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Windowed Fourier transform applied in the wavelength domain to process the spectral interference signals

P. Hlubina *, J. Luňáček, D. Ciprian, R. Chlebus

Department of Physics, Technical University Ostrava, 17. Listopadu 15, 708 33 Ostrava-Poruba, Czech Republic

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Abstract

We report on processing the spectral interference signals by a new method based on a windowed Fourier transform applied in the wavelength domain. First, the numerical simulations are performed to demonstrate high precision of the phase retrieval from the spectral signal. Second, the feasibility of the method is confirmed in processing experimental data from a dispersive Michelson interferometer comprising a cube beamsplitter made of BK7 glass. From the retrieved spectral phase difference, the effective thickness of the beamsplitter is determined precisely.

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1. Introduction

White-light spectral interferometry utilizing a broadband source in combination with a standard Michelson or Mach–Zehnder interferometer has been widely used in various research areas including distance and displacement measurements [1–5], profilometry [6–9], material characterization [5,10–16] and optical communications [17,18].

White-light spectral interferometry is based on the observation of spectrally resolved interference fringes (channeled spectrum) obviously recorded far from a stationary phase point [19] and involves measurement of the phase or period of the spectral fringes. The channeled spectrum can be recorded in the frequency (wavenumber) or wavelength domain and can be stationary or non-stationary (without or with variations of the periodicity of the fringes). In profilometry, for example, the sample height profile can be determined by simply differentiating the spectral phase difference (spectral phase) retrieved from a single interferogram recorded in the wavenumber domain [6–9]. Similarly, the group dispersion of a sample under study placed in the interferometer can be obtained by simply differentiating the frequency-dependent spectral phase [17]. If the spectral interferogram is recorded in the wavelength domain (by the CCD array with an equal wavelength sampling), it can be transformed into the frequency domain replacing the wavelength sampling with an equal frequency sampling using suitable algorithms [8,17,20]. The algorithms can also be used to transform non-stationary interferograms into stationary ones. The stationary wavelength domain channeled spectra are still appropriate to retrieve the spectral phase as a function of the wavelength [12,18].

There exist a large number of algorithms suitable for the phase reconstruction from the recorded channeled spectrum. For the spectrum recorded in the frequency (wavenumber) domain, these include a five-point [2,10] and a seven-point [7,9] algorithms, a Fourier transform algorithm [5,8] and especially for the non-stationary channeled spectrum, a wavelet transform algorithm [15]. For the spectrum

* Corresponding author. Tel.: +420 597 323 134; fax: +420 597 323 139.
E-mail address: petr.hlubina@vsb.cz (P. Hlubina).

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recorded in the wavelength domain, the algorithms include a two-point algorithm [11], a Fourier transform algorithm [12,13,18], a phase-locked loop method [12] and especially for the non-stationary channeled spectrum, the Kalman filtering method [14,21]. In the latter case, the spectral phase is reconstructed from the spectral interference signal, which is obtained from the recorded channeled spectrum subtracting the effect of the reference non-channeled spectrum.

In this paper, a new method of processing the non-stationary spectral interference signals is presented, which is based on a windowed Fourier transform [22,23] effectively applied in the wavelength domain. First, we performed the numerical simulations to demonstrate high precision of the phase retrieval from the spectral signal. The feasibility of the method was confirmed in processing experimental data from a dispersive Michelson interferometer to determine precisely the effective thickness of a cube beamsplitter made of BK7 glass. The spectral phase was retrieved and the effective thickness was obtained as the slope of the linear dependence of the retrieved optical path difference on the refractive index of the glass [13]. We utilized the technique to measure the effective thickness of two different cube beamsplitters.

2. Theoretical background

In this section we show how a windowed Fourier transform (WFT) is defined in the spatial domain [22,23] and how the spatial fringe pattern is related to the spectral interference signal. We also demonstrate the feasibility of the WFT in processing the simulated spectral interference signal, including noise, to retrieve precisely the spectral phase.

