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Generalized beams in *ABCDGH* systems

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Abstract

For a generalized beam at the source plane passing through co-located aperture and a propagation path consisting of an off-axis x–y asymmetric *ABCDGH* system, the receiver plane irradiance expression is derived using the Collins integral. A collection of source beam profiles that are obtainable from the generalized beam formulation are illustrated. Plots are given for viewing the progress of selected generalized beam types along the propagation axis, containing a single thin lens, co-centric and misaligned in the x-direction. The received power falling onto a finite aperture surface is calculated for various misalignment situations.

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1. Introduction

The expression of Huygens Fresnel diffraction integral in terms of *ABCDGH* matrix elements was initiated by Collins [1], hence bearing his name. Since then, *ABCDGH* matrix formulation has become a versatile tool for studying the propagation characteristics of beams in free apace. More popularly used version is the reduced *ABCD* system, where G and H elements of the matrix are eliminated, if there are no misalignments of optical items encountered on the propagation paths, or no axes transformations are envisaged.

Cai and Lin have made several contributions in this area by investigating the propagation properties of beams with elliptical flat-topped profiles [2], hollow elliptical Gaussian beams [3] and partially coherent elliptical flattened Gaussian beams [4] for both aligned and misaligned optical systems. A series of papers were published [5–8] in which *ABCDGH* matrices for a wide variety of optical components and propagation environments were derived and listed along with the application of the diffraction integral for *ABCDGH* matrix arrangements. Further studies were conducted for the cases of specialized beams such as Bessel–Gaussian [9], Hermite–Laguerre–Gaussian [10] and Gaussian-Schell model [11] beams passing through *ABCD* matrices. The effects of tilt and jitter of on-path optical elements within the context of *ABCD* ray matrices were investigated in [12,13]. *ABCD* transfer matrix was applied in [14] to study the propagation of elegant Hermite-cosine beams for unapertured and apertured cases. *ABCD* systems were also used for the treatment of off-axial beams [15]. It was shown in [16] that *ABCDGH* diffraction integral is also valid for a curved optical axis.

In the more general sense, there are two methods to treat the propagation of a light beam in a misaligned optical system, one is *ABCDGH*, another is *ABCDEFGH* [1,3]. Here it is the former that we study. Recently, we presented [17] results on the investigation of Hermite hyperbolic/sinusoidal beams propagating in *ABCD* systems containing x–y asymmetry. In this paper, we make the following extensions to our earlier work [17]:

(a) Via the concept of generalized beam, ability to care for a wide range properties and selection of beams under one single formulation,

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(b) the use of $ABCDGH$ matrix instead of the conventional $ABCD$, thus the capability to account for optical component and axes misalignments,
(c) calculation of power falling onto an aperture placed on the receiver plane and its variation against optical component misalignments.

2. Formulation

The source field expression of the generalized beam is [18]:

$$u_s(\mathbf{s}) = u_s(s_x, s_y) = \sum_{\ell=1}^{N}\sum_{(n,m)} A_{\ell nm}\exp(-\mathrm{i}\theta_{\ell nm})H_n(a_{x\ell n}s_x + b_{x\ell n}) \times \exp[-(0.5k\alpha_{x\ell n}s_x^2 + \mathrm{i}V_{x\ell n}s_x)]H_m(a_{y\ell m}s_y + b_{y\ell m}) \times \exp[-(0.5k\alpha_{y\ell m}s_y^2 + \mathrm{i}V_{y\ell m}s_y)], \quad (1)$$

where (s_x, s_y) designates the decomposition of the vector $\mathbf{s}$ into x and y components. $A_{\ell nm}$ and $\theta_{\ell nm}$ are, respectively the amplitude and the phase of the ℓnm component of the source field, $H_n(a_{x\ell n}s_x + b_{x\ell n})$ and $H_m(a_{y\ell m}s_y + b_{y\ell m})$ are Hermite polynomials defining the beam distribution for s_x and s_y directions, where n and m are the order, $a_{x\ell n}$ and $a_{y\ell m}$ stand for the width, $b_{x\ell n}$ and $b_{y\ell m}$ are the complex displacement parameters,

$$\alpha_{x\ell n} = 1/(k\alpha_{sx\ell n}^2) + \mathrm{i}/F_{x\ell n}, \quad \alpha_{y\ell m} = 1/(k\alpha_{sy\ell m}^2) + \mathrm{i}/F_{y\ell m}. \quad (2)$$

Here $\alpha_{sx\ell n}$ and $\alpha_{sy\ell m}$ are Gaussian source sizes, $F_{x\ell n}$ and $F_{y\ell m}$ are the source focusing parameters along s_x and s_y directions, $k = 2\pi/\lambda$ is the wave number with λ being the wavelength and $\mathrm{i} = (-1)^{0.5}$. $V_{x\ell n}$ and $V_{y\ell m}$ are the complex parameters used to create physical location displacement and phase rotation for the source field or a combination of both [19]. Furthermore by appropriately setting them as purely real or imaginary quantities, and implementing a summation over two terms, i.e. $N = 2$, we are successively able to attain sinusoidal and hyperbolic Gaussian beams discussed earlier [17].

