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Changes in the spectrum of twist anisotropic Gaussian Schell-model beams propagating through turbulent atmosphere

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Abstract

Based on the extended Huygens–Fresnel principle, the spectrum of twist anisotropic Gaussian Schell-model (TAGSM) beams propagating through turbulent atmosphere is derived analytically by using the partially coherent complex curvature tensor. The relative spectral shift of TAGSM beams propagating through turbulent atmosphere is closely related with the strength of atmospheric turbulence, the beam's parameter and the radial coordinate. The on-axis spectral shift of TAGSM beams propagating through turbulent atmosphere changes from blue shift to red shift with the increasing of the propagation distance z, and at a certain propagation distance z, there exists a rapid transition of the spectrum at the critical position r_c.

Keywords: Twist anisotropic Gaussian Schell-model beams; Spectral shift; Partially coherent complex curvature tensor; Turbulent atmosphere

1. Introduction

In 1986 [1], Wolf pointed out that the spectrum of light which is emitted from a spatially partially coherent source with a wide spectral bandwidth undergoes spectral shift during propagation. This phenomenon is referred to as correlation-induced spectral changes, which take place even if the light propagation through free space [2]. Later on, it was found that the spectrum of the partially coherent beam in the diffracted field will be changed as well [3,4]. This phenomenon is referred to as diffraction-induced spectral changes. The physical mechanism of the spectral changes is different from Doppler effects.

On the other hand, the atmospheric turbulence can significantly alert the prosperities of light propagating through it [5–7]. In fact, turbulent atmosphere is a diffracted field. So partially coherent beams propagating through turbulent atmosphere have spectral shift, caused by the correlation of the source and the diffraction of atmospheric turbulence. In the domain of partially coherent beams, the twist anisotropic Gaussian Schell-model (TAGSM) beams represents a general type of partially coherent beams. The theory commonly used to study the propagation and transformation of TAGSM beams is based on the Wigner distribution function [8]. However, this method is not appropriate for study spectral shift phenomena. In 2002, Lin et al. introduced a new method called the tensor method [9–12] for treating the propagation and transformation of TAGSM beams, which is more convenient.

The purpose of the present paper is to study the spectral changes of TAGSM beams passing through turbulent atmosphere by using tensor method. The influences of the turbulence, beam's initial parameter and the radial coordinate are studied analytically and numerically.

2. Spectrum of partially coherent TAGSM beams propagating through turbulent atmosphere in terms of the tensor method

In this section, we will outline briefly the tensor method for partially coherent TAGSM beams. The cross-spectral density of the TAGSM beams located on the input plane $z = 0$ takes the following form [2]

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$$W(\tilde{r},0;\omega) = S_0(\omega)\exp\left[\frac{i\omega}{2c}\tilde{r}^T M_s^{-1}\tilde{r}\right] \tag{1}$$

where M_s^{-1} is a 4×4 partially coherent complex curvature tensor given by

$$M_s^{-1} = \begin{bmatrix} R^{-1} - \left(\frac{ic}{2\omega}\right)(\sigma_I^2)^{-1} - \left(\frac{ic}{\omega}\right)(\sigma_g^2)^{-1} & \left(\frac{ic}{\omega}\right)(\sigma_g^2)^{-1} + \mu J \\ \left(\frac{ic}{\omega}\right)(\sigma_g^2)^{-1} + \mu J^T & -R^{-1} - \left(\frac{ic}{2\omega}\right)(\sigma_I^2)^{-1} - \left(\frac{ic}{\omega}\right)(\sigma_g^2)^{-1} \end{bmatrix} \tag{2}$$

where σ_I^2 is a transverse spot width matrix, μ is a real-valued constant named the twist factor, σ_g^2 is a transverse coherence width matrix, R^{-1} is a wave front curvature matrix. σ_I^2, σ_g^2, R^{-1} are all 2×2 matrices with transposition symmetry, given by

$$(\sigma_I^2)^{-1} = \begin{bmatrix} 1/\sigma_{I11}^2 & 1/\sigma_{I12}^2 \\ 1/\sigma_{I12}^2 & 1/\sigma_{I22}^2 \end{bmatrix}, \quad (\sigma_g^2)^{-1} = \begin{bmatrix} 1/\sigma_{g11}^2 & 1/\sigma_{g12}^2 \\ 1/\sigma_{g12}^2 & 1/\sigma_{g22}^2 \end{bmatrix},$$

$$R^{-1} = \begin{bmatrix} 1/R_{11} & 1/R_{12} \\ 1/R_{12} & 1/R_{22} \end{bmatrix}.$$

J is a transposition anti-symmetry matrix given by $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Based on the extended Huygens–Fresnel integral, we can obtain the following propagation formula for the cross-spectral density of a partially coherent beam through a turbulent atmosphere [11]

$$\begin{aligned} W_0(\tilde{\rho},z;\omega) = {} & \frac{k^2}{4\pi^2 z^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W_s(\tilde{r},0;\omega) \\ & \times \exp\left[-\frac{ik}{2z}(r_1-\rho_1)^2 + \frac{ik}{2z}(r_2-\rho_2)^2\right] \\ & \times \langle \exp[\Psi(r_1,\rho_1,z;\omega) + \Psi^*(r_2,\rho_2,z;\omega)]\rangle \, dr_1\, dr_2 \end{aligned} \tag{3}$$

