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An indirect algorithm of Fresnel diffraction

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ABSTRACT

An indirect algorithm for Fresnel diffraction calculation at any distance is presented in this paper. Using this indirect algorithm, the Fresnel diffraction on a near plane can be realized in two steps, the first is to acquire the diffractive field on a far field and the second is to obtain the diffractive field on a near plane using an inverse Fresnel integral. Compared with conventional algorithms, this algorithm is valid at any range of distance. The sampling theorem, as the basis of this indirect algorithm, is discussed in detail. Theoretical calculations show good agreements with experimental results.

1. Introduction

Accurate calculation of Fresnel diffraction has not been perfectly solved since a long time ago [1]. It is well known that Fresnel integral can be expressed in a form of Fourier transform (FT), therefore, fast FT (FFT) has been widely applied in the analysis of Fresnel diffraction [2–4], in which single FFT (S-FFT) is usually involved. Alternatively, Fresnel integral can be formulated in a form of convolution integral and a double FFT (D-FFT) is required. Theoretical analysis [2] demonstrates that S-FFT is more suitable for calculating the diffraction on a far plane, however, when the diffraction distance exceeds a certain range, only the amplitude of diffraction field can be obtained precisely by this method. Through D-FFT method, both the amplitude and the phase of diffraction field can be calculated correctly, whereas, its application is limited in the analysis of Fresnel diffraction for a short distance for it is difficult to represent the spread of diffraction field against the diffraction distance.

Since a Fresnel integral can be written as a fractional Fourier transform which can be calculated quickly through D-FFT, the Fresnel integral at any diffractive distance can be evaluated by changing the order of fractional Fourier transform. However, D-FFT in fractional Fourier transform has the same problem as S-FFT has when the diffraction distance exceeds a certain range. Thus, the requirement for accurate evaluations of Fresnel diffraction still remains [5].

In this paper, an indirect algorithm is proposed to calculate the diffractive field at any distance. A detailed discussion is given on the sampling theorem for this algorithm, and the simulation results are verified with experiment results.

2. Fresnel integral transform and its inverse

In a paraxial approximation, the convolution form of Fresnel integral can be stated as

$$U(x,y)=\frac{\exp(ikd)}{i\lambda d}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U_0(x_0,y_0)\times\exp\left\{\frac{ik}{2d}\left[(x-x_0)^2+(y-y_0)^2\right]\right\}\mathrm{d}x_0\,\mathrm{d}y_0 \qquad (1)$$

where $i=\sqrt{-1}, k=2\pi/\lambda$, and λ is wavelength; $U_0(x_0,y_0)$ is the optical wave in x_0y_0 plane, $U(x,y)$ is the diffracted wave in xy plane at a distance d away from x_0y_0 plane.

The inverse formulation for $U(x,y)$ is given by

$$U_0(x_0,y_0)=\frac{\exp(-ikd)}{-i\lambda d}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U(x,y)\times\exp\left\{-\frac{ik}{2d}\left[(x_0-x)^2+(y_0-y)^2\right]\right\}\mathrm{d}x\,\mathrm{d}y \qquad (2)$$

Eqs. (1) and (2) give a mutual transform of optical waves in two spatial planes, which are perpendicular to the wave propagation direction. $\mathscr{F}\{\}$ and $\mathscr{F}^{-1}\{\}$ denote Fourier transform and its inverse respectively, so these two equations can be rewritten as

$$U(x,y)=\frac{\exp(ikd)}{i\lambda d}\exp\left[\frac{ik}{2d}(x^2+y^2)\right]\times\mathscr{F}\left\{U_0(x_0,y_0)\exp\left[\frac{ik}{2d}(x_0^2+y_0^2)\right]\right\}_{\frac{x}{\lambda d},\frac{y}{\lambda d}} \qquad (3)$$

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$$U_0(x_0,y_0)=\frac{\exp(-ikd)}{-i\lambda d}\exp\left[-\frac{ik}{2d}(x_0^2+y_0^2)\right]\times\mathscr{F}^{-1}\left\{U(x,y)\exp\left[-\frac{ik}{2d}(x^2+y^2)\right]\right\}_{\frac{x_0}{\lambda d},\frac{y_0}{\lambda d}} \tag{4}$$

Thus, the calculations of Fresnel integral and its inverse can be performed by means of S-FFT. For a discrete calculation, if the amount of sampling points is $N\times N$, for the calculation of Eq. (3) we have [2]

(1) The calculating extent of diffractive field $U(x,y)$ is $\Delta L=\frac{\lambda dN}{\Delta L_0}$, where ΔL_0 and ΔL are the calculating extents of $U_0(x_0,y_0), U(x,y)$, respectively. When diffracting distance d approaches to zero, the calculation of diffraction cannot be performed.
(2) When $\Delta L_0<\sqrt{\lambda dN}$, only the intensity distribution of Fresnel diffractive field can be calculated correctly.
(3) If the amplitude and the phase of diffractive field need to be evaluated correctly, it should be $\Delta L_0=\Delta L=\sqrt{\lambda dN}$.

It is not difficult to see that above conclusions are also valid for Eq. (4), just change $U(x,y)$ and ΔL_0 to be $U_0(x_0,y_0)$ and ΔL.