The source field described by Eq. (1), after traveling in the free space environment containing optical items, whose collection is represented by the optical box, will reach the receiver plane located at an axial distance of L. Fig. 1 illustrates the associated geometry. With the help of Collins integral, the field at the receiver plane, $u_r(\mathbf{p}) = u_r\ (p_x, p_y)$, is attained [1,7] in the way shown below

$$u_r(\mathbf{p}) = u_r(p_x, p_y) = -\mathrm{i}k\exp\{-0.5\mathrm{i}[H_x(B_xG_x - A_xH_x)/B_x + H_y(B_yG_y - A_yH_y)/B_y]/k\} \times \exp[\mathrm{i}(0.5kD_xp_x^2 + H_xp_x)/B_x] \times \exp[\mathrm{i}(0.5kD_yp_y^2 + H_yp_y)/B_y]/(2\pi B_x^{0.5}B_y^{0.5}) \times \int_{t_{y1}}^{t_{y2}}\int_{t_{x1}}^{t_{x2}} \mathrm{d}^2\mathbf{s}\,u_s(\mathbf{s})\exp[0.5\mathrm{i}k(A_xs_x^2/B_x + A_ys_y^2/B_y)] \times \exp\{\mathrm{i}[(B_xG_x - A_xH_x - kp_x)s_x/B_x + (B_yG_y - A_yH_y - kp_y)s_y/B_y]\}. \quad (3)$$

As understood from Fig. 1, $\mathbf{p}$ is the positional vector on the receiver plane, also decomposed into $\mathbf{p} = (p_x, p_y)$. The terms, t_{x1}, t_{x2}, t_{y1} and t_{y2} appearing in the integral limits of Eq. (3) for s_x and s_y directions, refer to the dimensions of a rectangular aperture that may be placed on an arbitrary position of the source plane. The parameters A_x, A_y, B_x, B_y, D_x, D_y, G_x, G_y, H_x and H_y are the respective x and y elements of the $ABCDGH$ matrix defining the optical content of the involved propagation geometry, such that if for example the box illustrated in Fig. 1 were to be a single thin lens, misaligned in the x-direction by an amount, x_0, having a tilt of θ_x with respect to x-axis, and this lens was characterized by the parameters α_{ex} and α_{ey}, then the following matrices would hold for the entire link from source to receiver

$$\begin{pmatrix} A_x & B_x & 0 \\ C_x & D_x & 0 \\ G_x & H_x & 1 \end{pmatrix} = \begin{pmatrix} 1+\mathrm{i}\alpha_{ex}\cos\theta_xL_2 & L_1+L_2(1+\mathrm{i}\alpha_{ex}\cos\theta_xL_1) & 0 \\ \mathrm{i}\alpha_{ex}\cos\theta_x & 1+\mathrm{i}\alpha_{ex}\cos\theta_xL_1 & 0 \\ \mathrm{i}kx_0\alpha_{ex}\cos\theta_x & \mathrm{i}kx_0\alpha_{ex}\cos\theta_xL_1 & 1 \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} A_y & B_y & 0 \\ C_y & D_y & 0 \\ G_y & H_y & 1 \end{pmatrix} = \begin{pmatrix} 1+\mathrm{i}\alpha_{ey}L_2/\cos\theta_x & L_1+L_2(1+\mathrm{i}\alpha_{ex}L_1/\cos\theta_x) & 0 \\ \mathrm{i}\alpha_{ex}/\cos\theta_x & 1+\mathrm{i}\alpha_{ey}L_1/\cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

The $ABCDGH$ approach is able to account for any number of cascaded sections with different compositions. Thus, Eqs. (4) and (5) are obtained by multiplying the individual $ABCDGH$ matrix representation of each section, namely L_1, the lens and L_2, in the reverse order [20]. The parameters α_{ex} and correspondingly α_{ey} are related to effective transmission apertures, α_{esx} and α_{esy} and focal lengths, F_{ex} and F_{ey}, of the thin lens via

$$\alpha_{ex} = 1/(k\alpha_{esx}^2) + \mathrm{i}/F_{ex}, \quad \alpha_{ey} = 1/(k\alpha_{esy}^2) + \mathrm{i}/F_{ey}. \quad (6)$$

To solve the Collins integral, first Eq. (1) is substituted for $u_s(\mathbf{s})$ in Eq. (3), then all Hermite polynomials are expanded in series, subsequently a term by term integration based on the derivation steps outlined in [17] is performed, in the end, leading to the following result

$$u_r(\mathbf{p}) = -\mathrm{i}k\exp\{-0.5\mathrm{i}[H_x(B_xG_x - A_xH_x)/B_x + H_y(B_yG_y - A_yH_y)/B_y]/k\} \times \exp[\mathrm{i}(0.5kD_xp_x^2 + H_xp_x)/B_x] \times \exp[\mathrm{i}(0.5kD_yp_y^2 + H_yp_y)/B_y]/(2\pi B_x^{0.5}B_y^{0.5}) \times \sum_{\ell=1}^{N}\sum_{(n,m)} A_{\ell nm}\exp(-\mathrm{i}\theta_{\ell nm})\exp\{-0.5[Q_{Vx}^2/(B_xQ_{Ax}) + Q_{Vy}^2/(B_yQ_{Ay})]\}S_{Vx}S_{Vy}, \quad (7)$$