$\langle\rangle$denotes averaging over the ensemble of turbulent media, and can be expressed as

$$\begin{aligned} & \langle \exp[\Psi(r_1,\rho_1,z;\omega) + \Psi^*(r_2,\rho_2,z;\omega)]\rangle \\ & = \exp[-0.5 D_\Psi(r_1-r_2)] = \exp\left[-\frac{1}{\rho_0^2}(r_1-r_2)^2\right] \end{aligned} \tag{4}$$

where $D_\psi(r_1-r_2)$ is the phase structure function in Rytov's representation and $\rho_0^2 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent atmosphere. C_n^2 is the structure constant of the refraction index, which decribes how strong the turbulence strength. In the derivation of the Eq. (4), we have employed a quadratic approximation for Rytov's phase structure function.

After some rearrangement, we can express Eq. (3) in the following tensor form [11]

$$\begin{aligned} W_0(\tilde{\rho},z;\omega) = {} & \frac{k^2}{4\pi^2[\det(\widetilde{B})]^{1/2}} \\ & \times \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W_s(\tilde{r},0;\omega) \\ & \times \exp\left[-\frac{ik}{2}(\tilde{r}^T\widetilde{B}^{-1}\tilde{r} - 2\tilde{r}^T\widetilde{B}^{-1}\tilde{\rho} + \tilde{\rho}^T\widetilde{B}^{-1}\tilde{\rho})\right] \\ & \times \exp\left[-\frac{ik}{2}\tilde{r}^T\widetilde{P}\tilde{r}\right] d\tilde{r} \end{aligned} \tag{5}$$

where $\tilde{\rho}^T = (\rho_1^T, \rho_2^T)$, $\widetilde{B} = \begin{pmatrix} zI & 0 \\ 0 & -zI \end{pmatrix}$, $\widetilde{P} = \frac{2}{ik\rho_0^2}\begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$, and I is a 2×2 unit matrix.

Substituting Eq. (1) into Eq. (5), we obtain the following expression for the cross-spectral density of a TAGSM in output plane (after some vector integration and tensor operation)

$$\begin{aligned} W_0(\tilde{\rho},z;\omega) = {} & S_0(\omega)[\det(\widetilde{I} + \widetilde{B}M_s^{-1} + \widetilde{B}\widetilde{P})]^{-1/2} \\ & \times \exp\left[-\frac{ik}{2}\tilde{\rho}^T M_o^{-1}\tilde{\rho}\right] \end{aligned} \tag{6}$$

where $M_o^{-1} = [(M_s^{-1} + \widetilde{P})^{-1} + \widetilde{B}]^{-1}$ is the partially coherent complex curvature tensor in the output plane and $\widetilde{I}$ is a 4×4 unit matrix. Eq. (6) provides a convenient way for analyzing the spectral shift of TAGSM beam in a turbulent atmosphere.

The spectrum of the TAGSM beams propagating through turbulent atmosphere can be obtained by setting $\rho_1 = \rho_2$ in Eq. (6), i.e.

$$\begin{aligned} S(\rho_1,z;\omega) & = W_0(\rho_1 = \rho_2, z;\omega) \\ & = S_0(\omega)[\det(\widetilde{I} + \widetilde{B}M_s^{-1} + \widetilde{B}\widetilde{P})]^{-1/2} \\ & \quad \times \exp\left[-\frac{ik}{2}\tilde{\rho}^T M_o^{-1}\tilde{\rho}\right] \end{aligned} \tag{7}$$

Eq. (7) shows that $S(\rho_1,z;\omega)$ depends on the initial spectrum $S_0(\omega)$, position parameters of observation point (ρ_1, z), the atmospheric turbulence $\widetilde{P}$ and the beams initial parameter M_s^{-1} which including wave front curvature, transverse coherence width, transverse spot width, and twist parameter. We give numerical results in next section.

3. Numerical examples and analysis

3.1. The normalized on-axis spectrum

Numerical calculations were performed to illustrate how the spectrum of TAGSM beams in turbulent atmosphere. Let us assume the initial spectrum $S_0(\omega)$ is of the Lorentz type, i.e.

$$S_0(\omega) = \frac{S_0\delta^2}{(\omega-\omega_0)^2 + \delta^2} \tag{8}$$

where S_0 is a constant, ω_0 is the central frequency of the initial spectrum, and δ is the half-width at half-maximum of the initial spectrum. In the following, the parameters used in the numerical calculation are $\omega_0 = 3.2\times 10^{15}$ rad/s, $\delta = 0.6\times 10^{15}$ rad/s, $S_0 = 1$.

Substituting Eq. (8) into Eq. (7), we can obtain the spectrum of the TAGSM beams at any propagation distance in turbulent atmosphere. Fig. 1 shows the on-axis normalized spectrum $S(\omega)$ at several propagation distances. The parameters used in the calculation are $\mu = 0$,