Available online at www.sciencedirect.com
ScienceDirect
Optics Communications 281 (2008) 2893–2906
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

A collective variable approach for optical solitons in the cubic–quintic complex Ginzburg–Landau equation with third-order dispersion

S.I. Fewo [a], T.C. Kofane [a,b,*]

[a] Département de Physique, Faculté des Sciences, Université de Yaoundé I, B.P 812 Yaoundé, Cameroon
[b] The Abdus Salam International Centre for Theoretical Physics, P.O. Box 506 – Strada Costiera, II-34014 Trieste, Italy

Received 23 June 2007; received in revised form 26 November 2007; accepted 14 December 2007

Abstract

Considering the theory of electromagnetic, especially from the Maxwell equations, a basic equation modeling the propagation of ultrashort optical solitons in optical fibers is derived, namely a cubic–quintic complex Ginzburg–Landau equation (CQGLE) with third-order dispersion (TOD). Considering this one-dimensional CQGLE, we derive the equations of motion of pulse parameters called collective variables (CVs), of a pulse propagating in dispersion-managed (DM) fiber optic-links. Equations obtained are investigated numerically in order to view the evolution of pulse parameters along the propagation distance. A fully numerical simulation of the CQGLE finally tests the results of the CV theory. It appears chaotic pulses, attenuate pulses and stable pulses under some parameter values.

Keywords: Complex Ginzburg–Landau equation; Dissipative systems; Dispersion-managed solitons; Optical fibers; Collective variables

1. Introduction

It has been known for a long time that localized solutions can exist for some nonlinear partial differential equations. The best known example of such solutions is the soliton [1,2]. The concept of solitons describes various physical phenomena ranging from solitary waves on a water surface to ultrashort optical pulses from a laser. The study of optical solitons is interesting for its fundamental aspect as well as for its important applications [3–5]. The generation of a train of soliton pulses from continuous wave light in optical fibers was first suggested by Hasegawa [3,4], and first realized experimentally by Mollenauer et al. [5]. Optical solitons may soon be the primary carriers for long- and short- distance information transmission because, unlike pulses in a linear dispersive fiber, solitons are self-confined, propagating for a long-distance without changing shape [5]. A well-known example of an equation which admits pulse-like soliton solutions is the nonlinear Schrödinger equation (NLSE) [6]. Optical solitons propagating in optical fibers may induce a host of nonlinear phenomena such as parametric wave mixing, stimulated Raman scattering, or self-steepening [7–10]. Also, among other related topics, the modulational instability arising in the cubic–quintic NLSE has been investigated recently for pulse propagation [11].

From the standpoint of possible applications, the optical pulse transmission line is an useful example. For long-distance communication systems, compensating for attenuation of pulses inherent in the fiber, is an important issue. One approach

* Corresponding author. Address: Département de Physique, Faculté des Sciences, Université de Yaoundé I, B.P 812 Yaoundé, Cameroon. Tel.: +237 75476443.
E-mail addresses: sergefewo@yahoo.fr (S.I. Fewo), tckofane@yahoo.com (T.C. Kofane).

0030-4018/$ - see front matter
doi:10.1016/j.optcom.2007.12.035

is the use of periodically spaced phase-sensitive amplifiers. Each of such amplifiers exhibits an associated reference phase. The part of the signal in phase with this reference phase is amplified, while the out-of-phase component is attenuated [12–14]. In the second approach, the losses can be compensated by the erbium-doped amplifiers [8]. When frequency and intensity dependent gain and loss have to be taken into account, the governing equation is the cubic CGLE [15]. One of the most analyzed is the quintic CGLE, which plays an important role in many branches of physics, including binary fluid convection, phase transition and many phenomena in optics where it is often used to model several types of passively mode-locked lasers with saturable absorbers, parametric oscillators, transverse soliton effects in wide aperture lasers and wave propagation in nonlinear optical fibers with gain and spectral filtering [16–18]. The quintic CGLE has also been used to describe pattern formation near a hopf bifurcation and has become a paradigmatic model for the study of spatiotemporal chaos. Recently, this equation has also been theoretically and numerically investigated, where it has been noted a stabilizing effect of the nonlinear refractive index [19]. The one-dimensional quintic CGLE possesses a rich variety of soliton solutions, including coherent structures such as pulses (solitary waves), fronts (shock waves), sources, sinks [20,21] and new fascinating types of pulsating soliton. Namely, different types of localized pulsating solutions such as plain pulsating, exploding (erupting), creeping and chaotic solutions have been found [22].

