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Weakenings and redshifts induced by random propagation times

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Abstract

The crossing by light of a turbulent medium results in a random propagation time. We show that the broadening of monochromatic sources leads to a redshift for a part of the transmitted wave and a spreading for the other part. Gaussian shaped spectra and Gaussian propagation times allow to perform computations about the properties of the received wave. In particular, when it is possible to give a strong enough random character to the propagation, the shift parameter z can reach a sizeable amount, taking smooth hypotheses about the parameters of used Gaussian.

Keywords: Redshift; Random delays; Spectral width; Gaussian process

1. Introduction

Doppler effect has long been used as a measure of velocity. It is the case for the speed of cars using backscattering of radar waves, of liquids with acoustic waves or laser beams, or for stars with the emitted light. The discovery of quasars and other luminous galaxies has shown that the Doppler effect could not be the only cause of spectral shifts. Other theories have been put forward to explain anomalous redshifts, and are being much debated nowadays. This theme of research explains the study of optical devices which are able to create spectral changes [1,3,16].

In this paper, we show that it is possible to create redshifts from random propagation delays. We consider a narrowband wave $\mathbf{Z} = \{Z(t), t \in \mathbb{R}\}$ which is emitted by some device or not. This wave goes through a medium turbulent enough to explain a (independent) random propagation time $\mathbf{A} = \{A(t), t \in \mathbb{R}\}$. The observer receives a delayed process $\mathbf{U}$ defined by

$$U(t) = Z(t - A(t)). \tag{1}$$

Spectral characteristics of $\mathbf{U}$ depend on the statistical properties of the emitted process $\mathbf{Z}$ and on the random propagation time $\mathbf{A}$.

When applied on pure spectral rays, random propagation delays can be used to explain phenomena like the weakening of the acoustic waves [8,12], the widening and the particular shapes of spectra in the cases of backscattering of radar waves on trees [8], HF propagation [9], backscattering of radar waves on the sea, or laser propagation [11].

In sections hereafter, the initial process $\mathbf{Z}$ will be assumed narrowband with Gaussian spectral shape before propagating. Such a process will be referred as monochromatic (purely monochromatic when the bandwidth is zero). Also, the probability laws of the random propagation time will be assumed Gaussian. These hypotheses are currently used in literature and they allow to develop fair computations. We will calculate the spectral transformations as a function of the statistics of both processes, and we will show that redshifts naturally occur with amplitude depending mainly on the spectral width of the emitted process. Furthermore, a noise appears, which influence depends on the coherence properties of the propagation time.

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2. Gaussian delays and Gaussian spectra

(1) Firstly, we assume that $\mathbf{Z} = \{Z(t), t \in \mathbb{R}\}$ is a zero mean stationary process with power spectrum $s_Z(\omega)$ defined by [2,14]

$$E[Z(t)Z^*(t-\tau)] = \int_{-\infty}^{\infty} e^{i\omega\tau} s_Z(\omega)\,d\omega \tag{2}$$

where $E[..]$ is for the mathematical expectation or ensemble mean and * for the complex conjugate. Secondly, the propagation time $\mathbf{A} = \{A(t), t \in \mathbb{R}\}$ is assumed independent of the emitted process $\mathbf{Z}$ and with stationary statistics of order one and two defined by

$$\begin{cases} \psi(\omega) = E[e^{i\omega A(t)}] \\ \phi(\tau,\omega) = E[e^{i\omega(A(t)-A(t-\tau))}]. \end{cases}$$

The characteristic functions $\psi(\omega)$ and $\phi(\tau,\omega)$ are the Fourier transforms of the probability laws of the random variables (r.v) $A(t)$ and $A(t) - A(t-\tau)$. Then, the process $\mathbf{U} = \{U(t), t \in \mathbb{R}\}$ defined in (1) by $U(t) = Z(t - A(t))$ can be written as the sum of two uncorrelated processes $\mathbf{G} = \{G(t), t \in \mathbb{R}\}$ and $\mathbf{V} = \{V(t), t \in \mathbb{R}\}$ such as [7]

$$\begin{cases} U(t) = G(t) + V(t) \\ s_G(\omega) = |\psi(\omega)|^2 s_Z(\omega) \\ E[V(t)V^*(t-\tau)] = \int_{-\infty}^{\infty} (\phi(\tau,\omega) - |\psi(\omega)|^2) e^{i\omega\tau} s_Z(\omega)\,d\omega \end{cases} \tag{3}$$

where $s_G(\omega)$ is the spectral density of $\mathbf{G}$. From the point of view of the signal theory, $\mathbf{G}$ is the result of a linear invariant filtering of $\mathbf{Z}$ in a filter of complex gain (or frequency response) $\psi(\omega)$. The process $\mathbf{V}$ is not correlated with $\mathbf{Z}$ and consequently with $\mathbf{G}$. Then, $\mathbf{V}$ has to be considered as a noise due to the loss of coherence of the wave. In $\mathbf{U}$, the noise $\mathbf{V}$ is added to the filtered process $\mathbf{G}$ which holds the information on $\mathbf{Z}$. Because the total powers of $\mathbf{U}$ and $\mathbf{Z}$ are equal, the appearance of the process $\mathbf{V}$ implies that the total power of $\mathbf{G}$ is less than this of $\mathbf{Z}$. This fact can be interpreted as a weakening of the emitted process.