3. Indirect algorithm of Fresnel diffraction

Base on above analysis of Eqs. (3) and (4), the calculation of diffractive field at distance d can be performed in two steps. First, $U_1(x_1,y_1)$ at diffracted distance d_1 is evaluated by Eq. (3). Second, $U(x,y)$ at distance $d_2=d_1-d$ is extracted via Eq. (4), that is

$$U(x,y)=\frac{\exp(-ikd_2)}{-i\lambda d_2}\exp\left[-\frac{ik}{2d_2}(x^2+y^2)\right]\times\mathscr{F}^{-1}\left\{U_1(x_1,y_1)\exp\left[-\frac{ik}{2d_2}(x_1^2+y_1^2)\right]\right\}_{\frac{x}{\lambda d_2},\frac{y}{\lambda d_2}} \tag{5}$$

For $U_1(x_1,y_1)$ to satisfy the sampling theorem, there is $\Delta L=\Delta L_0=\sqrt{\lambda d_1 N}$. According to Ref. [2], the phase change, which is caused by quadratic-phase factors in Eq. (5), between two adjacent sampling points should be less than or equal to π. As the steep changes of phase is usually at the edge of sampling region, the sampling for $\mathscr{F}^{-1}\{\}$ can be expressed as

$$\frac{\partial}{\partial m}\frac{k}{2d_2}((m\Delta x)^2+(n\Delta y)^2)\bigg|_{m,n=N/2}\le\pi \tag{6}$$

$$(m,n=-N/2,-N/2+1,\ldots,N/2-1)$$

Let $\Delta x=\Delta y$, then

$$\Delta x^2\le\frac{\lambda d_2}{N} \tag{7}$$

Obviously, for $(\Delta L_0/N)^2=\frac{\lambda d_1}{N}>\frac{\lambda d_2}{N}$, the diffractive field can not be directly calculated from $U_1(x_1,y_1)$ using Eq. (5). However, through an interpolation method the $U_1(x_1,y_1)$ can be expressed as a discrete function with an extent of $\sqrt{\lambda d_2 N}$, and this satisfies the sampling theorem when N is fixed. Notice that the diffraction distance d is not involved in above calculation, the diffraction extent is $\sqrt{\lambda d_2 N}$. Choosing a appropriat d_2 in Eq. (5), the amplitude and phase of diffractive field in the whole Fresnel domain can be evalatuated by S-FFT.

Define $\Delta x=\Delta y=\sqrt{\lambda d_1 N}/N$ and $m=n$ in Eq. (6), then

$$|m,n|\le\frac{d_2 N}{2d_1} \tag{8}$$

Since $d=d_1-d_2$, for a smaller d, the calculable extent is wider when concerning the sampling theorem. From the other aspect, if calculated total power of diffracted field is very close to that of incident wave, the diffraction evaluation by Eq. (5) with $\Delta x=\Delta y=\sqrt{\lambda d_1 N}/N$ will be quite precise. From the FFT calculation, the extent of diffractive field by Eq. (5) is

$$\Delta L'=\lambda N d_2/\sqrt{\lambda N(d_2+d)} \tag{9}$$

For the quadratic-phase term in Eq. (5), it always meets the sampling theorem by choosing the sampling spacing $\Delta L'/N$. Once $\Delta L', \lambda, N$ and d are given, the distance d_2 and $d_1=d_2+d$ can be calculated by Eq. (9), therefore, the diffraction calculation can be performed.

4. Theoretical simulation and experiment results

For simplicity, the simulations of the indirect algorithm without interpolation and the experiment results have been compared with the results of D-FFT. In the experiment, the aperture of x_0y_0 plane is a round hole with a diameter of 60 mm, cross which a metallic filament with a diameter of 1 mm at the center. A quasi base mode CO_2 laser beam with a Gaussian radius of 7.2 mm illuminates the aperture. The laser power is 500 W, wavelength λ = 10.6 μm and the wave front radius of laser beam $R\approx$ 5000 mm. Diffraction patterns are recorded by a thermal paper, which is located on xy plane at a distance d in the light propagation direction. Fig. 1 gives its energy absorption which is a grey response curve [6]. In Fig. 1, the dashed line corresponds to burned areas on the paper.

Choose N = 256, ΔL_0 = 20 mm, the simulated pattern and experiment spots on x_0y_0 plane are shown in Fig. 2.

Fig. 3 show the pattern obtained on thermal papers, the simulations by the indirect algorithm and the simulations by D-FFT at distance d. The experiment results show good agreements with two simulations pattern. The reason for the variations of simulated patterns and experiments, when d = 90 mm and d = 381 mm, might be caused by discrete calculation errors and the spectral overlapping during FFT operation.

When the diffractive field approaches the object plane, S-FFT needs an enormous sampling number N for designed ΔL_0 to satisfy

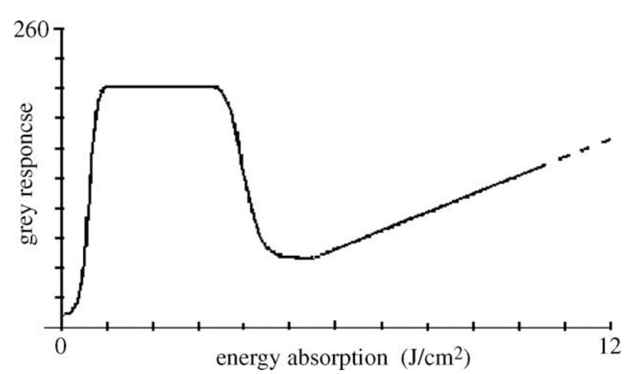

Fig. 1. Energy absorption curve on thermal paper.

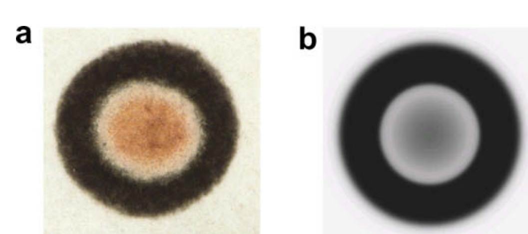

Fig. 2. Experiment pattern of laser beam on thermal paper (a) and simulated pattern (b), the figure size is 20 mm × 20 mm, the heating time is 15 ms.