To enlarge the information capacity, it is necessary to transmit ultrashort optical soliton pulses at high bit rate in the subpicosecond and femtosecond regime. However, several new effects, such as third-order dispersion (TOD) and others become important. The TOD effect has been considered in many physical systems modeled by the well-known NLSE. On the contrary, very few works including the complex TOD parameter have been done. Let's mention that the evolution of the pulsating, erupting and creeping solitons [22] under the effects of TOD has been analyzed by Song et al. [23]. They have found that even small TOD can eliminate the periodicity of the pulsating and creeping solitons and transform them into new fixed-shape solitons, while for the erupting soliton, the TOD can cause it to crack on only one side or to transform into chaotic structure. As pointed out by Song et al. [23], these new fixed-shape solitons are meaningful for practical application, such as to realize experimentally the undistorted transmission of femtosecond pulses in optical fibers. Thus, it is quite interesting to carry out some works on an important equation modeling many complex phenomena, in particular the propagation of optical solitons in optical fibers, namely the mentioned CQGLE with TOD. Therefore, the equation of interest for the propagation in optical fibers is the CQGLE with complex TOD parameter, which reads in the dimensionless form

$$i\psi_z + (p_r(z) + ip_i(z))\psi_{tt} + (q_r(z) + iq_i(z))|\psi|^2\psi + (c_r(z) + ic_i(z))|\psi|^4\psi = i(\gamma_r(z) + i\gamma_i(z))\psi + i(d_{3r} + id_{3i})\psi_{ttt} \tag{1}$$

where $\psi(z,t)$ is the envelope amplitude of the electric field, t is the retarded time and z the propagation distance. The parameters p_r, p_i, q_r, q_i, c_r, c_i, γ_r, γ_i, d_{3r} and d_{3i} are real constants. The parameter p_r measures the wave dispersion, p_i the spectral filtering. The parameters q_r and q_i represent the nonlinear coefficient and the nonlinear gain-absorption coefficient, respectively. We noticed that nonlinear gain helps to suppress the growth of radiative background (linear mode) which always accompanies the propagation of nonlinear stationary pulses in real fiber links. The parameters c_r and c_i stand for the higher-order correction terms to the nonlinear refractive index and the nonlinear amplification–absorption, respectively. The parameters γ_r and γ_i represent the linear gain and the frequency shift, respectively. The complex parameter d_3 $(d_{3r} + id_{3i})$ is the TOD coefficient.

Recent research demonstrates that DM solitons for data transmission design will substantially increase the capacity of the fiber-optic link [23–25]. Basically, the DM technique consists of using a transmission line with a periodic map, such that each period is built up by two types of fiber of generally different lengths and opposite group velocity dispersion (GVD) [24–31]. Dispersion-managed solitons are attracting considerable interest in optical communication systems because of their superb characteristics which are not observed with conventional solitons. Furthermore, transmission performance are sometimes degraded by perturbations (as linear waves). Actually, the propagation of pulse in fiber links is always destabilized. The use of transmission control methods such as guiding filters [32–34] were studied in order to stabilize DM solitons propagation. In addition, it is shown that nonlinear gain is expected to be more beneficial to DM solitons than to conventional solitons [35], in order to stabilize DM soliton transmission. Sufficiently strong periodic DM allows for stationary propagation of nonlinear return-to-zero pulses with finite energy when the average dispersion is close or even equal to zero [29–31,36]. For this reason, as can be seen in Eq. (1), the constant parameters p, q, c and γ are commonly expressed as functions of z (without loosing their constant character), i.e., $p = p_r(z) + ip_i(z)$, $q = q_r(z) + iq_i(z)$, $c = c_r(z) + ic_i(z)$ and $\gamma = \gamma_r(z) + i\gamma_i(z)$, respectively.

We note that various analytical treatments have been proposed [37–39] in order to describe the main characteristics of the pulse evolution. Among these treatments, a well studied method is the so-called variational method involving a Gaussian trial function which provides explicit (although approximate) analytical expressions for the pulse compression/decompression factor, the maximum pulse amplitude and the induced frequency chirp [37]. The purpose and optimization of the soliton transmission systems are fundamentally based on the general particle-like nature of solitons. This particle-like behaviour has led to the formulation of collective variable (CV) techniques for DM soliton, to obtain more insight into