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An experimental evaluation of state estimation with fluid dynamical models in process tomography

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Abstract

In this paper we perform an experimental evaluation of a state estimation approach in process tomography. In particular, we concentrate on the case where a system with rapidly moving target is imaged with electrical impedance tomography. We show experimental results which confirm that non-stationary estimation with proper fluid dynamical models works well even in cases where stationary estimates are completely useless.

Keywords: Process tomography; Electrical impedance tomography (EIT); State; Estimation

1. Introduction

In process tomography the aim is to monitor industrial processes on the basis of indirect observations from the boundary of the target. Techniques used in process monitoring are basically the same as in medical imaging. The variety of modalities include electrical, optical, X-ray and nuclear tracer techniques. In many applications one is often interested in imaging targets that change very rapidly. That is the case for example in mixing [1–3] and mass transport [4–6] applications. If the target changes at a very high rate in comparison with the rate of measurements, the stationary tomographic reconstructions are usually inadequate. In such cases the reconstructions may be improved by using the state estimation approach [7]. In state estimation approaches the temporal behavior of the target is modeled and the model is used in the image reconstruction to provide further information on the target.

In [8,9] we applied the state estimation approach to electrical impedance tomography (EIT) in the case of moving fluids. We modeled the dynamics of the system using the Navier–Stokes equations and the convection–diffusion (CD) equation. The resulting stochastic evolution model together with the observation model of EIT constituted the state-space representation of the system. The reconstruction of conductivity distributions was based on this representation, and the algorithms used in the image reconstruction were of the Kalman Filter type. The cited numerical studies have shown that the use of state estimation with suitable evolution models may improve the estimates considerably.

The aim of this paper is to provide an experimental validation of the state estimation approach in EIT. State estimation has already been applied to real EIT data in papers [10–12]. However, in these papers the random walk model has been used as the evolution model, instead of more realistic models.

In this paper we consider an experimental EIT measurement set up consisting of a saline-filled tank with a rotating impeller and a saline-filled table tennis ball floating in the tank. EIT measurements are carried out and the state estimation approach with an appropriate evolution model is used in the reconstruction of the conductivity distribution within the tank. The evolution model is based on approximate fluid dynamical modeling of the system and on a convection–diffusion model. The time-dependent internal structure, i.e. the impeller, is also taken into account in the reconstruction.

2. State estimation in EIT

In EIT conductive targets are monitored using electrical boundary measurements. Electric current is injected into the

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target using electrodes attached to the boundary of the target. The resulting voltages between the electrodes are measured and the internal conductivity distribution is reconstructed on the basis of the voltage measurements.

The reconstruction problem has a nature of an ill-posed inverse problem – even in a stationary case – and hence special estimation methods and appropriate modeling of the measurements are always required. An additional difficulty arises from time-varying targets because the voltage measurements at different times do not correspond to the same target. Hence, the use of data corresponding to multiple current injection patterns may lead to severe inaccuracies. On the other hand, when using ordinary (stationary) reconstruction algorithms, a single current injection does not (usually) yield adequate information for reconstructing the conductivity distribution. In order to tackle the problem of non-stationarity, we write EIT in state-space formalism, and utilize fluid dynamical modeling in the reconstruction.

In Section 2.1 we review the observation model of EIT. We also point out the difference between the stationary and the non-stationary reconstruction problems. In Section 2.2 we introduce one fluid dynamical model, the convection–diffusion model, which is used for modeling the time-dependence of the target in this paper. Finally, in Section 2.3 we write the reconstruction problem of EIT in the form of a state estimation problem and introduce two algorithms, Kalman filter and fixed-lag Kalman smoother that can be used for solving the problem.

