_1, \sqrt{N} g_2, \delta, \Omega$, the probability for the atoms undergoing a single-photon transition is negligible. If the atoms are initially not populated in the intermediate state $|i\rangle$ they will not populate this state during the interaction. In this case the state $|i\rangle$ can be adiabatically eliminated and $H_1$ can be replaced by the effective Hamiltonian

$$H_{1,e} = \sum_{j=1}^{N} [\beta_1 a^+ a |g_j\rangle\langle g_j| + \beta_2 a a^+ |e_j\rangle\langle e_j| + \lambda a^{+2} \mathrm{e}^{-\mathrm{i}\delta t} |g_j\rangle\langle e_j| + \lambda a^2 \mathrm{e}^{\mathrm{i}\delta t} |e_j\rangle\langle g_j|], \tag{6}$$

where $\beta_1 = g_1^2/\varDelta, \beta_2 = g_2^2/\varDelta$, and $\lambda = g_1 g_2/\varDelta$. The first two terms describes the photon-number-dependent Stark shifts, while the third and fourth terms describe the two-photon transition. On the other hand, $H_2$ can be replaced by the effective Hamiltonian

$$H_{2,e} = \sum_{j=1}^{N} \varepsilon(|g_j\rangle\langle g_j| + |e_j\rangle\langle e_j| + |g_j\rangle\langle e_j| + |e_j\rangle\langle g_j|), \tag{7}$$

where $\varepsilon = \Omega^2/\varDelta'$. In this case, the Stark shifts for the upper level $|e\rangle$ and lower level $|g\rangle$ are identical so that the Stark shifts only induce a trivial common phase shift. Define the new atomic basis [16]

$$|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle), |-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle). \tag{8}$$

Then, we can rewrite $H_{1,e}$ and $H_{2,e}$ as

$$H_{1,e} = \frac{1}{2}\sum_{j=1}^{N} \Big\{ \beta_1 a^+ a (I_j + \sigma_j^+ + \sigma_j^-) + \beta_2 a a^+ (I_j - \sigma_j^+ - \sigma_j^-) + \lambda \mathrm{e}^{-\mathrm{i}\delta t} a^{+2} (\sigma_{z,j} + \sigma_j^+ - \sigma_j^-) + \lambda \mathrm{e}^{\mathrm{i}\delta t} a^2 (\sigma_{z,j} + \sigma_j^- - \sigma_j^+)] \Big\}, \tag{9}$$

$$H_{2,e} = \sum_{j=1}^{N} \varepsilon(\sigma_{z,j} + I_j), \tag{10}$$

where $I_j = (|+_j\rangle\langle +_j| + |-_j\rangle\langle -_j|), \sigma_{z,j} = (|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|), \sigma_j^+ = |+_j\rangle\langle -_j|$ and $\sigma_j^- = |-_j\rangle\langle +_j|$.

The time evolution of this system is decided by Schrödinger's equation:

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H|\psi(t)\rangle. \tag{11}$$

Performing the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_{2,e}t}|\psi'(t)\rangle, \tag{12}$$