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Focusing of a vortex carrying beam with Gaussian background by an apertured system in presence of coma

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Abstract

In this paper, diffraction pattern of a vortex carrying beam with a Gaussian background has been studied by using Fresnel–Kirchhoff diffraction integral, in the presence of third-order coma. Results of intensity distribution and encircled energy at the Gaussian plane have been presented for two values of the topological charge. Positional shift and splitting of the dark core have been investigated in detail. It is noticed that the diffraction pattern of a beam with double topological charge is affected more by comatic aberration in comparison to the beam with single topological charge. We have also verified our results by using the optical transfer function approach. Propagation of an apertured Gaussian background vortex beam through a π-phase shifter has also been studied for two values of the topological charge.

Keywords: Optical vortex; Laguerre–Gaussian beam; Diffraction theory; Coma; Intensity; Encircled energy

1. Introduction

The propagation of optical vortices in the linear and non-linear media has attracted much attention of several groups in recent years [1–5]. The motion of the vortex is influenced by intensity and phase gradient, and the gradient of the background beam acts as a driving force for the motion of the vortex. Rozas et al. [3] have investigated the propagation dynamics of vortices and compared the similarity with the hydrodynamic vortex phenomena. The term optical vortex is used to represent the helical structure of constant phase surface with a point of undefined phase and zero amplitude in the heart of the surface. This point is referred to as a 'singular point'. In addition, the accumulated phase change around this point is an integral multiple of 2π. This integral multiple is referred to as the 'topological charge' m. Sign of the topological charge represents handedness of the helix. The superposition of vortex beams with Gaussian background leads to dependence of topological charge on the relative width and amplitudes of the beam in free-space propagation [6]. A specific feature of such a beam is that it carries the mechanical angular momentum with respect to the propagation axis, called 'orbital angular momentum' [7,8]. When an optical vortex is hosted within a Gaussian beam, the resulting beam exhibits an annular intensity profile with a dark core whilst maintaining a helical phase structure. A typical example of such a light field is a Laguerre–Gaussian (LG) beam [8,9] which is solution of wave equation in the cylindrical coordinates system. This family of solutions is referred to as Laguerre–Gaussian (LG) mode which is rotationally symmetric and exhibits an azimuthal angular dependence of the complex form $\exp(im\theta)$, where θ is the azimuthal coordinate in the transverse plane.

Applications of vortices have drawn considerable attention in the field of image processing [10–12]. The vortex filter is used in a 4-f set-up for image processing and use of such a filter gives isotropic results for edge enhancement.

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Recently Guo et al. [13] have proposed Laguerre–Gaussian spatial filter (LGSF) for the edge enhancement. Point spread function (PSF) of the imaging system with an LGSF presents smaller sub oscillations than that with angular phase mask (spiral phase plate). The mechanical characteristic of vortex beam has also been used in the trapping and manipulation of microparticles [7] and the presence of aberrations in the system has adverse effect on the performance [14–16]. Optical vortices are also helpful in examining weak background signals hidden in the glare of bright coherent signal [17,18], and in optical correlators [19].

Fair amount of literature is available on the diffraction of non-singular beams through unapertured and apertured systems. In a number of papers, researchers have presented results on the effects of aberrations, in terms of the optical transfer function (OTF) and/or PSF. But the studies are mostly confined to the cases of optical systems in the absence of vortices [20–26]. Recently diffraction of optical vortices has also drawn considerable attention of several groups [27–32]. Cai and He [29] have investigated propagation of Laguerre–Gaussian beam through a slightly misaligned paraxial optical system.

Recently effects of aberration on the spatial structure of vortex at the observation plane have attracted attention of a few groups [33–39]. Far-field diffraction patterns of optical vortex beam with uniform amplitude distribution have been investigated by Singh et al., in the presence of astigmatism [34], coma [35], and spherical aberration [36]. Wada et al. [37,38] have investigated the role of astigmatism, and comatic aberration on the propagation characteristics of a Laguerre–Gaussian beam. However, intensity distribution and encircled energy for Laguerre–Gaussian beam has not so far been studied for an apertured system in the presence of coma. In the present paper, Fresnel–Kirchhoff diffraction integral has been used to compute intensity distribution at the focal plane for two values of the topological charge and the results are crosschecked/verified by OTF route. Encircled energy is also computed as one of the parameters for the focusing system.