2.1. Observation model

In EIT, alternating currents I_ℓ are applied to electrodes on the surface of the object, and the resulting voltages between different pairs of electrodes are measured. The conductivity distribution σ within the object is reconstructed on the basis of the voltage measurements. We model the observations by using the complete electrode model (CEM) which is known to be so far the most accurate model used in EIT [13]. The CEM consists of the following equations:

$$\nabla\cdot(\sigma\nabla u)=0,\quad x\in\Omega \tag{1}$$

$$u+z_\ell\sigma\frac{\partial u}{\partial n}=U_\ell,\quad x\in e_\ell,\quad \ell=1,2,\ldots,L \tag{2}$$

$$\int_{e_\ell}\sigma\frac{\partial u}{\partial n}\,\mathrm{d}S=I_\ell,\quad x\in e_\ell,\ \ \ell=1,2,\ldots,L \tag{3}$$

$$\sigma\frac{\partial u}{\partial n}=0,\quad x\in\partial\Omega\backslash\cup_{\ell=1}^{L}e_\ell \tag{4}$$

where $u=u(x)$ is the electric potential, e_ℓ the ℓth electrode, z_ℓ the contact impedance between the ℓth electrode and contact material, U_ℓ the potential on ℓth electrode, I_ℓ the injected current, n the outward unit normal and L is the number of the electrodes. In addition, the charge conservation law

$$\sum_{\ell=1}^{L}I_\ell=0 \tag{5}$$

needs to be fulfilled. Further, in order to determine uniquely the potentials u and U_ℓ based on the CEM, the reference level of potential needs to be fixed. This is achieved, e.g. by writing

$$\sum_{\ell=1}^{L}U_\ell=0 \tag{6}$$

We approximate the complete electrode model numerically using the finite element method (FEM), see [13–15]. The resulting finite dimensional observation model is of the form

$$V_t=U_t(\sigma_t)+\upsilon_t, \tag{7}$$

where t is a discrete time index, V_t the observed voltages resulting from one current injection pattern, $\sigma_t\in\mathbb{R}^{\mathrm{N}}$ a finite-dimensional approximation of the conductivity distribution at time t, $U_t(\sigma_t)$ a non-linear mapping between the conductivity and voltages and υ_t is observation error. If we further linearize the observation model (7) we obtain

$$V_t=U_t(\sigma_*)+J_t(\sigma_t-\sigma_*)+\upsilon_t, \tag{8}$$

where the matrix J_t is the Jacobian corresponding to the model $U_t\,(\sigma)$ and the vector σ_* is a linearization point.

In a *stationary case* it is assumed that the conductivity within the target does not change during the measurements, i.e., $\sigma_1=\sigma_2=\cdots=\sigma_{\mathrm{T}}=:\sigma$. Thus, the observation models corresponding to different current injection patterns $I_1, I_2, \ldots, I_{\mathrm{T}}$ can be combined into one stationary model

$$V=U(\sigma)+\upsilon, \tag{9}$$

where

$$V=\begin{bmatrix}V_1\\V_2\\\vdots\\V_{\mathrm{T}}\end{bmatrix},\qquad U(\sigma)=\begin{bmatrix}U_1(\sigma)\\U_2(\sigma)\\\vdots\\U_{\mathrm{T}}(\sigma)\end{bmatrix}\quad\text{and}\quad \upsilon=\begin{bmatrix}\upsilon_1\\\upsilon_2\\\vdots\\\upsilon_{\mathrm{T}}\end{bmatrix}. \tag{10}$$

Thus, in a stationary case the conductivity distribution σ is reconstructed based on the observation model (9). The reconstruction problem is known to be ill-posed, and hence spatial prior information of the target is needed to be utilized in the reconstruction. Spatial prior information is typically incorporated into the problem formulation by using the Tikhonov regularization scheme. The regularized solution is of the form

$$\widehat{\sigma}=\arg\min_{\sigma}\{||V-U(\sigma)||^2+\alpha R(\sigma)\},$$

where $R(\sigma)>0$ is a functional that favors certain *a priori* known features in the minimization and $\alpha>0$ is a regularization parameter. The regularizing functional is usually selected as $R(\sigma)=||L(\sigma-\sigma_{\mathrm{prior}})||^2$, where the regularization matrix L is typically a discrete differential operator, a choice which yields smooth estimates, and σ_{prior} is a prior guess for σ.

In the non-stationary case, however, the assumption that the conductivity distribution is non-varying during a set of different current patterns is no longer valid and therefore the stationary