(2) To fix the ideas and to give an accurate meaning to the used parameters, we place ourselves in the Gaussian frame together through the shape of the spectrum $s_Z(\omega)$ and through the probability law of the propagation delay $\mathbf{A}$

$$\begin{cases} s_Z(\omega) = \frac{1}{a\sqrt{2\pi}} \exp[-(\omega-\omega_0)^2/2a^2] \\ \psi(\omega) = \exp[im\omega - \sigma^2\omega^2/2] \\ \phi(\tau,\omega) = \exp[-\sigma^2\omega^2(1-\rho(\tau))] \\ \sigma^2\rho(\tau) = \mathrm{Cov}[A(t), A(t-\tau)]. \end{cases} \tag{4}$$

The spectrum $s_Z(\omega)$ is normalized to a unit total power, and $\omega_0 > 0$ is the central frequency. a is the spectral half-width of the emitted process $\mathbf{Z}$. $m, \sigma, \rho(\tau)$ are the parameters of the propagation delay $\mathbf{A}$. m is the mean (which does not intervene anywhere in what follows, besides in a crosscorrelation function). σ (the standard deviation) gives the range of $\mathbf{A}$, and $\rho(\tau)$ (a correlation coefficient) measures the celerity of variations of $\mathbf{A}$. Hypotheses on $\rho(\tau)$ will be taken in Section 4. The formulas of (4) will be used in the following Sections to study both processes $\mathbf{G}$ and $\mathbf{V}$ which result in the randomness of the propagation time.

3. The redshift

(1) $\mathbf{G}$ is the part of the received process $\mathbf{U}$ which is (linearly) linked to the emitted process $\mathbf{Z}$. Using the expressions of $s_G(\omega), \psi(\omega)$ given in (3) and (4), we obtain

$$\begin{cases} s_G(\omega) = \frac{1}{a\sqrt{2\pi}} \exp[-\omega_G\omega_0\sigma^2 - (\omega-\omega_G)^2/2b^2] \\ \omega_G = \omega_0/(1+2a^2\sigma^2), \quad b = a/\sqrt{1+2a^2\sigma^2}. \end{cases} \tag{5}$$

ω_G is the gravity center of $s_G(\omega)$. Because $\omega_G < \omega_0$, $\mathbf{G}$ is redshifted in respect to $\mathbf{Z}$. The redshift parameter z is defined by

$$z = \frac{\lambda_G - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega_G}{\omega_G} = 2a^2\sigma^2 \tag{6}$$

and does not depend on ω_0 ($\lambda = 2\pi c/\omega$ is the wavelength). Obviously, it is a significant property. Because the parameters a (the half-width of the emitted ray) and σ (the standard deviation of the propagation time) are arbitrary quantities, the model ends to an arbitrary large redshift. Furthermore, $b = a/\sqrt{1+z} < a$, which shows that the ray is narrowed in absolute value, but not in relative value because

$$\frac{b}{\omega_G} = \frac{a}{\omega_0}\sqrt{1+z} > \frac{a}{\omega_0}.$$

For small z, both quantities a and b can be confused. The total power P_G of the ray is given by (remind that the emitted power P_Z is unit)

$$P_G = \int_{-\infty}^{\infty} s_G(\omega)\,d\omega = \frac{1}{\sqrt{1+z}} \exp\left[-\frac{(\omega_0\sigma)^2}{1+z}\right]. \tag{7}$$

Note that $P_G = \exp[-(\omega_0\sigma)^2]$ is for the pure monochromatic case ($a = 0$) with no redshift ($z = 0$), whatever the propagation time randomness. From the signal theory point of view, $\mathbf{G}$ is a linear (Gaussian) filtering which highlights a part of the $\mathbf{Z}$ spectrum (around ω_G) and cancels the power outside an interval around ω_G. We will discuss this point.

(2) (7) can be written as

$$P_G = \frac{1}{\sqrt{1+z}} \exp\left[-\frac{1}{2\delta^2}\frac{z}{1+z}\right], \quad \begin{cases} \delta = a/\omega_0 \\ z = 2(a\sigma)^2. \end{cases} \tag{8}$$

δ represents the relative half-width of the emitted wave. Table 1 below gives the values of $-10\log P_G$ (P_G in $-$dB) as a function of z (the redshift parameter) and of δ (the relative raywidth). P_G is formally independent of ω_0, but, in real situations, it is possible that each ray has its particular width, so that P_G may be linked to ω_0 through a. For small z, a good approximation is given by $P_G \cong \exp[-z/2\delta^2]$.

Table 1 shows that a sizeable redshift for $\mathbf{G}$ seems possible only when the spectral width of $\mathbf{Z}$ is sufficient (the missing values are above 500). Nevertheless, we remind