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Scheme for implementing perfect quantum dense coding with three-atom W-class state in cavity QED

Zhao-Hui Peng *, Chun-Xia Jia

Department of Physics, Hunan University of Science and Technology, Xiangtan 411201, PR China

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Abstract

We propose an experimentally feasible scheme of implementing perfect quantum dense coding with three-atom W-class state in cavity QED. In this scheme atoms interact simultaneously with a highly detuned cavity field and the cavity is only virtually excited, thus the scheme is insensitive to the cavity decay, which is very important in view of experiment. Moreover, we also propose a scheme of transmitting three bits of classical information by sending one qubit and one classical bit with 3-qubit W-class and GHZ states.

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Keywords: Quantum dense coding; W-class state; Cavity QED

1. Introduction

Quantum entanglement is a fundamental element for quantum communication and quantum computation. Dense coding [1] is one of the important applications of quantum entanglement in quantum communication, which can transmit two bits of classical information by sending only one qubit with the help of entanglement between the sender (Alice) and the receiver (Bob). It has been experimentally demonstrated in optical system [2,3], nuclear magnetic resonance (NMR) system [4] and ion trap system [5]. As one of possible candidates for engineering quantum entanglement, the cavity quantum electrodynamics (QED) system has made many applications in quantum information processing (QIP). In most previous schemes the cavity is used to store the quantum information and transfer it back to the atomic system, thus the cavity decay is one of the main obstacles to implement QIP in cavity QED. Recently Zheng and Guo [6] have proposed a novel scheme in which two identical atoms simultaneously interact with a nonresonant cavity field and the effective Hamiltonian of the system which is obtained by adiabatically eliminating the atomic coherence only includes the terms which describe the photon-number dependent Stark shifts and the dipole coupling between the two atoms in the large detuning regime. The cavity is only virtually excited, thus the efficient decoherence time is greatly prolonged. Osnaghi et al. [7] have experimentally implemented the scheme in which two Rydberg atoms crossing a nonresonant cavity are entangled by energy exchange and demonstrated that the scheme is essentially insensitive to the thermal field and cavity decay. Following this idea, various schemes of generating entangled states [8–10], quantum teleportation [11,12] and dense coding [13] have been proposed in cavity QED. In a word, the method of Ref. [6] has opened a new prospect for quantum entanglement and QIP.

It has been shown that there are two inequivalent classes of tripartite entanglement states, the Greenberger–Horne–Zeilinger (GHZ) class and the W-class [14]. Under

* Corresponding author. Tel.: +86 13017323520; fax: +86 732 8291 433.
E-mail address: raul121991@126.com (Z.-H. Peng).

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stochastic local operation and classical communication (SLOCC), the W-class states cannot be converted to the GHZ-class states. The GHZ state can be used for perfect teleportation and dense coding, but the W state cannot [15]. Many schemes of perfect dense coding with the GHZ state have been proposed [16–19]. Due to its robustness against loss of qubit (i.e., if we trace out one particle from the W-class state, there is still genuine entanglement between the remaining two particles), how to implement perfect dense coding with the W-class state is a promising and challenging subject. Recently Gorbachev et al. [20,21] have shown that any quantum channel, which can be obtained from the GHZ state by two particle unitary operation, is suitable for perfect teleportation and dense coding. Then they have introduced a unitary nonlocal operation which can convert the GHZ state into the W-class state and showed that the W-class state can also be used for perfect quantum teleportation and dense coding [20,21]. Agrawal and Pati [22] have shown that there also exists a class of W state which can be used for perfect quantum teleportation and dense coding. They have demonstrated there is one ebit of entanglement shared between Alice and Bob when they use this class of W state. At the same time Zheng [23] proposed a perfect scheme of splitting quantum information with the same W-class state in ion trap. It has been shown that the W-class state may be a good candidate for perfect QIP. So it may be very interesting to investigate how to implement perfect dense coding with the W-class state in cavity QED.