2. Optical geometry and Laguerre–Gaussian pupil

Coordinate system employed in the diffraction integral (Fig. 1) has the origin at O. The position of a point on the exit pupil plane of radius a is represented by r_p and the normalized position coordinate is $\rho = r_p/a$. The quantities (ρ, θ) give the polar coordinate of a point in the exit pupil plane. The position coordinate of a point in the observation plane, separated by f from the exit pupil plane, is denoted by (r, ϕ, f). The diffraction image centered at the Gaussian image point P' is aberration-free, if the converging wave emanating from the exit pupil has its center of curvature at point P'. In the presence of aberration, the center of curvature of wave shifts from point P' because of the deviation of the actual wave front from the ideal wave front at the exit pupil. The actual wave front and the Gaussian reference sphere, both pass through the center of the exit pupil, and the origin of coordinate system lies at the center of the exit pupil. In the coordinate geometry, we have chosen two mutually parallel planes orthogonal to z-axis, namely the exit pupil plane, and the Gaussian plane (or focal plane). The z-axis coincides with the optical axis of the focusing system.

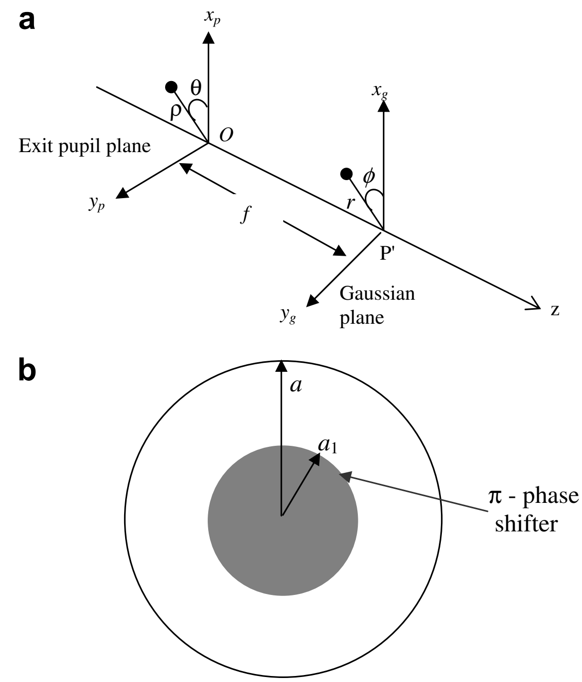

Fig. 1. (a) The coordinate system employed in the diffraction integral. (b) Geometric configuration of π-phase shifter.

The Laguerre–Gaussian modes are the solution of wave equation in the circular coordinate system. The transverse distribution of the field contains Laguerre polynomial with an azimuthal phase factor. The Laguerre–Gaussian beams are given [8,9] as

$$E_{pm}(r,\phi,z) = \frac{E_0}{(1+z^2/z_R^2)^{1/2}}\left[\frac{r\sqrt{2}}{w(z)}\right]^m L_p^m\left[\frac{2r^2}{w^2(z)}\right] \times \exp\left[-\frac{r^2}{w^2(z)}\right]\exp\frac{-\mathrm{i}kr^2z}{2\{z^2+z_R^2\}}\exp(-\mathrm{i}m\phi) \times \exp\left[\mathrm{i}(2p+m+1)\tan^{-1}\frac{z}{z_R}\right]\exp[\mathrm{i}kz] \quad (1)$$

where z_{R} is the Rayleigh range, $w(z)$ is the radius of the beam, $L_p^{|m|}(x)$ is the Laguerre polynomial with p and m as the radial and angular mode numbers respectively, and E_0 is a constant. In the case when the waist plane coincides with $z = 0$ plane, the complex amplitude given by Eq. (1) simplifies as

$$E(\rho,\theta,z=0) = E_0(\sqrt{2}\gamma\rho)^{|m|}L_p^{|m|}[2\gamma^2\rho^2]\exp(-\gamma^2\rho^2) \times \exp(-\mathrm{i}m\theta) \quad (2)$$

where ρ is the radial distance of a point from its center normalized by its radius a and θ is the azimuthal coordinate on the $z = 0$ plane, and