2.1. Windowed Fourier transform

A 1D spatial domain fringe pattern can be generally expressed as

$$f(x) = a(x) + b(x)\cos[\phi(x)] + n_f(x), \tag{1}$$

where $f(x)$, $a(x)$ and $b(x)$ are the recorded intensity, background intensity, and fringe amplitude, respectively; $\phi(x)$ is the fringe phase distribution, and $n_f(x)$ is the noise, which is assumed to be additive. To retrieve the phase information $\phi(x)$ from the recorded fringe intensity $f(x)$, a WFT will be used. The WFT and inverse WFT can be written as

$$F(u,\xi) = \int_{-\infty}^{+\infty} f(x)g(x-u)\exp(-\mathrm{j}\xi x)\mathrm{d}x, \tag{2}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,\xi)g(x-u)\exp(\mathrm{j}\xi x)\mathrm{d}\xi\,\mathrm{d}u, \tag{3}$$

where $F(u,\xi)$ denotes the WFT spectrum and $g(x)$ is a window, which can be chosen as a Gaussian function

$$g(x) = \exp(-x^2/2\sigma^2), \tag{4}$$

where the parameter σ controls the width of the Gaussian window. By combining Eqs. (2) and (3), we obtain

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}[f(x)\otimes h(x,\xi)]\otimes h(x,\xi)\mathrm{d}\xi, \tag{5}$$

where $h(x,\xi) = g(x)\exp(\mathrm{j}\xi x)$ and $\otimes$ denotes a convolution, which is implemented with respect to the variable x. Next, an approach based on a windowed Fourier filtering with modifications of Eq. (5) is used [22,23]

$$\bar{f}(x) = \frac{1}{2\pi}\int_{a}^{b}\overline{[f(x)\otimes h(x,\xi)]}\otimes h(x,\xi)\mathrm{d}\xi. \tag{6}$$

The modifications include the thresholding denoted by the overline and the setting the integration limits from a to b. The thresholding means that if $|\,f(x)\otimes h(x,\xi)\,|$ is smaller than a certain threshold T, it is treated as noise and is removed. The integration limits mean that only the desired spectrum in a limited range is selected.

After fringe pattern $\bar{f}(x)$ is synthetized, the phase $\phi(x)$ can be retrieved as

$$\phi(x) = \mathrm{angle}[\bar{f}(x)]. \tag{7}$$

The phase can be unwrapped using a simple procedure.

2.2. Spectral interference signal

A 1D fringe pattern in the wavelength domain, which can be recorded for example at the output of a Michelson interferometer, has the general form [4]

$$I_\mathrm{M}(\lambda) = I_\mathrm{M}^{(0)}(\lambda)\{1 + V_I(\lambda)\cos[(2\pi/\lambda)\Delta_\mathrm{M}(\lambda)]\}, \tag{8}$$

where $I_\mathrm{M}^{(0)}(\lambda)$ is the reference spectrum, $V_I(\lambda)$ is the wavelength-dependent overall visibility of the spectral interference fringes and $\Delta_\mathrm{M}(\lambda)$ is the wavelength-dependent optical path difference (OPD) between two beams in the Michelson interferometer. The form (8) can be related to the form of the 1D spatial fringe pattern given by Eq. (1) simply replacing the spatial coordinate x by the wavelength λ. Because we are retrieving only the phase, we evaluate the spectral interference signal $S_\mathrm{M}(\lambda)$ defined as [14,16]

$$S_\mathrm{M}(\lambda) = I_\mathrm{M}(\lambda)/I_\mathrm{M}^{(0)}(\lambda) - 1, \tag{9}$$

which can be represented in the form:

$$S_\mathrm{M}(\lambda) = c(\lambda)\cos[\phi(\lambda)] + n_S(\lambda), \tag{10}$$

where $S_\mathrm{M}(\lambda)$ and $c(\lambda)$ are the measured spectral interference signal and spectral fringe visibility, respectively; $\phi(\lambda)$ is the spectral phase to be retrieved, and $n_S(\lambda)$ is the noise, which is assumed to be additive.

2.3. Numerical simulation and processing

In this subsection we show the feasibility of the WFT in processing the simulated spectral interference signal, including the noise, related to a specified interferometer configuration. To simulate the signal $S_\mathrm{M}(\lambda)$, we consider a slightly dispersive Michelson interferometer with two