It is well known that the preparation and measurement of Bell states or multi-qubit entangled state is essential for quantum dense coding and quantum teleportation. In the early experiment four Bell states could not been distinguished completely [2,24]. Up to 2001, Shih et al. [25] had implemented the complete Bell state measurement with the nonlinear interaction in optical system. Recently, Schuck et al. [3] have implemented the complete deterministic Bell state measurement with only linear optical elements. For multi-qubit entangled state, Pan et al. [26] proposed to identify only two of the GHZ states in optical system. Yang and Han [27] proposed to implement local measurement for a set of n-qubit maximally entangled GHZ states in cavity QED. Roos et al. [28] implemented the control and measurement of three-qubit maximally entangled GHZ state and W state in ion trap. In this paper we will propose a feasible scheme of generating the W-class state, distinguishing mutually orthogonal W-class states and then implementing perfect quantum dense coding in cavity QED. The distinct advantage of our protocol is that it does not require the transfer of quantum information between the atoms and cavity due to large detuning between the atomic transition frequency and the cavity frequency, thus it is insensitive to the cavity decay. Moreover, we also propose to transmit three bits of classical information by sending one qubit and one classical bit with this three-qubit W-class state and the GHZ state.

2. Generation of the W-class state

We consider three identical three-level atoms, whose states are denoted by $|g\rangle$, $|e\rangle$ and $|f\rangle$ simultaneously interacting with a single-mode cavity field. The transition frequency between the states $|e\rangle$ and $|f\rangle$ is highly detuned from the cavity frequency and thus the state $|f\rangle$ is not affected during the atom-cavity interaction. The Hamiltonian for the system is given by

$$H = H_0 + H_i, \tag{1}$$

where

$$H_0 = \omega a^\dagger a + \omega_0 \sum_{j=1}^{3} S_Z^j, \tag{2}$$

$$H_i = g \sum_{j=1}^{3} (a^\dagger S_j^- + a S_j^+), \tag{3}$$

where $S_Z^j = (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)/2$, $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$ with $|g_j\rangle$ and $|e_j\rangle$ being the ground and excited states of the jth atom, respectively, $a^\dagger$ and a are the creation and annihilation operators for the cavity field, g is the atom-cavity coupling strength. In the interaction picture the Hamiltonian for the system can be written as

$$H_i = g \sum_{j=1}^{3} (\mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^- + \mathrm{e}^{\mathrm{i}\delta t} a S_j^+), \tag{4}$$

where δ is the detuning between the atomic transition frequency ω_0 and the cavity field frequency ω. In the case of $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. Then the effective Hamiltonian, which can be obtained by adiabatically eliminating the atomic coherence, is given by [9]

$$H = \lambda \left[\sum_{i,j=1}^{3} (s_j^+ s_i^- a a^+ - s_j^- s_i^+ a^+ a) \right], \tag{5}$$

where $\lambda = g^2/\delta$. It is noted that this effective Hamiltonian is different from the also well-known ones in the dispersive limit [29]. Assume that the cavity field is initially in the vacuum state, the Hamiltonian reduces to

$$H = \lambda \left(\sum_{j=1}^{3} |e\rangle_{jj}\langle e| + \sum_{i,j=1,i\neq j}^{3} s_j^+ s_i^- \right). \tag{6}$$

Assume that atoms 1–3 are initially in the state $|g\rangle_1 |g\rangle_2 |e\rangle_3$, the state evolution of the system can be represented by [9]

$$|W(t)\rangle = \frac{\mathrm{e}^{-\mathrm{i}3\lambda t} + 2 - 1}{3} |g\rangle_1 |g\rangle_2 |e\rangle_3 + \frac{\mathrm{e}^{-\mathrm{i}3\lambda t} - 1}{3} \times (|g\rangle_1 |e\rangle_2 |g\rangle_3 + |e\rangle_1 |g\rangle_2 |g\rangle_3). \tag{7}$$

With the different choices of the evolution time, one can get various three-atom W-class states. If we choose the evolution time $\lambda t = [\pi - \arccos(1/8)]/3$, the three-atom W-class state is

$$|W\rangle = \frac{1}{\sqrt{2}} \mathrm{e}^{\mathrm{i}\theta} |g\rangle_1 |g\rangle_2 |e\rangle_3 + \frac{1}{2} (|g\rangle_1 |e\rangle_2 |g\rangle_3 + |e\rangle_1 |g\rangle_2 |g\rangle_3), \tag{8}$$